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A Study on Generalised Aluthge Transformation

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Abstract

In this paper, various properties of $T(s,t) = |T|^s\, U\, |T|^t$ defined more generally for any s and t such as s ≥ 0 and t ≥ 0 the Aluthge transform of an operator T are studied.

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1 Introduction

In [1] A. Aluthge introduced the operator $\widetilde{T} = |T|^{1/2}$ U $|T|^{1/2}$ for an operator T with its polar decomposition T = U $|T| = |T^*|$ U . [9] Takashi Yoshino defined more generally for any s and t such as s ≥ 0 and t ≥ 0 T (s , t) = $|T|^s$ U $|T|^t$ the p - hyponormality of the Aluthge transform of T.[5] introduced a very interesting class of bounded linear Hilbert space operators Class A . Class A operators have been studied by many researchers for example [2 , 3 , 6, 10 ,11, 12]. Recently Jeon and Kim [7] introduced quasiclass A operators as an extension of the notion of class A operators. In [8] Tanahashi, Jeon , Kim and Uchiyama considered an extension of quasiclass A operators, quasiclass (A, k) operators. [4] H. Crawford Rhaly introduced posinormal operators in Hilbert space. In this article we are interested in some of the properties of the Althuge transform T (s,t).

2 Preliminary Notes

Let T be a bounded linear operator on a Hilbert space H. In [1], A. Aluthge introduced the operator $\widetilde{T}$ for an operator T with its polar decomposition T = U $|T|$ = $|T^*|$ U and [9] has introduced T (s , t) = $|T|^s$ U $|T|^t$.

Definition 2.1 *: [5] An operator T belongs to class A iff* $\left(T^* |T|^2 T\right)^{1/2} \geq T^*T$.

Definition 2.2 *: [7] An operator T belongs to quasiclass A iff* $T^* \left(|T^2| - |T|^2\right) T \geq 0$.

Definition 2.3 *: [8] T in B(H) is called a k- quasiclass A operator for a positive integer k if* $T^{*^k} \left(|T^2| - |T|^2\right) T^k \geq 0$.

Definition 2.4 *: [4] An operator T in a Hilbert space H is called posinormal if* $TT^* \leq c^2T^*T$ *for some* $c > 0$.

Definition 2.5 *: [4] An operator T in a Hilbert space H is quasiposinormal if* $(TT^*)^2 \leq c^2{T^*}^2T^2$.

Definition 2.6 *: [1] A bounded linear operator on a Hilbert space H is p - hyponormal if* $(T^*T)^p \geq (TT^*)^p, p > 0$.

3 Posinormal Operators

Theorem 3.1 :
Let T be p - hyponormal for some p such that p >0. Then for any s , t such as max (s, t) $\leq$ *p, T(s , t) is posinormal.*
Proof:
Since T is p - hyponormal $|T|^{2p} \geq |T^*|^{2p}$. *T is posinormal if* $TT^* \leq c^2T^*T$ *for some c> 0.*
Now,

$T(s,t)T^*(s,t) - c^2T^*(s,t)T(s,t)$

$$
\begin{aligned}
&= |T|^s U |T|^t (|T|^s U |T|^t)^* - c^2(|T|^s U |T|^t)^*(|T|^s U |T|^t) \\
&= |T|^s U |T|^t |T|^t U^* |T|^s - c^2 |T|^t U^* |T|^s |T|^s U |T|^t \\
&= |T|^s U |T|^{2t} U^* |T|^s - c^2 |T|^t U^* |T|^{2s} U |T|^t \\
&= |T|^s |T^*|^{2t} |T|^s - c^2 |T|^t |T^*|^{2s} |T|^t \\
&= |T|^s (|T^*|^{2p})^{t/p} |T|^s - c^2 |T|^t (|T^*|^{2p})^{s/p} |T|^t \\
&\leq |T|^s (|T|^{2p})^{t/p} |T|^s - c^2 |T|^t (|T|^{2p})^{s/p} |T|^t \\
&\leq |T|^{2(s+t)} - c^2 |T|^{2(s+t)} \\
&= (1-c^2) |T|^{2(s+t)} \\
&\leq 0, c > 0.
\end{aligned}
$$

$\Rightarrow$ *T (s , t) is posinormal.*

Theorem 3.2 :
If T is a p - hyponormal operator then T (s , t) is quasiposinormal.
Proof:
T is quasiposinormal if

$(TT^*)^2 \leq c^2 T^{*^2} T^2.$

$(T(s,t)T^*(s,t))^2 - c^2 T^{*^2}(s,t)T^2(s,t)$

$$
\begin{aligned}
&= (|T|^s (|T^*|^{2p})^{t/p} |T|^s)^2 - c^2 |T|^{2t} U^* |T|^{4s} U |T|^{2t} \\
&\leq (|T|^{2(s+t)})^2 - c^2 |T|^{2t} (|T|^{2p})^{2s/p} |T|^{2t} \\
&= |T|^{4(s+t)} - c^2 |T|^{4(s+t)} \\
&= (1-c^2) |T|^{4(s+t)} \\
&\leq 0, c > 0
\end{aligned}
$$

$\Rightarrow$ *T (s , t) is quasiposinormal.*

4 Class A Operators

Theorem 4.1 :
If T is a p - hyponormal operator in a Hilbert space H, then T (s,t) is of class A.
Proof:
An operator T is in class A if $\left(T^* |T|^2 T\right)^{1/2} \geq T^*T$
or $\left(T^* |T|^2 T\right) \geq (T^*T)^2$

Now,
$T^*(s,t)\left|T(s,t)\right|^2 T(s,t)$

$$\begin{aligned}
&= T^*(s,t)T^*(s,t)T(s,t)T(s,t)\\
&= T^{*^2}(s,t)T^2(s,t)\\
&= |T|^{2t} U^* |T|^{2s} |T|^{2s} U |T|^{2t}\\
&= |T|^{2t} U^* |T|^{4s} U |T|^{2t}\\
&\geq |T|^{2t} |T^*|^{4s} |T|^{2t}\\
&\geq |T|^{4(s+t)}\\
&= (T^*(s,t)T(s,t))^2
\end{aligned}$$

$\Rightarrow$ *$T(s,t)$ is in class A.*

Theorem 4.2 :
If T is p- hyponormal then T (s,t) is in quasiclass A.
Proof:
An operator T belongs to quasi class A if $T^*\left(|T^2| - |T|^2\right)T \geq 0$.
$T^*(s,t)\left|T^2(s,t)\right| T(s,t) - T^*(s,t)\left|T(s,t)\right|^2 T(s,t)$

$$\begin{aligned}
&= T^*(s,t)(T^{*^2}(s,t)T^2(s,t))^{1/2}T(s,t) - T^{*^2}(s,t)T^2(s,t)\\
&\geq T^*(s,t)(|T|^{4(s+t)})^{1/2}T(s,t) - |T|^{4(s+t)}\\
&\geq T^*(s,t)T^*(s,t)T(s,t)T(s,t) - |T|^{4(s+t)}\\
&\geq |T|^{4(s+t)} - |T|^{4(s+t)}\\
&= 0
\end{aligned}$$

$\Rightarrow$ *T(s , t) belongs to quasiclass A.*

Theorem 4.3 :
If T is p - hyponormal then T (s,t) belongs to k - quasiclass A.
Proof:
An operator T is k quasiclass A if $T^{*^k}\left(|T^2| - |T|^2\right)T^k \geq 0$.
$T^{*^k}(s,t)\left|T^2(s,t)\right| T(s,t) - T^*(s,t)\left|T(s,t)\right|^2 T^k(s,t)$

$$\begin{aligned}
&= T^{*^k}(s,t)(T^{*^2}(s,t)T^2(s,t))^{1/2}T(s,t) - T^{*^k}(s,t)T^*(s,t)T(s,t)T^k(s,t)\\
&\geq T^{*^k}(s,t)(|T|^{4(s+t)})^{1/2}T^k(s,t) - T^{*^{(k+1)}}(s,t)T^{(k+1)}(s,t)\\
&\geq T^{*^k}(s,t)T^*(s,t)T(s,t)T^k(s,t) - |T|^{(k+1)(s+t)}\\
&\geq |T|^{(k+1)(s+t)} - |T|^{(k+1)(s+t)}\\
&= 0
\end{aligned}$$

$\Rightarrow$ *T(s , t) belongs to k quasiclass A.*

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