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On Weakly (g, m)-Continuous Functions

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Abstract

The purpose of this paper is to introduce a new notion of weakly (g, m)-continuous functions as functions from a generalized topological space into a set satisfying some minimal conditions. We obtain some characterizations and several properties of functions.

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1 Introduction

Á. Császár [1] introduced the concepts of generalized neighborhood systems and generalized topological spaces. He also introduced the concepts of continuous functions and associated interior and closure operators on generalized neighborhood systems and generalized topological spaces. In particular, he investigated characterizations for the generalized continuous function by using a closure operator defined on generalized neighborhood systems. In [2], he introduced and studied the notions of g-α-open sets, g-semi-open sets, g-preopen sets and g-β-open sets in generalized topological spaces. The concept of minimal structure (briefly m-structure) was introduced by V. Popa and T. Noiri [6] in 2000. Also they introduced the notion of m_X-open set and m_X-closed set and characterize those sets using m_X-closure and m_X-interior operators respectively. Further they introduced M-continuous functions and studied some of its basic properties. In this paper, we introduce a new notion of weakly (g, m)-continuous functions as functions from a generalized topological space (X, g_X) into a set satisfying some minimal conditions. We obtain several characterizations and properties of such functions.

2 Preliminaries

We recall some notions and notations defined in [1]. Let X be a nonempty set and g_X be a collection of subsets of X. Then g_X is called a *generalized topology* (briefly GT) on X iff $\emptyset \in g_X$ and $G_i \in g_X$ for $i \in I \neq \emptyset$ implies $G = \cup_{i\in I} G_i \in g_X$. We call the pair (X, g_X) a *generalized topological space* (briefly GTS) on X. The elements of g_X are called g_X*-open* sets and the complements are called g_X*-closed* sets. Set $gO(X) = \{U \subseteq X : U \in g_X\}$ and $gO(x) = \{U \in g_X : x \in U\}$. The closure of a subset A of X, denoted by $c_X(A)$, is the intersection of generalized closed sets including A. And the interior of A, denoted by $i_X(A)$, is the union of generalized open sets contained in A.

Theorem 2.1. [1] *Let (X, g_X) be a generalized topological space. Then*

(1) $c_X(A) = X - i_X(X - A)$;

(2) $i_X(A) = X - c_X(X - A)$.

Proposition 2.2. [4] *Let (X, g_X) be a generalized topological space and $A \subseteq X$. Then*

(1) $x \in i_X(A)$ if and only if there exists $V \in gO(x)$ such that $V \subseteq A$.

(2) $x \in c_X(A)$ if and only if $V \cap A \neq \emptyset$ for every $V \in gO(x)$.

Proposition 2.3. *Let (X, g_X) be a generalized topological space. For subsets A and B of X, the following properties holds:*

(1) $c_X(X - A) = X - i_X(A)$ and $i_X(X - A) = X - c_X(A)$;

(2) If $(X - A) \in g_X$, then $c_X(A) = A$ and if $A \in g_X$, then $i_X(A) = A$;

(3) If $A \subseteq B$, then $c_X(A) \subseteq c_X(B)$ and $i_X(A) \subseteq i_X(B)$;

(4) $A \subseteq c_X(A)$ and $i_X(A) \subseteq A$;

(5) $c_X(c_X(A)) = c_X(A)$ and $i_X(i_X(A)) = i_X(A)$.

Definition 2.4. [5] Let X be a nonempty set and $P(X)$ the power set of X. A subfamily m_X of $P(X)$ is called a *minimal structure* (briefly *m-structure*) on X if $\emptyset \in m_X$ and $X \in m_X$

By (X, m_X), we denote a nonempty set X with an m-structure m_X on X and it is called an m-space. Each member of m_X is said to be m_X*-open* and the complement of an m_X-open set is said to be m_X*-closed*.

Definition 2.5. [5] Let X be a nonempty set and m_X an m-structure on X. For a subset A of X, the m_X*-closure* of A and the m_X*-interior* of A are defined as follows:

(1) $m_X\text{-}Cl(A) = \cap\{F : A \subseteq F, X - F \in m_X\}$;

(2) $m_X\text{-}Int(A) = \cup\{U : U \subseteq A, U \in m_X\}$.

Lemma 2.6. [3] *Let X be a nonempty set and m_X a minimal structure on X. For subset A and B of X, the following properties hold:*

(1) $m_X\text{-}Cl(X - A) = X - (m_X\text{-}Int(A))$ and $m_X\text{-}Int(X - A) = X - (m_X\text{-}Cl(A))$;

(2) If $(X - A) \in m_X$, then $m_X\text{-}Cl(A) = A$ and if $A \in m_X$, then $m_X\text{-}Int(A) = A$;

(3) $m_X\text{-}Cl(\emptyset) = \emptyset$, $m_X\text{-}Cl(X) = X$, $m_X\text{-}Int(\emptyset) = \emptyset$ and $m_X\text{-}Int(X) = X$;

(4) If $A \subseteq B$, then $m_X\text{-}Cl(A) \subseteq m_X\text{-}Cl(B)$ and $m_X\text{-}Int(A) \subseteq m_X\text{-}Int(B)$;

(5) $A \subseteq m_X\text{-}Cl(A)$ and $m_X\text{-}Int(A) \subseteq A$;

(6) $m_X\text{-}Cl(m_X\text{-}Cl(A)) = m_X\text{-}Cl(A)$ and $m_X\text{-}Int(m_X\text{-}Int(A)) = m_X\text{-}Int(A)$.

Lemma 2.7. [3] *Let X be a nonempty set with a minimal structure m_X and A a subset of X. Then $x \in m_X\text{-}Cl(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \in m_X$ containing x.*

Definition 2.8. [3] An m-structure m_X on a nonempty set X is said to have *property $\mathcal{B}$* if the union of any family of subsets belong to m_X belong to m_X.

Lemma 2.9. [5] *Let X be a nonempty set and m_X an m-structure on X sastisfying property $\mathcal{B}$. For a subset A of X, the following properties hold:*

(1) $A \in m_X$ if and only if $m_X\text{-}IntA = A$;

(2) If A is m_X-closed if and only if $m_X\text{-}Cl(A) = A$;

(3) $m_X\text{-}Int(A) \in m_X$ and $m_X\text{-}Cl(A)$ is m_X-closed.

3 Weakly (g,m)-continuous functions

In this section, we introduce weakly (g,m)-continuous functions and investigate some of their characterizations.

Definition 3.1. A function $f : (X, g_X) \to (Y, m_Y)$ is said to be (g,m)-*continuous* at a point $x \in X$ if for each m_Y-open set V containing $f(x)$, there exists a g_X-open set U containing x such that $f(U) \subseteq V$. A function $f : (X, g_X) \to (Y, m_Y)$ is said to be (g,m)-*continuous* if it has this property at each point $x \in X$.

Theorem 3.2. *For a function $f : (X, g_X) \to (Y, m_Y)$, the following properties are equivalent:*

(1) f is (g, m)-continuous;

(2) $f^{-1}(V)$ is g-open in X for every m_Y-open set V of Y;

(3) $f(c_X A) \subseteq$-m_Y-$Cl(f(A))$ for every subset A of X;

(4) $c_X(f^{-1}(B)) \subseteq f^{-1}(m_Y$-$Cl(B))$ for every subset B of Y;

(5) $f^{-1}(m_Y$-$Int(B)) \subseteq i_X(f^{-1}(B))$ for every subset B of Y;

(6) $f^{-1}(K)$ is g_X-closed in X for every m_Y-closed set K of Y.

Proof. (1) $\Rightarrow$ (2): Let $V \in m_Y$ such that $x \in f^{-1}(V)$. Then $f(x) \in V$. There exists an g_X-open subset U of X containing x such that $f(U) \subseteq V$. Since $U \in g_X$, we have $x \in i_X(f^{-1}(V))$.

(2) $\Rightarrow$ (3): Let A be any subset of X. Let $x \in c_X(A)$ and $V \in m_Y$ containing $f(x)$. Then $x \in i_X(f^{-1}(V))$. There exists $U \in g_X$ such that $x \in U \subseteq f^{-1}(V)$. Since $x \in c_X(A)$, by Proposition 2.2, $U \cap A \neq \emptyset$ and $\emptyset \neq f(U \cap A) \subseteq f(U) \cap f(A) \subseteq V \cap f(A)$. Since $V \in m_Y$ containing $f(x)$, $f(x) \in m_Y$-$Cl(f(A))$.

(3) $\Rightarrow$ (4): Let B be any subset of Y. By (3), $f(c_X(f^{-1}(B)) \subseteq m_Y$-$Cl(f(f^{-1}(B)))$. Hence, we have $c_X(f^{-1}(B)) \subseteq f^{-1}(m_Y$-$Cl(B))$.

(4) $\Rightarrow$ (5): Let B be any subset of Y. By (4), we have $X - i_X(f^{-1}(B)) = c_X(X - f^{-1}(B)) = c_X(f^{-1}(Y - B)) \subseteq f^{-1}(m_Y$-$Cl(Y - B)) = f^{-1}(Y - (m_Y$-$Int(B))) = X - f^{-1}(m_Y$-$Int(B))$. Hence, $f^{-1}(m_Y$-$Int(B)) \subseteq i_X(f^{-1}(B))$.

(5) $\Rightarrow$ (6): Let K be any m_Y-closed set of Y. Then $Y - K = m_Y$-$Int(Y - K)$ because $Y - K$ is m_Y-open. By (5), $X - f^{-1}(K) = f^{-1}(Y - K) \subseteq i_X(f^{-1}(Y - K)) = i_X(X - f^{-1}(K)) = X - c_X(f^{-1}(K))$. Hence, $c_X(f^{-1}(K)) \subseteq f^{-1}(K)$.

(6) $\Rightarrow$ (2): Obvious.

(2) $\Rightarrow$ (1): Let $V \in m_Y$ containing $f(x)$. By (2), $x \in i_X(f^{-1}(V)$ and hence there exists $U \in g_X$ containing x such that $x \in U \subseteq f^{-1}(V)$. Therefore, $f(U) \subseteq V$ and f is (g, m)-continuous at x. □

Definition 3.3. A function $f : (X, g_X) \to (Y, m_Y)$ is said to be *weakly* (g, m)-*continuous* at a point $x \in X$ if for each m_Y-open set V containing $f(x)$, there exists a g_X-open set U containing x such that $f(U) \subseteq m_Y$-$Cl(V)$. A function $f : (X, g_X) \to (Y, m_Y)$ is said to be *weakly* (g, m)-*continuous* if it has this property at each point $x \in X$.

Remark 1. *From the above definitions of (g, m)-continuity and weakly (g, m)-continuity, we have the following implication but the reverse relation may not be true in general:*

$$(g, m)\text{-continuous} \Rightarrow \text{weakly } (g, m)\text{-continuous.}$$

Example 3.4. *Let $X = \{a, b\} = Y$, $g_X = \{\emptyset, X\}$ and $m_Y = \{\emptyset, \{a\}, Y\}$. Let $f : (X, g_X) \to (Y, m_Y)$ be the identity function. Then f is weakly (g, m)-continuous but it is not (g, m)-continuous.*

Theorem 3.5. *A function $f : (X, g_X) \to (Y, m_Y)$ is weakly (g, m)-continuous at x if and only if each m_Y-open set V containing $f(x)$, $x \in i_X(f^{-1}(m_Y\text{-}Cl(V)))$.*

Proof. Let f be weakly (g, m)-continuous at x and V an m_Y-open set containing $f(x)$. Then, there exists an open set U containing x such that $f(U) \subseteq m_Y\text{-}Cl(V)$. Then we have $x \in U \subseteq f^{-1}(m_Y\text{-}Cl(V))$ and hence $x \in i_X(f^{-1}(m_Y\text{-}Cl(V)))$.

Conversely, let V be an m_Y-open set containing $f(x)$ such that $x \in i_X(f^{-1}(m_Y\text{-}Cl(V)))$. Put $U = i_X(f^{-1}(m_Y\text{-}Cl(V)))$. Then U is g_X-open in X, $x \in U$ and $f(U) \subseteq m_Y\text{-}Cl(V)$. □

Theorem 3.6. *A function $f : (X, g_X) \to (Y, m_Y)$ is weakly (g, m)-continuous if and only if $f^{-1}(V) \subseteq i_X(f^{-1}(m_Y\text{-}Cl(V)))$ for every m_Y-open set V of Y.*

Proof. Let $x \in f^{-1}(V)$. Then $f(x) \in V$. Since f is weakly (g, m)-continuous at x, by Theorem 3.5 we have $x \in i_X(f^{-1}(m_Y\text{-}Cl(V)))$ and hence $f(V) \subseteq i_X(f^{-1}(m_Y\text{-}Cl(V)))$.

Conversely, let V be an m_Y-open set of Y containing $f(x)$. Then, we have $x \in f^{-1}(V) \subseteq i_X(f^{-1}(m_Y\text{-}Cl(V)))$. By Theorem 3.5, f is weakly (g, m)-continuous. □

Definition 3.7. [7] Let S be a subset of (X, m_X). A point $x \in X$ is called an *m_θ-adherent point* of S if $m_X\text{-}Cl(U) \cap S \neq \emptyset$ for every m_X-open set U containing x.

The set of all m_θ-adherent points of S is called the *m_θ-closure* of S and is denoted by $m_X\text{-}Cl_\theta(S)$. If $A = m_X\text{-}Cl_\theta(A)$, then A is called *m_θ-closed.* The complement of a m_θ-closed set is said to be *m_θ-open.* The union of all m_θ-open sets contained in A is called the *m_θ-interior* of A and is denoted by $m_X\text{-}Cl_\theta(A)$.

Lemma 3.8. [7] *Let A be a subset of an m-space (X, m_X). Then the following properties hold:*

(1) If A is m_X-open in X, then $m_X\text{-}Cl(A) = m_X\text{-}Cl_\theta(A)$;

(2) If m_X has property $\mathcal{B}$, then $m_X\text{-}Cl_\theta(A)$ is m_X-closed in X for every subset A of X.

Theorem 3.9. *Let (Y, m_Y) satisfy the property $\mathcal{B}$. For a function $f : (X, g_X) \to (Y, m_Y)$, the following are equivalent:*

(1) f is weakly (g, m)-continuous;

(2) $f(c_X(A)) \subseteq m_X\text{-}Cl_\theta(f(A))$ for every subset A of X;

(3) $c_X(f^{-1}(B)) \subseteq f^{-1}(m_X\text{-}Cl_\theta(B))$ for every subset B of Y;

(4) $c_X(f^{-1}(V)) \subseteq f^{-1}(m_Y\text{-}Cl(V))$ for every m_Y-open set V of Y.

Proof. (1) ⇒ (2): Let A be any subset of X. Suppose that $x \in c_X(A)$ and G be any m_Y-open set containing $f(x)$. Since f is weakly (g, m)-continuous, there exists an g_X-open set U containing x such that $f(U) \subseteq m_Y\text{-}(G)$. Since $x \in c_X(A)$, we have $U \cap A \neq \emptyset$. It follows that $\emptyset \neq f(U) \cap f(A) \subseteq m_Y\text{-}Cl(G) \cap f(A)$. Hence $m_Y\text{-}Cl(G) \cap f(A) \neq \emptyset$ and $f(x) \in m_X\text{-}Cl_\theta(f(A))$.

(2) ⇒ (3): Let B be any subset of Y. We obtain $f(c_X(f^{-1}(B)) \subseteq m_Y\text{-}Cl_\theta(f(f^{-1}(B)))$ and hence $c_X(f^{-1}(B)) \subseteq f^{-1}(m_Y\text{-}Cl_\theta(B))$.

(3) ⇒ (4): Let V be an m_Y-open set of Y. By Lemma 3.8, $m_Y\text{-}Cl_\theta(V) = m_Y\text{-}Cl(V)$. Thus the proof is obvious.

(4) ⇒ (1): Let V be any m_Y-open set containing $f(x)$. Since $V \cap (Y - (m_Y\text{-}Cl(V)) = \emptyset$, clearly $f(x) \notin m_Y\text{-}Cl(Y - (m_Y\text{-}Cl(V)))$ and hence $x \notin f^{-1}(m_Y\text{-}Cl(Y - (m_Y\text{-}Cl(V))))$. Since (Y, m_Y) satisfy the property $\mathbb{B}$, $Y - (m_Y\text{-}Cl(V)) = m_Y\text{-}Int(Y - V) \in m_Y$ and by (4), $x \notin c_X(f^{-1}(Y - (m_Y\text{-}Cl(V))))$. Therefore, there exists an g_X-open set U containing x such that $U \cap f^{-1}(Y - (m_Y\text{-}Cl(V))) = \emptyset$; hence $f(U) \cap (Y - (m_Y\text{-}Cl(V))) = \emptyset$. This shows that $f(U) \subseteq m_Y\text{-}Cl(V)$. Therefore, f is weakly (g, m)-continuous. □

Definition 3.10. A subset A of a m-space (X, m_X) is said to be

(1) m_X-regular open if $A = m_X\text{-}Int(m_X\text{-}Cl(A))$;

(2) m_X-semi-open if $A \subseteq m_X\text{-}Cl(m_X\text{-}Int(A))$;

(3) m_X-preopen if $A \subseteq m_X\text{-}Int(m_X\text{-}Cl(A))$;

(4) m_X-α-open if $A \subseteq m_X\text{-}Int(m_X\text{-}Cl(m_X\text{-}Int(A)))$;

(5) m_X-β-open if $A \subseteq m_X\text{-}Cl(m_X\text{-}Int(m_X\text{-}Cl(A)))$.

The complement of a m_X-regular open (resp. m_X-semi-open, m_X-preopen, m_X-α-open, m_X-β-open) set is called *m_X-regular closed* (resp. *m_X-semi-closed*, *m_X-preclosed*, *m_X-α-closed*, *m_X-β-closed*).

Lemma 3.11. *Let (X, m_X) be a m-space and A a subset of X.*

(1) A is m_X-regular closed if and only if $A = m_X\text{-}Cl(m_X\text{-}Int(A))$;

(2) *A is m_X-semi-closed if and only if m_X-$Int(m_X$-$Cl(A)) \subseteq A$;*

(3) *A is m_X-preclosed if and only if m_X-$Cl(m_X$-$Int(A)) \subseteq A$;*

(4) *A is m_X-β-closed if and only if m_X-$Int(m_X$-$Cl(m_X$-$Int(A))) \subseteq A$.*

Theorem 3.12. *For a function $f : (X, g_X) \to (Y, m_Y)$, where m_Y has property $\mathcal{B}$, the following properties are equivalent:*

(1) *f is weakly (g, m)-continuous;*

(2) *$f^{-1}(U) \subseteq i_X(f^{-1}(m_Y$-$Cl(U)))$ for every m_Y-open subset U of Y;*

(3) *$c_X(f^{-1}(m_Y$-$Int(F))) \subseteq f^{-1}(F)$ for every m_Y-closed subset F of Y;*

(4) *$c_X(f^{-1}(m_Y$-$Int(m_Y$-$Cl(A)))) \subseteq f^{-1}(m_Y$-$Cl(A))$ for every subset A of Y;*

(5) *$f^{-1}(m_Y$-$Int(A)) \subseteq i_X(f^{-1}(m_Y$-$Cl(m_Y$-$Int(A))))$ for every subset A of Y;*

(6) *$c_X(f^{-1}(U)) \subseteq f^{-1}(m_Y$-$Cl(U))$ for very m_Y-open subset U of Y.*

Proof. (1) $\Rightarrow$ (2): Let U be any m_Y-open subset of Y and $x \in f^{-1}(U)$. There exists an g_X-open subset V of X containing x such that $f(V) \subseteq m_Y$-$Cl(U)$. Since $x \in V \subseteq f^{-1}(m_Y$-$Cl(U))$, $x \in i_X(f^{-1}(m_Y$-$Cl(U)))$. Hence $f^{-1}(U) \subseteq i_X(f^{-1}(m_Y$-$Cl(U)))$.

(2) $\Rightarrow$ (3): Let F be a m_Y-closed subset of Y. Then $Y - F$ is m_Y-open subset of Y and, by (2), $f^{-1}(Y - F) \subseteq i_X(f^{-1}(m_Y$-$Cl(Y - F))) = i_X(f^{-1}(Y - (m_Y$-$Int(F)))) = X - c_X(f^{-1}(m_Y$-$Int(F)))$. Thus $c_X(f^{-1}(m_Y$-$Int(F))) \subseteq f^{-1}(F)$.

(3) $\Rightarrow$ (4): Let A be a subset of Y. Since m_Y-$Cl(A)$ is m_Y-closed in Y, from (3), it follows $c_X(f^{-1}(m_Y$-$Int(m_Y$-$Cl(A)))) \subseteq f^{-1}(m_Y$-$Cl(A))$.

(4) $\Rightarrow$ (5): Let A be a subset of Y. From (4), it follows $f^{-1}(m_Y$-$Int(A)) = X - f^{-1}(m_Y$-$Cl(Y - A)) \subseteq X - c_X(f^{-1}(m_Y$-$Int(m_Y$-$Cl(Y - A)))) = i_X(f^{-1}(m_Y$-$Cl(m_Y$-$Int(A))))$. Thus we get the result.

(5) $\Rightarrow$ (6): Let U be a m_Y-open subset of Y. Suppose $x \notin f^{-1}(m_Y$-$Cl(U))$. Then $f(x) \notin m_Y$-$Cl(U)$ and so there exists an m_X-open set V containing $f(x)$ such that $U \cap V = \emptyset$ and so m_Y-$Cl(V) \cap U = \emptyset$. By (5), $x \in f^{-1}(V) \subseteq i_X(f^{-1}(m_Y$-$Cl(V)))$. There exists an g_X-open set G containing x such that $x \in G \subseteq f^{-1}(m_Y$-$Cl(V))$. Since m_Y-$Cl(V) \cap U = \emptyset$ and $f(G) \subseteq m_Y$-$Cl(V)$, we have $G \cap f^{-1}(U) = \emptyset$ and so $x \notin c_X(f^{-1}(U))$. Hence $c_X(f^{-1}(U)) \subseteq f^{-1}(m_Y$-$Cl(U))$.

(6) $\Rightarrow$ (1): Let $x \in X$ and U be a m_Y-open subset of Y containing $f(x)$. From $U = m_Y$-$Int(U) \subseteq m_Y$-$Int(m_Y$-$Cl(U))$ and (6), $x \in f^{-1}(U) \subseteq f^{-1}(m_Y$-$Int(m_Y$-$Cl(U))) = X - f^{-1}(m_Y$-$Cl(Y - (m_Y$-$Cl(U)))) \subseteq X - c_X(f^{-1}(Y -

$(m_Y\text{-}Cl(U)))) = i_X(f^{-1}(m_Y\text{-}Cl(U)))$. So there exists an g_X-open subset V of X containing x such that $V \subseteq f^{-1}(m_Y\text{-}Cl(U))$. Hence f is weakly (g, m)-continuous. □

Theorem 3.13. *For a function $f : (X, g_X) \to (Y, m_Y)$, where m_Y has property $\mathcal{B}$, the following properties are equivalent:*

(1) f is weakly (g, m)-continuous;

(2) $c_X(f^{-1}(m_Y\text{-}Int(F))) \subseteq f^{-1}(F)$ for every m_Y-regular closed subset F of Y;

(3) $c_X(f^{-1}(m_Y\text{-}Int(m_Y\text{-}Cl(G)))) \subseteq f^{-1}(m_Y\text{-}Cl(G))$ for every m_Y-β-open subset G of Y;

(4) $c_X(f^{-1}(m_Y\text{-}Int(m_Y\text{-}Cl(G)))) \subseteq f^{-1}(m_Y\text{-}Cl(G))$ for every m_Y-semi-open subset G of Y.

Proof. (1) ⇒ (2): Let F be any m_Y-regular closed subset of Y. Then m_Y-$Int(F)$ is m_Y-open, by Theorem 3.12(6), we have $c_X(f^{-1}(m_Y\text{-}Int(F))) \subseteq f^{-1}(m_Y\text{-}Cl(m_Y\text{-}Int(F)))$. Since F is m_Y-regular closed, we have $c_X(f^{-1}(m_Y\text{-}Int(F))) \subseteq f^{-1}(m_Y\text{-}Cl(m_Y\text{-}Int(F))) \subseteq f^{-1}(F)$.

(2) ⇒ (3): Let G be any m_Y-β-open set. Then $m_Y\text{-}Cl(G) \subseteq m_Y\text{-}Cl(m_Y\text{-}Int(m_Y\text{-}Cl(G))) \subseteq m_Y\text{-}Cl(G)$, so that m_Y-$Cl(G)$ is m_Y-regular closed. From (2), it follows $c_X(f^{-1}(m_Y\text{-}Int(m_Y\text{-}Cl(G)))) \subseteq f^{-1}(m_Y\text{-}Cl(G))$.

(3) ⇒ (4): Since every m_Y-semi-open set is m_Y-β-open, it is obvious.

(4) ⇒ (1): Let U be any m_Y-open subset of Y. Then from (4), $c_X(f^{-1}(U) \subseteq c_X(f^{-1}(m_Y\text{-}Int(m_Y\text{-}Cl(U)))) \subseteq f^{-1}(m_Y\text{-}Cl(U))$. Hence by Theorem 3.12(6), f is weakly (g, m)-continuous. □

Theorem 3.14. *For a function $f : (X, g_X) \to (Y, m_Y)$, the following properties are equivalent:*

(1) f is weakly (g, m)-continuous;

(2) $c_X(f^{-1}(m_Y\text{-}Int(m_Y\text{-}Cl(G)))) \subseteq f^{-1}(m_Y\text{-}Cl(G))$ for every m_Y-preopen subset G of Y;

(3) $c_X(f^{-1}(G)) \subseteq f^{-1}(m_Y\text{-}Cl(G))$ for every m_Y-preopen subset G of Y;

(4) $f^{-1}(G) \subseteq i_X(f^{-1}(m_Y\text{-}Cl(G)))$ for every m_Y-preopen subset G of Y.

Proof. (1) ⇒ (2): Let G be any m_Y-preopen subset of Y. Then $m_Y\text{-}Cl(G) = m_Y\text{-}Cl(m_Y\text{-}Int(m_Y\text{-}Cl(G)))$, so m_Y-$Cl(G)$ is m_Y-regular closed. From Theorem 3.13(2), it follows that $c_X(f^{-1}(m_Y\text{-}Int(m_Y\text{-}Cl(G)))) \subseteq f^{-1}(m_Y\text{-}Cl(G))$.

$(2) \Rightarrow (3)$: Let G be any m_Y-preopen subset of Y. Then $G \subseteq m_Y\text{-}Int(m_Y\text{-}Cl(G))$ and by (2), we have $c_X(f^{-1}(G)) \subseteq c_X(f^{-1}(m_Y\text{-}Int(m_Y\text{-}Cl(G)))) \subseteq f^{-1}(m_Y\text{-}Cl(G))$.

$(3) \Rightarrow (4)$: Let G be any m_Y-preopen subset of Y. By (3), it follows that $f^{-1}(G) \subseteq f^{-1}(m_Y\text{-}Int(m_Y\text{-}Cl(G))) = X - f^{-1}(m_Y\text{-}Cl(Y - (m_Y\text{-}Cl(G)))) = X - c_X(f^{-1}(Y - (m_Y\text{-}Cl(G)))) = i_X(f^{-1}(m_Y\text{-}Cl(G)))$. Hence we have (4).

$(4) \Rightarrow (1)$: Since every m_Y-open set is m_Y-preopen, from (4), and Theorem 3.12(2), it follows f is weakly (g,m)-continuous. □

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