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# Jungck Type Contractions and its Application

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**Abstract.** In this paper we established some coincidence theorems and derived common fixed point theorems for coincidently commuting mappings, which generalized the results of Mihet [9].

**Keywords:** Probabilistic Metric Space, Contraction Mapping, Jungck type contraction

## 1. Introduction

The Banach fixed point theorem guarantees the existence of unique fixed point under a contraction mapping on a complete metric space. A similar result does not hold in a complete Menger Probabilistic metric space. The problem is that the triangular function in such spaces is not enough to guarantee that the sequence of iterates of a point under a certain map is Cauchy sequence. Two different approaches have been pursued. One is to identify those triangle functions which guarantee that the sequence of iterates is a Cauchy sequence. The other is to modify the original definition of contraction map. First this was done by Hicks [5]. Fixed point theorems using contractive conditions in Menger Probabilistic metric space have been studied by many authors, a few to be named, Constantin and Istrătescu [1], Hadžić and Pap [4] and Schweizer and Sklar [12] etc. In fact Sehgal and Reid [13] introduced the concept of contraction mapping in probabilistic metric space and proved several significant results.

In 1967 R. Machuca [7] established the following coincidence theorem.

**Theorem 1.1.**

Let X be a topological space satisfying the first axiom of countability; and Y a complete metric space. Let $f, g : X \to Y$ be continuous maps satisfying the following conditions.

(i) $f(X) \subset g(X)$,

(ii) f is a compact mapping with f(X) closed, or g is a compact mapping with g(X) closed and for some $k \in (0,1)$, $d(fx, fy) \le kd(gx, gy)$ $\forall x, y \in X$.

Then f and g have coincidence point in X.

Goebel [3] generalizing the Theorem1.1 proved the following.

**Theorem 1.2.**

Let X be an arbitrary set and Y a complete metric space. Let $f, g : X \to Y$ be mappings satisfying the following conditions,

(i) $f(X) \subset g(X)$,

(ii) g(X) is a complete subspace of Y and there exists $k \in (0,1)$, such that

$$d(fx, fy) \le kd(gx, gy) \ \forall x, y \in X.$$

Then f and g have coincidence point in X.

In 1976 Jungck [6] proved a fixed point theorem for a pair of commuting mappings. Subsequently coincidence theorems and fixed point theorems for contractive type mappings on metric, probabilistic, uniform and other spaces were established, (see for instance, [1], [3] )

In this paper we shall establish some coincidence theorems on an arbitrary set with values in generalized Menger space and derive fixed-point theorems for mappings commuting only at coincidence point. The results of this paper generalize the well-known results of Hadžić [4] and Mihet [9].

## 2. Preliminaries

In this section we recall some useful facts from the probabilistic metric space theory. For more details see ([1], [4]) and the paper of Mihet [9].

**Definition 2.1.**

A triangular norm t is a mapping $t:[0,1]\times[0,1]\rightarrow[0,1]$ such that t is nondecreasing, commutative, associative and satisfying t (a, 1) = a $\forall a\in[0,1]$.

The mappings $T_L, T_p, T_M : [0,1]\times[0,1]\rightarrow[0,1]$ defined by respectively $T_L(a,b)=\max\,(a+b-1,0)$, $T_p(a,b)=ab$ and $T_M(a,b)=\min\{a,b\}$ are the examples of t–norm. The following family of t–norms are,

Sugeno – Weber family $(T_{\lambda,}{}^{sw})_{\lambda\in(-1,\infty)}$ defined by

$$T_{\lambda,}{}^{sw}(x,y)=\max\left(0,\frac{x+y-1+\lambda xy}{1+\lambda}\right)$$

Domby family $(T_{\lambda,}{}^{D})_{\lambda\in(0,\infty)}$ defined by

$$T_{\lambda,}{}^{D}(x,y)=\left(1+\left(\left(\frac{1-x}{x}\right)^{\lambda}+\left(\frac{1-y}{y}\right)^{\lambda}\right)^{1/\lambda}\right)^{-1}$$

Aczel- Alsina family $(T_{\lambda,}{}^{AA})_{\lambda\in(0,\infty)}$ defined by

$$T_{\lambda,}{}^{AA}(x,y)=e^{-\left(|\log x|^{\lambda}+|\log y|^{\lambda}\right)^{1/\lambda}}$$

**Definition 2.2 [4].**

The above t–norm T is said to be of Hadžić (H- type) ( written as $T\in H$ ) if the family $\{T^n\}_{n\in N}$ of its iterates defined, for each $x\in[0,1]$ inductively by, $T^o(x)=1$, $T^{n+1}(x)=T\left(T^n(x),x\right)\ \forall n\geq 0,$ whenever T n is defined, is equicontinuous at x = 1, that is, $\forall\ \varepsilon\in(0,1)\ \exists\ \delta\in(0,1)$ such that $x>1-\delta\Rightarrow T^n(x)>1-\varepsilon,\ \ \forall n\geq 1.$

In 1994, Radu [11] gave some properties of t–norm of H – type.

If there exist a strictly increasing sequence $(b_n)$ in (0, 1) such that $\lim_{n\to\infty} b_n=1$ and $T(a_n,b_n)=b_n\ \forall n$, then T is of H – type.

If T is continuous and $T\in H$, then there exist a sequence $(b_n)$ such that $\lim_{n\to\infty} b_n=1$.

The t-norm $T_M$ is a trivial example of t-norm of H-type; but there are t-norms T of
H-type with $T \neq T_M$ (see, [4]).

**Definition 2.3.**

If T is a t-norm and $(x_1, x_2, ......x_n) \in [0,1]^n$, then $T_{i=1}^n x_i = 1$ if n = 0 and $T_{i=1}^n x_i = T(T_{i=1}^{n-1} x_i, x_i)$ for $n \geq 1$. If $(x_i)_{i \in N}$ is sequence in [0, 1], then $T_{i=1}^\infty x_i = \lim_{n\to\infty} T_{i=1}^n x_i$ and $T_{i=n}^\infty x_i = T_{i=1}^\infty x_{n+i}$.

**Proposition 2.1 [4].**

(i) If $T \geq T_L$, then
$\lim_{n\to\infty} T_{i=1}^\infty x_{n+1} = 1 \Leftrightarrow \sum 1 - x_n < \infty$ and $T = T_\lambda^{sw}$.
(by $T \geq T_L$ mean $a \geq c,\ b \geq d \Rightarrow T(a,b) \geq T(c,d)$).
(ii) If $T \in H$, then for every sequence $(x_n)_{n \in N}$ in [0, 1],
$\lim_{n\to\infty} x_n = 1 \Rightarrow \lim_{n\to\infty} T_{i=1}^\infty x_{n+i} = 1$.
If $T \in \{T_\lambda^D, T_\lambda^{AA} T\}$, then
$\lim_{n\to\infty} T_{i=1}^\infty x_{n+i} = 1 \Leftrightarrow \sum (1 - x_n)^\lambda < \infty$.

**Remark 2.1.**

If T is a t-norm and $(x_n)_{n \in N}$ is a sequence in [0, 1] such that $\lim_{n\to\infty} x_n = 1$ and $\lim_{n\to\infty} T_{i=1}^\infty x_{n+i} = 1$, then $\sup_{a<1} T(a,a) = 1$.
would like to repeat the definition of distribution functions and PM spaces for the interval $[0, \infty]$.

**Definition 2.4.**

Let $L_+$ be the class of distribution function $F : [0,\infty] \to [0,\infty]$ with the property,

$(i)$ $F(0) = 0,$

$(ii)$ $F$ is non decreasing,

and $(iii)$ $F$ is left continuous on $(0,\infty)$.

Suppose $D_+$ is subset of $L_+$ containing functions F such that $\lim_{x\to\infty} F(x) = 1$. The specific distribution function $\varepsilon_0$, defined by,

$\varepsilon_0(x) = 0$ if $x = 0$

$= 1$ if $x > 0,$

lies in D + .

**Definition 2.5.**

A probabilistic metric space (PM space) is an ordered pair (X, F), where X is a nonempty set and $F : X \times X \to L_+$ is a mapping such that, by denoting $F(p,q)$ by $F_{p,q}$, we have,

(I) $F_{p,q}(x) = 1 \ \forall \ x > 0$ iff $p = q,$

(II) $F_{p,q}(0) = 0,$

(III) $F_{p,q} = F_{q,p},$

and (IV) $F_{p,q}(x) = 1$ and $F_{q,r}(y) = 1 \Rightarrow F_{p,r}(x+y) = 1.$

We note that $F_{p,q}(x)$ is value of the function $F_{p,q} = F(p,q) \in L_+$ at $x \in R$.

**Definition2.6:**

A Menger PM space is a triple (X, F; T), where (X, F) is a PM space and T is t-norm such that,

$$F_{p,r}(x+y) \geq t\left(F_{p,q}(x), F_{q,r}(y)\right) \ \ \forall \ \ x, y \geq 0 .$$

**Definition 2.7.**

A generalized Menger PM space is a triple (X,F;T) satisfying the following properties,

(I) $F_{p,q}(x) = \varepsilon_0 \ \forall\ x > 0$ iff $p = q$,

(II) $F_{p,q} = F_{q,p}$,

and (III) $F_{p,r}(x+y) \geq T(F_{p,q}(x), F_{q,r}(y)) \quad \forall\ p,q,r \in X$ and $\forall x, y > 0$.

**Remark 2.2.**

Note that a Menger PM space is generalized Menger PM space with the property $\text{Range}(F) \subset D_+$. If (X, F; T) is generalized Menger Probabilistic metric space with $\sup T(x,x) = 1$, $0 < x < 1$, then (X, F; T) is a Hausdorff topological space in the topology T induced by the family of $(\in, \lambda)$ neighborhoods, $\{U_p(\in,\lambda) : p \in X, \in > 0, 0 < \lambda > 1\}$ where, $U_p(\in,\lambda) = \{x \in X : F_{x,p}(\in) > 1-\lambda\}$.
This topology is called F-topology.

**Definition 2.8.**

A sequence $\{p_n\}$ in X is said to be F- convergent to $p \in X$ iff $\forall\ \varepsilon > 0$ and $\forall \lambda \in (0,1)$, there exists an integer M such that $F_{p_n,p}(\varepsilon) > 1-\lambda\ \forall n \geq M$. Again $\{p_n\}$ is a Cauchy sequence if $\forall\ \varepsilon > 0$ and $\forall\ \lambda \in (0,1)\ \exists$ an integer M such that $F_{p_n,p_m}(\varepsilon) > 1-\lambda\ \forall\ m,n \geq M$.
In [11] Radu proved the following theorem.

**Theorem 2.1 [11].**

Every contraction mapping in complete Menger space
(X, F; T) with T continuous has a fixed point if and only if T is of Hadžić-type.
In 2005 Mihet [9] proved the following lemma, which is very important in proving whether a given sequence is an F- Cauchy sequence.

**Lemma 2.1 [9].**

Let (X, F; T) be a generalized Menger Probabilistic metric space and $\{p_n\}$ a Cauchy sequence in X such that, for some $k \in (0,\ 1)$,
$F_{p_n, p_{n+1}}(kx) \geq F_{p_{n-1}, p_n}(x) \ \ \forall\ x > 0.$
Further suppose there exists r > 1 such that,

$$\lim_{n\to\infty} T_{i=n}^{\infty}(F_{P_0,P_1}(r^i)) = 1.$$

Then $\{p_n\}$ is a F-Cauchy sequence.
Mihet [9] also defined a class function $\Phi$ of all continuous functions $\phi : [0,1]^4 \to R$, (where R is the set of real numbers) with the property
$\phi(u,v,v,u) \geq 0 \Rightarrow u \geq v.$
We present some interesting examples essentially due to Mihet [9].

**Example 2.1 [9].**

(i) If $a,b,c,d \in R$ and a + b + c + d = 0, then
$\phi(t_1,t_2,t_3,t_4) = at_1 + bt_2 + ct_3 + dt_4 \in \Phi$ if and only if $a + d > 0$.

(ii) Function $\phi$ defined by $\phi(t_1,t_2,t_3,t_4) = t_1^{\ 2} - t_2 t_3$ and more generally

(iii) $\phi(t_1,t_2,t_3,t_4) = t_1^{\ 2} - (at_2^{\ 2} + bt_3^{\ 2}) - t_2 t_3$ with a + b = 0
are in $\Phi$.

We shall need the following results.

**Lemma 2.2 (Mihet [9]).**

Let (X, F; T) be generalized Menger Probabilistic metric space and $\{p_n\}$ a sequence in X with T continuous in (a, 1) for all $a \in (0,1)$ such that,
$\{a_n\} \to a,\ \ \{b_n\} \to 1 \ \Rightarrow\ a_n b_n \to a.$
Then $p,q \in X$ and $p_n \to p \Rightarrow F_{P_n,q} \to F_{p,q}.$
Following theorem (generalized result of Hadžić) and corollaries were proved by Mihet [9].

**Theorem 2.2(Mihet [9]).**

Let (X, F; T) be an F - complete generalized Menger Probabilistic metric space with T continuous in (a, 1) for all $a \in (0,1),\ k \in (0,\ 1).$ Suppose $f : X \to X$ is a mapping such that,

$\varphi\left(F_{fp,fq}(kx), F_{p,q}(kx), F_{p,fp}(kx), F_{q,fq}(kx)\right) \geq 0 \quad \forall p,q \in X, \forall x > 0,$

where $\phi \in \Phi$ and there exists $p_0 \in X$ and r > 1 for which $\lim_{n\to\infty} T_{i=n}^{\infty}(F_{p_0,fp_0}(r^i)) = 1.$ Then f has a fixed point.

In 195 following important Lemma was proved Dedeic and Sarapa [2].

**Lemma 2.3.**

Let $\{p_n\}$ be a sequence In Menger space X. Suppose $k \in (0,1)$ and $p \neq q.$ Then there exists t > 0 such that $F_{p,q}(kt) < F_{p,q}(t)$ .

## 3. Main results

The following theorems, interestingly enough, ensure the existence of coincidence point and unique common fixed point of two mappings with values in generalized Menger space, ofcourse under certain conditions.

**Theorem 3.1.**

Let (X, F; T) be a generalized Menger space under a continuous t-norm T in (a, 1) $\forall a \in (0,1)$. Suppose $k \in (0,1)$, $\phi \in \Phi$ and $f, g : Y \to X$ are mappings such that,

$(i)\ \phi\left(F_{fp,fq}(kx), F_{gp,gq}(x), F_{fp,gp}(x), F_{fq,gq}(kx)\right) \geq 0 \quad \forall p,q \in Y, \forall x > 0,$

$(ii)\ f(Y) \subset g(Y),$

and $(iii)\ \exists p_0, p_1$ in $Y$ such that $fp_0 = gp_1$ and $\lim_{n\to\infty} T_{i=n}^{\infty} F_{fp_0,fp_1}(r^i) = 1,$ for $r > 1.$

Then f and g have a coincidence point.

PROOF. Let $p_0, p_1$ be point of Y such that $fp_0 = gp_1$ and

$$\lim_{n\to\infty} T_{i=n}^{\infty} F_{fp_0,fp_1}(r^i) = 1 \ldots \qquad (3.1)$$

Since $f(Y) \subset g(Y)$ and $fp_0 = gp_1,$ hence we can construct a sequence $\{p_n\}$ such that $fp_n = gp_{n+1}$. Let $z_n = fp_n$. Then by (3.1) $\lim_{n\to\infty} T_{i=1}^{n} F_{z_0,z_1}(r^i) = 1.$

Putting $p = p_n$ and $q = p_{n+1}$ in (i), we have

$$\phi\left(F_{fp_n, fp_{n+1}}(kx), F_{gp_n, gp_{n+1}}(x), F_{fp_n, gp_n}(x), F_{fp_{n+1}, gp_{n+1}}(kx)\right) \geq 0 \text{ ,}$$

i.e. $\phi\left(F_{z_n, z_{n+1}}(kx), F_{z_{n-1}, z_n}(x), F_{z_{n-1}, z_n}(x), F_{z_n, z_{n+1}}(kx)\right) \geq 0.$

Now by the property of $\phi$, we have,

$$F_{z_n, z_{n+1}}(kx) \geq F_{z_{n-1}, z_n}(x) \quad \forall n .$$

Therefore by Lemma 2.1 $\{z_n\} = \{fp_n\}$ is a Cauchy sequence. Suppose $g(Y)$ is F-complete.

Then there exists $p \in g(Y)$ such that $\{z_n\} \rightarrow p,$ and $\exists\, z \in Y$ such that $gz = p$.

Putting $p = p_n$ and $y = z$ in $(i)$, we get the inequality

$$\phi\left(F_{fp_n, fz}(kx), F_{gp_n, gz}(x), F_{fp_n, gp_n}(x), F_{fz, gz}(kx)\right) \geq 0,$$

i.e. $\phi\left(F_{z_n, fz}(kx), F_{z_{n-1}, gz}(x), F_{z_{n-1}, fz}(x), F_{gz, fz}(kx)\right) \geq 0.$

Taking limit as $n \rightarrow \infty$, we have,

$$\phi\left(F_{gz, fz}(kx), F_{gz, gz}(x), F_{gz, fz}(x), F_{fz, gz}(kx)\right) \geq 0,$$

i.e. $\phi\left(F_{gz, fz}(kx), 1, F_{gz, fz}(x), F_{gz, fz}(kx)\right) \geq 0.$

Again, using the property of $\phi$, we have,

$$F_{gz, fz}(kx) \geq 1 \text{ hence } fz = gz = p .$$

Therefore z is a coincidence point of f and g.

Thus if f(Y) is complete then $\{z_n\} \rightarrow p \in f(Y) \subset g(Y)$, hence as above z is a coincidence point of f and g.

**Theorem 3.2:**

Let (X, F; T) be a generalized Menger space under a continuous t-norm T in (a, 1) $\forall a \in (0,1)$. Suppose $k \in (0,1)$, $\phi \in \Phi$ and $f, g : X \rightarrow X$ are mappings such that,

$(i)\ \phi\left(F_{fp,fq}(kx), F_{gp,gq}(x), F_{fp,gp}(x), F_{fq,gq}(kx)\right) \ge 0 \quad \forall p,q \in Y, \forall x > 0,$

$(ii)\ f(X) \subset g(X),$

$(iii)\ \exists p_0, p_1$ such that $fp_0 = gp_1$ and $\lim_{n\to\infty} T_{i=n}^{\infty} F_{fp_0, fp_1}(r^i) = 1,$ for $r > 1,$

$(iv)$ Either $f(X)$ or $g(X)$ is $F-$ complete,

and $(v)$ $f$ and $g$ are commuting at their coincidence point.

Then f and g have a unique common fixed point.

PROOF. If we take Y = X in Theorem3.1, then we get $z_n = fp_n$ so that $\{z_n\}$ is a Cauchy sequence. Suppose g(X) is F-complete. Then $z_n \to p \in g(X)$ hence there exists $z \in X$ such that $g(z) = p$.

Putting $p = p_n, q = z$ in (i) we get $fz = gz = p$. Since f and g are commuting at their coincidence point hence fgz = gfz i.e. fp = gp.
Now putting p = z and q = fz in (i), we have,

$$\phi\left(F_{fz,ffz}(kx), F_{gz,gfz}(x), F_{gz,fz}(x), F_{ffz,gfz}(kx)\right) \ge 0,$$

i.e. $\phi\left(F_{p,fp}(kx), F_{p,fp}(x), 1, F_{fp,gp}(kx)\right) \ge 0.$

From the property of $\phi$, we have,

$$F_{p,fp}(kx) \ge F_{p,fp}(x) \quad \forall x > 0,$$

which is possible only if p = fp, otherwise by Lemma 2.3 we get a contradiction.
Therefore p is common fixed point of f and g.
For uniqueness suppose $p'$ and $q'$ are two common fixed points of f and g.
Then from (i),

$$\phi\left(F_{p',q'}(kx), F_{p',q'}(x), F_{p',p'}(x), F_{q',q'}(kx)\right) \ge 0,$$

i.e. $F_{p',q'}(kx) \ge 1$ yielding $p' = q'.$
This proves the uniqueness.

**Corollary 3.1.**

Let (X, F; T) be a generalized Menger space under a continuous t-norm $T \in H$. Suppose $k \in (0,1)$, $\phi \in \Phi$ and $f, g : X \to X$ are mappings such that,

$(i)\ \phi\left(F_{fp,fq}(kx), F_{gp,gq}(x), F_{fp,gp}(x), F_{fq,gq}(kx)\right) \geq 0 \quad \forall p, q \in X, \forall x > 0,$

$(ii)\ f(X) \subset g(X),$

$(iii)\ \exists p_0, p_1$ such that $fp_0 = gp_1$ for which $F_{fp_0, fp_1} \in D_+$,

$(iv)$ Either $f(X)$ or $g(X)$ is $F-$ complete,

and $(v)$ $f$ and $g$ are commuting at their coincidence point.

Then f and g have coincidence point as well as unique fixed point.

PROOF: Choosing a $\lambda > 1$, in Theorem 3.1 we have a sequence $\{z_n\}$ and a function $f_{z_0,z_1} \in D_+$, such that $\lim_{n\to\infty} F_{z_0,z_1}(\lambda^n) = 1$. Therefore from proposition (ii) of 2.1 $\lim_{n\to\infty} T_{i=n}^{\infty} F_{z_0,z_1}(\lambda^i) = 1$.
Now from Theorem 3.2. the result follows,

**Corollary 3.2 (Mihet [9])**

Let (X, F; T) be an F - complete generalized Menger Probabilistic metric space, where $T \in H$ (Hadžić type) is continuous t-norm and $\phi \in \Phi$. Suppose $f : X \to X$ is a mapping such that,

$\varphi\left(F_{fp,fq}(kx), F_{p,q}(kx), F_{p,fp}(kx), F_{q,fq}(kx)\right) \geq 0 \quad \forall p, q \in X, \forall x > 0,$

and there exists $p_0 \in X$ for which $F_{P_0, fP_0} \in D_+$. Then f has fixed point.
PROOF. Result follows from corollary 3.1, by taking g = I.

**Corollary 3.3.**

Let (X, F; $T_L$) be a generalized Menger space .Suppose $k \in (0,1)$, $\phi \in \Phi$ and $f, g : X \to X$ are mappings such that,

$(i)$ $\phi\left(F_{fp,fq}(kx), F_{gp,gq}(x), F_{fp,gp}(x), F_{fq,gq}(kx)\right) \geq 0 \quad \forall p,q \in X, \forall x > 0,$

$(ii)$ $f(X) \subset g(X),$

$(iii)$ $\exists p_0, p_1$ such that $fp_0 = gp_1$ for which $\sum_{n=1}^{\infty}(1\text{-}F_{fp_0,fp_1}(\lambda^n)) < \infty,\ \lambda > 1,$

$(iv)$ Either $f(X)$ or $g(X)$ is $F-$ complete,

and $(v)$ $f$ and $g$ are commuting at their coincidence point.

Then f and g have coincidence point as well as unique fixed point.

PROOF. The result is immediate consequence of Theorem 3.2 and proposition 2.1.

**Corollary 3.4 (Mihet [9]).**

Let (X, F; T) be an F-complete generalized Menger Probabilistic metric space and $\phi \in \Phi.$ Suppose $f : X \to X$ is a mapping such that,

$\varphi\left(F_{fp,fq}(kx), F_{p,q}(kx), F_{p,fp}(kx), F_{q,fq}(kx)\right) \geq 0 \quad \forall p,q \in X, \forall x > 0,$

and $\sum_{n=1}^{\infty}(1 - F_{p_0,fp_0}(r^n)) < \infty$ for some $p_0 \in X$ and r > 1.

Then f has a fixed point.

PROOF. Taking g = I (Identity mapping) in the corollary 3.3, we get the result.

**Corollary 3.5.**

Let (X, F; T) be a generalized Menger space under $T \in \{T_\lambda^D, T_\lambda^{AA}\}.$ Suppose $k \in (0,1),$ $\phi \in \Phi$ and $f, g : X \to X$ are mappings such that,

$(i)$ $\phi\left(F_{fp,fq}(kx), F_{gp,gq}(x), F_{fp,gp}(x), F_{fq,gq}(kx)\right) \geq 0 \;\; \forall p,q \in X, \forall x > 0,$

$(ii)$ $f(X) \subset g(X),$

$(iii)$ $\exists p_0, p_1$ such that $fp_0 = gp_1$ for which $\sum_{n=1}^{\infty}(1\text{-}F_{fp_0,fp_1}(\lambda^n))^{\lambda} < \infty,\; \lambda > 1,$

$(iv)$ $f(X)$ or $g(X)$ is $F-$ complete,

and $(v)$ $f$ and $g$ are commuting at their coincidence point.

Then f and g have coincidence point as well as unique fixed point.

PROOF. The result is immediate from Theorem 3.2 and proposition 2.1 combined.

**Corollary 3.6 [Mihet 9].**

Let (X, F; T) be a F-complete generalized Menger space under $T \in \{T_{\lambda}^{D}, T_{\lambda}^{AA}\}$. Suppose $k \in (0,1)$, $\phi \in \Phi$ and $f : X \to X$ is a mapping such that,

$\left(F_{fp,fq}(kx), F_{p,q}(kx), F_{p,fp}(kx), F_{q,fq}(kx)\right) \geq 0 \;\; \forall p,q \in X, \forall x > 0,$

and $\sum_{n=1}^{\infty}\left(1 - F_{p_0,fp_0}(r^n)\right)^{\lambda} < \infty$ for some $p_0 \in X$ and r > 1.

Then f has a fixed point.

PROOF. Taking g = I (Identity map) in the corollary 3.5, the result follows

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