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Common Fixed Point Theorem for Some New Generalized Contractive Mappings

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Abstract

In this paper, some new generalized contractive type condition for three mappings in metric spaces is defined. Some common fixed point results for these mappings are presented.

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1 Introduction

The study of common fixed points of three mappings satisfying contractive type conditions and the comparative study of various contractive definitions have attracted a great deal of research activity during recent years. In 1986 Jungck [3] generalized the notion of weakly commuting maps by introducing the concept of compatible maps. Since then many interesting fixed point theorems for compatible maps satisfying contractive type conditions have been obtained by various authors [1, 4, 5]. The authors in [2, 6, 7] have obtained some fixed point results for mappings satisfying integral type contractive condition. In this paper, we give some new generalized contractive type conditions for three mappings in metric space and prove some common fixed results for these mappings which generalizes the results in [2, 6, 7].

Throughout this paper, R^+ stands for non-negative reals, $A \in (0, +\infty)$, $R_A^+ = [0, A)$. Let $F : R_A^+ \to R$ satisfies that

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(i) $F(0) = 0$ and $F(t) > 0$ for each $t \in (0, A)$;
(ii) F is nondecreasing on R_A^+;
(iii) F is continuous.

Define $\Omega[0, A) = \{F : F \ satisfies \ (i) - (iii)\}$.

If $F \in \Omega[0, A)$ and $lim_{n\to\infty} F(\epsilon_n) = 0$, for $\epsilon_n \in R_A^+$, then $lim_{n\to\infty}\epsilon_n = 0$. For, if the result is false then there exists $\delta > 0$ and a subsequence $\{\epsilon_{n_i}\}$ of $\{\epsilon_n\}$ such that $\epsilon_{n_i} > \delta$ for each i. Hence $0 < F(\delta) \leq F(\epsilon_{n_i}) \to 0, \ i \to \infty$, a contradiction.

Example 1.1: Let $F(t) = t$, then $F \in \Omega[0, A)$ for each $A \in (0, +\infty]$.

Example 1.2: Suppose $\phi(t)$ is nonnegative, Lebesgue integrable on $[0, A)$ and satisfies $\int_0^\epsilon \phi(t)dt > 0$ for each $\epsilon \in (0, A)$. Let $F(t) = \int_0^t \phi(s)ds$, then $F \in \Omega[0, A)$.

Example 1.3: Suppose $\psi(t)$ is nonnegative, Lebesgue integrable on $[0, A)$ and satisfies $\int_0^\epsilon \psi(t)dt > 0$ for each $\epsilon \in (0, A)$. Suppose $\phi(t)$ is nonnegative, Lebesgue integrable on $[0, \int_0^A \psi(s)ds)$ and satisfies $\int_0^\epsilon \phi(t)dt > 0$ for each $\epsilon \in \left(0, \int_0^A \psi(s)ds\right)$. Let $F(t) = \int_0^{\int_0^t \psi(s)ds} \phi(u)du$, then $F \in \Omega[0, A)$.

Let $A \in (0, +\infty)$, $\psi : R_A^+ \to R^+$ satisfies that
(i) $\psi(t) < t$ for each $t \in (0, A)$;
(ii) ψ is nondecreasing and upper semicontinuous;
(iii) for each $t \in (0, A)$, $lim_{n\to\infty}\psi^n(t) = 0$.

Define $\Gamma[0, A) = \{\psi : \psi \ satisfies \ (i) - (iii)\}$.

For any $\psi \in \Gamma[0, A)$ and $t \in (0, A)$, $0 \leq \psi(0) \leq \psi(t) < t$ which implies $\psi(0) = 0$.

Let (X, d) be a metric space and let T and I be two self-mappings of X. Sessa [8] defines T and I to be weakly commiting if

$$d(TIx, ITx) \leq d(Tx, Ix),$$

for all $x \in X$.

Two commuting mappings of X of course weakly commute but converse is false as shown in the following example.

Example 1.4: Let $X = [0, 1]$ with the Euclidean metric and define the self-mappings T and I by

$$Tx = \frac{x}{2a + x}, \ Ix = \frac{x}{a}$$

for all x in X, where $a > 1$. Then

$$d(TIx, ITx) \leq \frac{ax + x^2}{a(2a + x)} = d(Tx, Ix)$$

for all x in X. Thus T and I weakly commute but they do not commute since

$$TIx = \frac{x}{2a^2 + x} > \frac{x}{a(2a + x)} = ITx$$

for all non-zero x in X.

Jungck [3] extended the concept of weak commutativity in the following way:

Definition 1.1: Let T and I be two mappings from a metric space (X, d) into itself. The mappings T and I are said to be compatible if

$$lim_{n\to\infty} d(TIx_n, ITx_n) = 0,$$

whenever $\{x_n\}$ is a sequence in X such that

$$lim_{n\to\infty} Tx_n = lim_{n\to\infty} Ix_n = t,$$

for some t in X.

It is obvious that two weakly commuting mappings are compatible but the converse is not true as shown in the following example.

Example 1.5: Let $X = [2, \infty)$, define mappings $T, I : X \to X$ by

$$Tx = x^2 + 1, Ix = 2x^2 + 1.$$

We observe that

$$Tx_n \to t,\ Ix_n \to t \Rightarrow t = 1$$

which is not in X. Therefore there does not exists any sequence $\{x_n\}$ in X such that $Tx_n \to t$ and $Ix_n \to t$ for some $t \in X$ and $|TIx_n - ITx_n|$ does not converges to 0. Thus mappings T and I are compatible. Now

$$|TIx - ITx| = 2x^4 - 1 > x^2 = |Tx - Ix|$$

Therefore T and I do not weakly commute.

2 Two Lemmas

Let (X, d) be a metric space, $D = sup\{d(x, y) : x, y \in X\}$, set $A = D$ if $D = \infty$ and $A > D$ if $D < \infty$. Let T, S and I be self-mappings of X. Assume

(H_1) $T(X) \subset I(X)$ and $S(X) \subset I(X)$.

(H_2) Suppose there exists $F \in \Omega[0, A)$ and $\psi \in \Gamma[0, F(A - 0))$ satisfying

$$F(d(Tx, Sy)) \le \psi(F(M(x, y)))$$

for all $x, y \in X$, where

$$M(x, y) = max\{d(Ix, Iy), d(Ix, Tx), d(Iy, Sy), \frac{1}{2}(d(Ix, Sy) + d(Iy, Tx))\}. \tag{2.1}$$

(H_3) I is continuous and $I(X)$ is complete.

Now let x_0 be an arbitrary point in X. Since (H_1) holds, we can define a sequence $\{Ix_n\}$ by

$$Ix_{2n+1} = Tx_{2n},\ Ix_{2n+2} = Sx_{2n+1} \tag{2.2}$$

for $n = 0, 1, 2, \ldots$. For simplicity we put $d_n = d(Ix_n, Ix_{n+1})$ for $n = 0, 1, 2, \ldots$. We then have following two lemmas.

Lemma 2.1: *If (H_1) and (H_2) hold then lim $d_n = 0$.*

Proof: We have

$$\begin{aligned} M(x_{2n}, x_{2n+1}) &= max\{d(Ix_{2n}, Ix_{2n+1}), d(Ix_{2n}, Tx_{2n}), d(Ix_{2n+1}, Sx_{2n+1}), \\ &\quad \frac{1}{2}(d(Ix_{2n}, Sx_{2n+1}) + d(Ix_{2n+1}, Tx_{2n}))\} \\ &= max\{d_{2n}, d_{2n+1}\}. \end{aligned}$$

If $max\{d_{2n}, d_{2n+1}\} = d_{2n+1}$, then

$$\begin{aligned} M(x_{2n}, x_{2n+1}) = d_{2n+1} &= d(Ix_{2n+1}, Ix_{2n+2}) = d(Tx_{2n}, Sx_{2n+1}), \\ F(d(Tx_{2n}, Sx_{2n+1})) &\leq \psi(F(M(x_{2n}, x_{2n+1}))) \\ &= \psi(F(d(Tx_{2n}, Sx_{2n+1}))) \\ &< F(d(Tx_{2n}, Sx_{2n+1})) \end{aligned}$$

which is a contradiction. Therefore $max\{d_{2n}, d_{2n+1}\} = d_{2n}$ and hence

$$d_{2n+1} < d_{2n}. \tag{2.3}$$

$$\begin{aligned} M(x_{2n}, x_{2n-1}) &= max\{d(Ix_{2n}, Ix_{2n-1}), d(Ix_{2n}, Tx_{2n}), d(Ix_{2n-1}, Sx_{2n-1}), \\ &\quad \frac{1}{2}(d(Ix_{2n}, Sx_{2n-1}) + d(Ix_{2n-1}, Tx_{2n}))\} \\ &= max\{d_{2n-1}, d_{2n}\}. \end{aligned}$$

If $max\{d_{2n-1}, d_{2n}\} = d_{2n}$, then

$$\begin{aligned} M(x_{2n}, x_{2n-1}) = d_{2n} &= d(Ix_{2n}, Ix_{2n+1}) = d(Sx_{2n-1}, Tx_{2n}) \\ F(d(Tx_{2n}, Sx_{2n-1})) &\leq \psi(F(M(x_{2n}, x_{2n-1}))) \\ &= \psi(F(d(Sx_{2n-1}, Tx_{2n}))) \\ &< F(d(Sx_{2n-1}, Tx_{2n})) \end{aligned}$$

which is a contradiction. Therefore $max\{d_{2n-1}, d_{2n}\} = d_{2n-1}$ and hence

$$d_{2n} < d_{2n-1} \tag{2.4}$$

From (2.3) and (2.4) we obtain, $d_{2n+1} < d_{2n} < d_{2n-1}$, therefore in general, $d_{n+1} < d_n$.
Thus $\{d_n\}$ is a decreasing sequence. Since F is nondecreasing, $F(d_{n+1}) < F(d_n)$.

$$\begin{aligned} F(d_{2n}) &= F(d(Sx_{2n-1}, Tx_{2n})) \\ &\leq \psi(F(M(x_{2n}, x_{2n-1}))) \\ &= \psi(F(d_{2n-1})). \\ F(d_{2n+1}) &= F(d(Tx_{2n}, Sx_{2n+1})) \\ &\leq \psi(F(M(x_{2n}, x_{2n+1}))) \\ &= \psi(F(d_{2n})). \end{aligned}$$

$$F(d_n) \leq \psi(F(d_{n-1})) \leq ... \leq \psi^n(F(d_0)). \tag{2.5}$$

Since $\psi^n(t) \to 0$ for any $t \in (0, A)$ and $F(\epsilon_n) \to 0 \Rightarrow \epsilon_n \to 0$, from (2.5) we observe that $lim\, d_n = 0$.

Lemma 2.2: *If* (H_1) *and* (H_2) *hold then the sequence* $\{Ix_n\}$ *defined by (2.2) is a Cauchy sequence.*
Proof: Suppose $\{Ix_n\}$ is not a Cauchy sequence, then there exists $\epsilon > 0$ and subsequences $\{Ix_{n_i}\}$ and $\{Ix_{m_i}\}$ of $\{Ix_n\}$ with $n_i < m_i$ such that $d(Ix_{n_i}, Ix_{m_i}) \geq 2\epsilon$, for each $i \in N$. We know by Lemma 2.1, $d_n = d(Ix_n, Ix_{n+1}) \to 0$ as $n \to \infty$. Therefore, when i is large enough
$d(Ix_{n_i}, Ix_{n_i+1}) < \frac{\epsilon}{2}$ and $d(Ix_{m_i-1}, Ix_{m_i}) < \frac{\epsilon}{2}$.
By triangle inequality, we have

$$\begin{aligned} d(Ix_{n_i+1}, Ix_{m_i}) &\geq d(Ix_{n_i}, Ix_{m_i}) - d(Ix_{n_i}, Ix_{n_i+1}) > \epsilon, \\ d(Ix_{n_i}, Ix_{m_i-1}) &\geq d(Ix_{n_i}, Ix_{m_i}) - d(Ix_{m_i-1}, Ix_{m_i}) > \epsilon, \\ d\,(Ix_{n_i+1}, Ix_{m_i-1})) &\geq d(Ix_{n_i}, Ix_{m_i}) - d(Ix_{n_i}, Ix_{n_i+1}) - d(Ix_{m_i-1}, Ix_{m_i}) > \epsilon. \end{aligned}$$

We may assume n_i are even numbers and m_i are odd numbers and $d(Ix_{n_i}, Ix_{m_i}) > \epsilon$, for all i. Set

$$k_i = min\{m_i : d(Ix_{n_i}, Ix_{m_i}) > \epsilon,\ m_i \text{ } is \text{ } odd \text{ } number\}.$$

We have

$$\begin{aligned} \epsilon &< d(Ix_{n_i}, Ix_{k_i}) \\ &\leq d(Ix_{n_i}, Ix_{k_i-2}) + d(Ix_{k_i-2}, Ix_{k_i-1}) + d(Ix_{k_i-1}, Ix_{k_i}) \\ &< \epsilon + d_{k_i-2} + d_{k_i-1} \to \epsilon \text{ } as \text{ } i \to \infty. \end{aligned}$$

Thus $d(Ix_{n_i}, Ix_{k_i}) \to \epsilon^+$. We obtain

$$\begin{aligned} d(Ix_{n_i}, Ix_{k_i}) - d_{n_i} - d_{k_i} &\leq d(Ix_{n_i+1}, Ix_{k_i+1}) \\ &\leq d(Ix_{n_i}, Ix_{k_i}) + d_{n_i} + d_{k_i}. \end{aligned}$$

If $i \to \infty$, $d_{n_i} \to 0$ and hence $d(Ix_{n_i+1}, Ix_{k_i+1}) \to \epsilon$. Since

$$\begin{aligned} M(x_{n_i}, x_{k_i}) &= max\{d(Ix_{n_i}, Ix_{k_i}), d(Ix_{n_i}, Tx_{n_i}), d(Ix_{k_i}, Sx_{k_i}), \\ &\quad \frac{1}{2}\left(d(Ix_{n_i}, Sx_{k_i}) + d(Ix_{k_i}, Tx_{n_i})\right)\} \\ &\le d(Ix_{n_i}, Ix_{k_i}) + max\{d_{n_i}, d_{k_i}\} \\ &\le d(Ix_{n_i}, Ix_{k_i}) + \delta_i, \end{aligned}$$

where $\delta_i \to 0$ as $i \to \infty$.

$$\begin{aligned} F(d(Ix_{n_i+1}, Ix_{k_i+1})) &= F(d(Tx_{n_i}, Sx_{k_i})) \\ &\le \psi(F(M(x_{n_i}, x_{k_i})) \\ &= \psi\left(F(d(Ix_{n_i}, Ix_{k_i}) + \delta_i)\right). \end{aligned}$$

Let $i \to \infty$, we get $F(\epsilon) \le \psi(F(\epsilon)) < F(\epsilon)$, which is a contradiction. Hence $\{Ix_n\}$ is a Cauchy sequence.

3 Existence Theorem

Theorem 3.1: *Suppose mappings $T, S, I : X \to X$ satisfy (H_1) - (H_3). If I is compatible with T or S. Then T, S and I have a unique common fixed point in X. Moreover the iterated sequence $\{Ix_n\}$ defined by (2.2) converges to the common fixed point of T, S and I.*

Proof: For any $x_0 \in X$, by Lemma 2.2, the sequence $\{Ix_n\}$ defined by (2.2) is a Cauchy sequence. Since $I(X)$ is complete, $\{Ix_n\}$ converges to some z in $I(X)$. Consequently, $T(x_{2n}) \to z$ and $S(x_{2n+1}) \to z$. Since I is continuous, $I(Ix_n) \to Iz, I(Sx_{2n+1}) \to Iz$. Now suppose I is compatible with S (argument is similar if I is compatible with T), then SIx_{2n+1} converges to Iz.

$$\begin{aligned} M(x_{2n}, Ix_{2n+1}) &= max\{d(Ix_{2n}, IIx_{2n+1}), d(Ix_{2n}, Tx_{2n}), d(IIx_{2n+1}, SIx_{2n+1}), \\ &\quad \frac{1}{2}\left(d(Ix_{2n}, SIx_{2n+1}) + d(IIx_{2n+1}), Tx_{2n})\right)\}. \end{aligned}$$

$$\begin{aligned} limM(x_{2n}, Ix_{2n+1}) &= max\{d(z, Iz), d(z, z), d(Iz, Iz), \frac{1}{2}(d(z, Iz) + d(Iz, z))\} \\ &= d(z, Iz). \\ F(d(Tx_{2n}, SIx_{2n+1})) &\le \psi(F(M(x_{2n}, Ix_{2n+1}))). \end{aligned}$$

Here F is continuous, ψ is upper semi-continuous and now taking limit as $n \to \infty$, we get

$$F(d(z, Iz)) \le \psi(F(d(z, Iz))) < F(d(z, Iz)) \Rightarrow z = Iz.$$

Also we have

$$limM(x_{2n}, z) = d(z, Sz)$$

and

$$F(d(Tx_{2n}, Sz)) \leq \psi(F(M(x_{2n}, z))).$$

Taking limit as $n \to \infty$, we obtain

$$F(d(z, Sz)) \leq \psi(F(d(z, Sz))) < F(d(z, Sz)) \Rightarrow z = Sz.$$

Since $M(z, z) = d(z, Tz)$,

$$F(d(Tz, z)) \leq \psi(F(M(z, z)) < F(d(Tz, z)) \Rightarrow z = Tz.$$

To prove the uniqueness, let $Tz = Sz = Iz = z$ and $Ty = Sy = Iy = y$. We obtain

$$M(z, y) = d(z, y),$$

and

$$F(d(z, y)) \leq \psi(F(M(z, y))) < F(d(z, y)) \Rightarrow z = y.$$

Thus T, S and I have a unique common fixed point.

We then have following Corollaries:

Corollary 3.1: *Suppose that the mappings $T, S, I : X \to X$ satisfy (H_1) - (H_3). If the mapping I weakly commute with T or S, then T, S and I have a unique common fixed point in X. Moreover the iterated sequence $\{Ix_n\}$ defined by (2.2) converges to the common fixed point of T, S and I.*

Corollary 3.2[7]: *Let X be a metric space and A as in Section 2. Suppose that $T, S : X \to X$; $F \in \Omega[0, A)$ and $\psi \in \Gamma[0, F(A - 0))$ satisfy*

$$F(d(Tx, Sy)) \leq \psi(F(M(x, y)))$$

for each $x, y \in X$, where

$$M(x, y) = max\{d(x, y), d(x, Tx), d(y, Sy), \frac{1}{2}(d(x, Sy) + d(y, Tx))\}.$$

Then T and S have a unique common fixed point in X. Moreover the iterated sequence $\{x_n\}$ defined by $x_{2n+1} = Tx_{2n}$, $x_{2n+2} = Sx_{2n+1}$ converges to the common fixed point of T and S.

Corollary 3.3: *Suppose that the mappings $T, I : X \to X$ satisfy $T(X) \subset I(X)$ and*

$$F(d(Tx, Ty)) \leq \psi(F(M(x, y)))$$

for each $x, y \in X$, where

$$M(x,y) = max\{d(Ix,Iy), d(Ix,Tx), d(Iy,Ty), \frac{1}{2}(d(Ix,Ty)+d(Iy,Tx))\}.$$

If I is continuous and compatible with T; $I(X)$ is complete, then T and I have a unique common fixed point in X. Moreover the iterated sequence $\{Ix_n\}$ defined by $Ix_n = Tx_{n-1}$ converges to the common fixed point of T and I.

Corollary 3.4: *Let X be a metric space and A as in section 2. Suppose ϕ is non-negative, Lebesgue integrable on $[0, A]$ and satisfies $\int_0^\epsilon \phi(t)dt > 0$ for each $\epsilon \in (0, A)$. Suppose $T, S, I : X \to X$ satisfy (H_1) and (H_3). Further suppose $\psi \in \Gamma\left(0, \int_0^A \phi(s)ds\right)$ satisfy*

$$\int_0^{d(Tx,Sy)} \phi(s)ds \leq \psi\left(\int_0^{M(x,y)} \phi(s)ds\right)$$

for $x, y \in X$ where $M(x, y)$ is as defined by (2.1). Then T, S and I have a unique common fixed point in X. Moreover the iterated sequence $\{Ix_n\}$ defined by (2.2) converges to the common fixed point of T, S and I.

Remark 3.1: Corollaries 3.2, 3.3 and 3.4 extends the results of Zhang [7].

Remark 3.2: Theorem 3.1 and Corollaries unify and extend many results in [6, 7].

Example 3.1: Let $X = \{\frac{1}{n}, n = 3, 4, 5, ...\} \cup \{0\}$ with the Euclidean metric d. Let $F(t) = t^{\frac{1}{t}}$, then $F \in \Omega[0, A)$, where $A = e > D = \frac{1}{3}$. Let $\psi(t) = \frac{t}{6}$, then $\psi \in \Gamma[0, e^{\frac{1}{e}})$. Suppose that S, T and I are self-mappings of X defined by

$$Tx = Sx = \begin{cases} 0, & if\ x = 0 \\ \frac{1}{2n}, & if\ x = \frac{1}{n},\ n \geq 3 \end{cases}$$

and

$$Ix = \begin{cases} 0, & if\ x = 0 \\ \frac{1}{n+1}, & if\ x = \frac{1}{n},\ n \geq 3, \end{cases}$$

It is obvious to see that $T(X) \subset I(X), S(X) \subset I(X)$; I is continuous and $I(X)$ is complete.

For $x = 0$, $d(TIx, ITx) \leq d(Tx, Ix)$ is true.

For $x = \frac{1}{n}$, $n \geq 3$

$$d(TIx, ITx) = \frac{1}{2(n+1)(2n+1)} < \frac{n-1}{2n(n+1)} = d(Tx, Ix).$$

Therefore (T, I) is weakly commuting pair and hence T and I are compatible. Now we prove that for each $x, y \in X$,

$$F(d(Tx, Sy)) \leq \psi(F(d(x, y))) \leq \psi(F(M(x, y))).$$

There are three possible cases:
Case(i) $x = y$. If $x = y = 0$, then

$$F(d(Tx, Sy)) = 0 = \psi(F(d(x, y))) \leq \psi(F(M(x, y))).$$

If $x = y = \frac{1}{n},\ n \geq 3$, then $M(x, y) = \left(\frac{n-1}{2n(n+1)}\right)$.

$$F(d(Tx, Sy)) = 0 \leq \frac{1}{6}\left(\frac{n-1}{2n(n+1)}\right)^{\left(\frac{2n(n+1)}{n-1}\right)} = \psi(F(M(x, y))).$$

Case(ii) If $x = 0, y = \frac{1}{n}$ or vice-versa then $M(x, y) = \frac{1}{n+1}$.

$$F(d(Tx, Sy)) = \left(\frac{1}{2n}\right)^{2n} \leq \frac{1}{6}\left(\frac{1}{n+1}\right)^{n+1} = \psi(F(M(x, y))).$$

Case(iii) If $x = \frac{1}{n}, y = \frac{1}{m}, n > m \geq 3$, then $d(Ix, Iy) = \frac{n-m}{(m+1)(n+1)}$ and

$$\begin{aligned} F(d(Tx, Sy)) &= \left(\frac{n-m}{2mn}\right)^{\left(\frac{2mn}{n-m}\right)} \\ &\leq \frac{1}{6}\left(\frac{n-m}{(m+1)(n+1)}\right)^{\left(\frac{(m+1)(n+1)}{n-m}\right)} \\ &= \psi(F(d(Ix, Iy))) \\ &\leq \psi(F(M(x, y))). \end{aligned}$$

Therefore T, S and I satisfies all conditions in Theorem 3.1, so they have a unique common fixed point $x^* = 0$.

Now we give an example which satisfies all conditions in Corollary 3.2 but does not satisfy the general contractive condition.

Example 3.2: In the above Example 3.1, suppose I is Identity mapping and define T and S by

$$Tx = Sx = \begin{cases} 0, & if\ x = 0 \\ \frac{1}{n+1}, & if\ x = \frac{1}{n}, \end{cases}$$

Analogous to Example 3.1, all the conditions in Corollary 3.2 are satisfied and have a unique common fixed point $x^* = 0$.

Now we show that T and S do not satisfy the general contractive condition

$$d(Tx, Sy) \leq cM(x, y)$$

for some $c \in [0, 1)$ and $x, y \in X$, where

$$M(x, y) = max\{d(x, y), d(x, Tx), d(y, Sy), \frac{1}{2}(d(x, Sy) + d(y, Tx))\}.$$

To see this, let $x = \frac{1}{n}, y = \frac{1}{n+1}$, then

$$d(T\frac{1}{n}, S\frac{1}{n+1}) = \frac{1}{(n+1)(n+2)},$$

and

$$M\left(\frac{1}{n}, \frac{1}{n+1}\right) = \frac{1}{n(n+1)}$$

$$\begin{aligned} sup_{x,y\in X, x\neq y}\left\{\frac{d(Tx,Sy)}{M(x,y)}\right\} &\geq sup_{n\in N}\left\{\frac{d(T\frac{1}{n}, S\frac{1}{n+1})}{M(\frac{1}{n}, \frac{1}{n+1})}\right\} \\ &= sup_{n\in N}\frac{1}{1+\frac{2}{n}} \\ &= 1. \end{aligned}$$

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