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Matricial Operators which Preserve Schauder Basis in p-Adic Analysis

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Abstract

In this work we give a generalization of the results established by W. Ruckle and L. W. Baric for the matrix transformations which preserve schauder basis in the classical case for a p-adic analysis. We give several characterizations of matricial operators which preserve Schauder bases in non archimedean Barrelled spaces.

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1 Introduction

W. Ruckle ([7], theorem2.1, p. 548) gave a characterization of a matrix $A = (a_{ij})_{i,j}$ which transforms a basis sequence $(x_i)_i$ to a basis sequence $(y_i)_i$ $y_i = \sum_{j=1}^{\infty} a_{ij}x_j$ for all $i \geq 1$ in a Frechet space E on a field K where $K = \mathbb{R}$ or $K = \mathbb{C}$. By A^T, the transpose matrix of A, he gave an other characterization in case which E is a Banach space ([7], theorem3.2, p.549). In ([1], theorem1 and theorem2, p.277, 278) L. W. Baric and W. Ruckle enhanced these results. These characterizations are based on the result established by A.Wilansky ([10], Theorem5, p. 211) and generalized by J. R. Retherford and C. W. MC. Arthur to a Hausdorff barrelled space ([6], theorem3.1 and theorem3.2, p. 40, 41). J. T. Marti ([4] , III. 2, theorem2 and corollary3, p. 31) gave a standard criterion for a biorthogonal system to be a basis in a F-space (Frechet space). In ([11], theorem1, p. 6) N. Zobin gave a characterization of a matrix A which transforms an absolute basis $(e_i)_i$ of a Frechet space to an absolute basis $(f_i)_i$.

In this work we generalize the results before on a locally K-convex barrelled space X, where K is a non-archimedean non-trivially valued field which is complete under the metric induced by the valuation $|.|$; we define several notions of preserving matricial operators and establish other results characterizing them. The following notions and notations will be used:

(X, τ) is a non archimedean (n.a) Hausdorff locally K-convex space over K, the topology τ is determined by a family $(\mathcal{P})$ of n.a seminorms. For fundamentals of locally K-convex space we refer to [5], [8] and [9]. A sequence $B = (x_i)_i$ in X is called a (topological) base of X if each $x \in X$ can be written uniquely as $x = \sum_{i=1}^{\infty} \lambda_i x_i$ whith $\lambda_i \in K$. If the coefficient functionals $f_n : X \longrightarrow K, \ \ x = \sum_{i=1}^{\infty} \lambda_i x_i \longmapsto \lambda_n$ for all $n \geq 1$ are continuous then the basis B is called a Schauder basis. If the topology of X can be determined by a family $(\mathcal{P})$ of n.a seminorms satisfying the condition if $x \in X$, $x = \sum_{i=1}^{\infty} \lambda_i x_i$ then $p(x) = \max_i \quad p(\lambda_i x_i)$ for all $p \in (\mathcal{P})$ the basis is called orthogonal. A sequence $B = (x_i)_i$ is called a basic sequence if it is a Schauder basis of $\overline{[x_i]}$ (it's closed linear span) and it is called complete if $\overline{[x_i]} = X$. A sequence $(x_i, f_i)_i$, $x_i \in X$, $f_i \in X'$, the topological dual of X, is said to be biorthogonal if $\langle x_i, f_j \rangle = \delta_{ij} \ \ i, j = 1, 2, ...$ For general properties of bases before we refer to [2] and [3]. For every Schauder basis $B = (x_i)_i$ the sequence $F = (f_i)_i$ is a Schauder basis of $(X', \sigma(X', X))$ where $\sigma(X', X)$ is the weak topology on X' ([2], lemma3, p. 402).

$(\omega, \tau_\omega) :=$ the linear space of all sequences in K endowed with the product topology τ_ω which is generated by the family of n.a seminorms (p_n) which is defined by $p_n(\lambda) = |\lambda_n|$ for all $\lambda = (\lambda_n)_n \in \omega$. A sequence space on K is a subspace of ω.

A sequence space(E, τ_E) on K, is called a K-space if for every n in $I\!N$ the functional map $\pi_n : (E, \tau_E) \longrightarrow K, \ \ (\lambda_i)_i \longmapsto \lambda_n$ is continuous. It is called a $FK-$space if it is a F-space and a K-space over K.

Let E be a non empty subset of ω, the β-dual of E is the set noted by E^β and defined as follows:

$E^\beta = \left\{ (\lambda_j)_j \in \omega : \lim_j \lambda_j \mu_j = 0 \ \mathit{for\ all}\, (\mu_j)_j \in E \right\}$.

Throughout $B = (x_i)_i$ is a Schauder basis of (X, τ) and $F = (f_i)_i$ is a weak Schauder basis associated.

Let $T \in B(X)$, then there exists an infinite matrix $A = (a_{ij})_{i,j}$ such that $a_{ij} \in K$ for all $i, \ j = 1, 2, ...$ and $T(x_i) = y_i = \sum_{j=1}^{\infty} a_{ij} x_j$ for all i in $I\!N$. Throughout we consider $T = (a_{ij})_{ij}$ it is called matricial operator and we denoted by ω_T, E_T, Λ and L the following spaces:

$\omega_T = \{(\lambda_i)_i \in \omega : \sum_i \lambda_i y_i \ \mathit{converges\ in}\ X\}$

$E_T = \{y \in X : \mathit{there\ exists}\, (\lambda_i)_i \in \omega_T : y = \sum_{i=1}^{\infty} \lambda_i y_i\}$

$\Lambda = \{(\lambda_i)_i \in \omega : \sum_i \lambda_i x_i \ \mathit{converges\ in}\ X\}$

$L = \overline{[y_i]}$

We provide ω_T and E_T with the topology $\overline{\tau}_T$ defined by the family of n.a seminorms $(\overline{p}_T)_{p\in(\mathcal{P})}$ where $\overline{p}_T$ is defined as: $\overline{p}_T\,(y) = \overline{p}_T\,((\lambda_i)_i) = \sup_i p\,(\lambda_i y_i)$ for all $(\lambda_i)_i \in \omega_T$, all $y = \sum_{i=1}^{\infty} \lambda_i y_i \in E_T$ and all $p \in (\mathcal{P})$. Over Λ we define the topology $\overline{\tau}$ with the family of n.a seminorms $(\overline{p})_p \in (\mathcal{P})$ where $\overline{p}$ is defined as follows: $\overline{p}\,((\lambda_i)_i) = p\,(x)$ for all $(\lambda_i)_i \in \Lambda$, all $x = \sum_{i=1}^{\infty} \lambda_i x_i \in X$ and all $p \in (\mathcal{P})$.

In §.2 we give several notions of preserving and equicontinuous operators and characterize them. We also give an analogue result to ([7], theorem1.21, p. 547) which characterizes the sequence that are a Schauder basis in a n.a barrelled space. In §.3 considering a barrelled space which is sequentially complete we give some characterizations of the matrix A which transforms a Schauder basis B into a topological basis or into a Schauder basis or into an orthogonal basis.

2 Preserving matricial operators

Definitions 1 *Let T be a matricial operator; T is called:*

(a). Semi-preserving if it transforms the Schauder basis B into a basic sequence,

(b). Preserving if it transforms the Schauder basis B into a Schauder basis of X,

(c). Topologically preserving if it transforms the Schauder basis B into a topological basis of X,

(d). Semi-preserving orthogonally if it transforms the Schauder basis B into an orthogonal basis of $(E_T, \overline{\tau}_T)$,

(e). Orthogonally-preserving if it transforms the Schauder basis B into an orthogonal basis of X.

Proposition 1 *$(\omega_T, \overline{\tau}_T)$ is a K-space for which $(e^i)_i$ is a topological basis and the mapping Φ_T^{-1} is continuous, where $\Phi_T : \left(E_T, \tau_{/E_T}\right) \longrightarrow (\omega_T, \overline{\tau}_T)$ $y = \sum_{i=1}^{\infty} \lambda_i y_i \longmapsto (\lambda_i)_i$ and $e^i = (\delta_{ij})_j$ for all $i \geq 1$ (δ_{ij} is the kroneker symbol).*

Proof. Let us prove that for all $n \in I\!N$ $\pi_n : \omega_T \longrightarrow K$ $(\lambda_i)_i \longmapsto \lambda_n$ is continuous. Let $(\lambda_i)_i \in \omega_T$, then for every $n \geq 1$ there exists $p \in (\mathcal{P})$ such that $p\,(y_n) \neq 0$ and $|\lambda_n| \leq \frac{\overline{p}_T(\lambda)}{p(y_n)}$, as π_n is linear then it is continuous.

Let $(\lambda_i)_i \in \omega_T$, then $\sum_i \lambda_i y_i$ converges in X, so for all $p \in (\mathcal{P})$ there exists $i_0 \in I\!N$ such that for all $i \succ i_0$ $p\,(\lambda_i y_i) \leq 1$ then for all $p \in (\mathcal{P})$ there exists $i_0 \in I\!N$ such that $\overline{p}_T\left(\sum_{i=1}^{i_0} \lambda_i e^i\right) \leq 1$. Therefore $\lim_{n\to\infty} \sum_{i=1}^{n} \lambda_i e^i = \lambda$ in $(\omega_T, \overline{\tau}_T)$. On the other hand, for all $y = \sum_{i=1}^{\infty} \lambda_i y_i \in E_T$ and all $p \in (\mathcal{P})$, $p\,(y) \leq \overline{p}_T\,(y)$, then Φ_T^{-1} is continuous. □

Remark 1 *Φ_T is an isometry from $(E_T, \overline{\tau}_T)$ towards $(\omega_T, \overline{\tau}_T)$.*

Proposition 2 *$(E_T, \overline{\tau}_T)$ is a Hausdorff space.*

Proof. Let $y = \sum_{i=1}^{\infty} \lambda_i y_i \in E_T$ such that $y \neq 0$, then there exists $p \in (\mathcal{P})$ such that $p(y) \neq 0$, then there exists $i_0 \in I\!N$ such that $p(\lambda_{i_0} y_{i_0}) \neq 0$ and so $\overline{p}_T\,(y) \neq 0$. □

Proposition 3 *If (X, τ) is sequentially complete, then $(E_T, \overline{\tau}_T)$ is complete.*

Proof. Let $(y^i)_{i \in I}$ be a Cauchy-net in $(E_T, \overline{\tau}_T)$ and let $y^i = \sum_{n=1}^{\infty} \lambda_n^i y_n$ for all $i \in I$, then for all $p \in (\mathcal{P})$ and all $\varepsilon \succ 0$ there exists $i_0 \in I$ such that for all $i,j \geq i_0$ $\overline{p}_T\,(y^i - y^j) \leq \varepsilon$; moreover (1) $\sup_n |\lambda_n^i - \lambda_n^j|\, p(y_n) \leq \varepsilon$, for all $i,j \geq i_0$, so for all $n \geq 1$, $(\lambda_n^i)_{i \in I}$ is a Cauchy-net in K, then for all $n \geq 1$, there exists $\lambda_n \in K$ such that $\lim_i \lambda_n^i = \lambda_n$ in K. In passing to the limit on j, in (1) we obtain

$$(2)\ \sup_n \left|\lambda_n^i - \lambda_n\right| p(y_n) \leq \varepsilon,\ \ for\ all\ i \geq i_0.$$

On the other hand, there exists $n_0 \in I\!N$ such that for all $n \geq n_0$ $|\lambda_n^{i_0}|\, p(y_n) \leq \varepsilon$. Then there exists $n_0 \in I\!N$ such that for all $n \geq n_0$

$$|\lambda_n|\, p(y_n) \leq \max\left(\left|\lambda_n^{i_0} - \lambda_n\right| p(y_n), \left|\lambda_n^{i_0}\right| p(y_n)\right) \leq \varepsilon.$$

Therefore$\sum_n \lambda_n y_n$ converges in (X, τ).
Let $y = \sum_n^{\infty} \lambda_n y_n$, then $y \in E_T$ and by (2) we have $y = \lim_i y^i$ in $(E_T, \overline{\tau}_T)$. □

Definition 1 *T is called equicontinuous if for every $p \in (\mathcal{P})$ there exists $q \in (\mathcal{P})$ such that for all $y = \sum_i^{\infty} \lambda_i y_i \in E_T$ and all $i \geq 1$ $p(\lambda_i y_i) \leq q(y)$.*

Remark 2 *Let $\Psi_n : X \longrightarrow E_T$, $y = \sum_{i=1}^{\infty} \lambda_i y_i \longmapsto \lambda_n y_n$ for all $n \geq 1$, then T is equicontinuous if and only if $(\Psi_n)_n$ is equicontinuous as a linear maps of (X, τ) in $(E_T, \overline{\tau}_T)$ i.e. for all $p \in (\mathcal{P})$ there exists $q \in (\mathcal{P})$ such that for all $y = \sum_{i=1}^{\infty} \lambda_i y_i \in E_T$, $\overline{p}_T\,(y) \leq q(y)$. Therefore $\overline{\tau}_T = \tau_{/E_T}$.*

Proposition 4 *If T is equicontinuous, then T is semi-preserving orthogonally.*

Proof. Let $y \in E_T$, then there exists $(\lambda_i)_i \in \omega$ such that $y = \sum_{i=1}^{\infty} \lambda_i y_i$, then $(y_i)_i$ is a topological basis of E_T, and for every $p \in (\mathcal{P})$, $\overline{p}_T\,(y) = \sup_i p(\lambda_i y_i) = \sup_i \overline{p}_T\,(\lambda_i y_i)$, then $(y_i)_i$ is an orthogonal basis of $(E_T, \overline{\tau}_T)$. □

Proposition 5 *Suppose that (X, τ) is sequentially complete. If T is equicontinuous then E_T is closed in (X, τ).*

Proof. By remark2 $\overline{E_T}^{\overline{\tau}_T} = \overline{E_T}^{\tau}$, then $E_T = \overline{E_T}^{\tau}$ because $(E_T, \overline{\tau}_T)$ is complete, by proposition3 . □

Theorem 1 *If (X,τ) is sequentially complete, T is complete ($L = X$) and equicontinuous, then T is preserving.*

Proof. for all $n \geq 1$, $y_n \in E_T$ then $[y_n] \subset E_T$, so $L \subset E_T$, E_T is closed in (X,τ), therefore $E_T = X$. Then $(y_n)_n$ is a Schauder basis of $(X, \overline{\tau}_T)$. Well $\overline{\tau}_T = \tau_{/E_T}$ (remark2) then $\overline{\tau}_T = \tau$, and so $(y_n)_n$ is a Schauder basis of (X,τ). □

Remark 3 *Under the conditions of theorem1, T is orthogonally preserving.*

If (X,τ) is barrelled we have the following theorem which is a converse of theorem1.

Theorem 2 *Suppose that (X,τ) is barrelled. Then if T is topologically preserving, then T is equicontinuous.*

Proof. Us $(y_n)_n$ is a topological basis of (X,τ), then $E_T = X$, and us $(y_n)_n$ is an orthogonal basis of $(E_T, \overline{\tau}_T)$, then it is equicontinuous ([2] proposition5, p. 400). It suffices to show that $\overline{\tau}_T = \tau$. We have $\tau \leq \overline{\tau}_T$ (by definition of $\overline{\tau}_T$ and in fact that $E_T = X$).

Conversely let $U = \{x \in X : \overline{p}_T(x) \leq 1\} = B_{\overline{p}_T}(0,1)$, then U is absolutely K-convex and closed in (X,τ); let $x = \sum_{n=1}^{\infty} \lambda_n y_n \in X$, then for every $p \in (\mathcal{P})$ there exists $n_0 \geq 1$ such that for all $n \geq n_0$ $p(\lambda_n y_n) \leq 1$; let $\lambda \in K$ such that $|\lambda| \succ \max(1, p(\lambda_1 y_1), p(\lambda_2 y_2), ..., p(\lambda_{n_0} y_{n_0}))$ then $\overline{p}_T(x) \leq \lambda$ so $x \in \lambda U$ and U is absorbing; us (X,τ) is barrelled then U is a neighbourhood of zero in (X,τ); therefore $\overline{\tau}_T \leq \tau$. □

Remark 4 *In a barrelled space, every topological basis is an orthogonal basis (then it is a Schauder basis). N.De Grande-De Kimpe has proved that in a barrelled space, every Schauder basis is an orthogonal basis ([2], proposition7, p. 401).*

Theorem 3 *Let (X,τ) be a locally K-convex, sequentially complete and barrelled space and $(z_j)_j$ be a complete sequence in X such that for all $j \in \mathbb{N}$ $z_j \neq 0$; then $(z_j)_j$ is a topological basis (it is a Schauder basis) if and only if for every $p \in (\mathcal{P})$ there exists $q \in (\mathcal{P})$ such that for all $j \in \mathbb{N}$ and all $z = \sum_{j=1}^{\infty} \lambda_j z_j \in X$, $p(\lambda_j z_j) \leq q(z)$.*

Proof. By theorem1 and theorem2. □

3 Conditions on the matrix A

In this paragraph (X,τ) is a space of type of theorem3 and the family $(\mathcal{P})$ verify $p\left(\sum_{i=1}^{\infty} \lambda_i x_i\right) = max_i\ p(\lambda_i x_i)$ for all $p \in (\mathcal{P})$.

Theorem 4 *Let (X,τ) be a locally K-convex, sequentially complete and barrelled space, then T is semi-preserving if and only if for every $p\in(\mathcal{P})$ there exists $q\in(\mathcal{P})$ such that for all $\lambda=(\lambda_j)_j\in\omega_T$ there exists $i_0\geq 1$ such that $\overline{p}(\lambda)\leq\left|\sum_{j=1}^{\infty}a_{i_0j}\lambda_j\right|q(x_{i_0})$.*

Lemma 1 *If $\sum_i\lambda_i y_i$ converges, then $\sum_j a_{ij}\lambda_j$ converges for every $i\in\mathbb{N}$ and in this case $\sum_{j=1}^{\infty}\lambda_j y_j=\sum_{i=1}^{\infty}\left(\sum_{j=1}^{\infty}a_{ij}\lambda_j\right)x_i$.*

Proof. For every $i\geq 1$, f_i is continuous, then

$$\begin{aligned} f_i\left(\sum_{j=1}^{\infty}\lambda_j y_j\right) &= \sum_{j=1}^{\infty}\lambda_j f_i(y_j)\\ &= \sum_{j=1}^{\infty}\lambda_j f_i\left(\sum_{k=1}^{\infty}a_{kj}x_k\right)\\ &= \sum_{j=1}^{\infty}\lambda_j\left(\sum_{k=1}^{\infty}a_{kj}f_i(x_k)\right)\\ &= \sum_{j=1}^{\infty}\lambda_j a_{ij}. \end{aligned}$$

Then $\sum_j a_{ij}\lambda_j$ is converging in X and we have

$$\sum_{j=1}^{\infty}\lambda_j y_j=\sum_{i=1}^{\infty}\left(\sum_{j=1}^{\infty}a_{ij}\lambda_j\right)x_i.$$

□

Proof. Of theorem. By theorem3, $(y_i)_i$ is a Schauder basis of $L=\overline{[y_i]}$ if and only if for all $p\in(\mathcal{P})$ there exists $q\in(\mathcal{P})$ such that for all $y=\sum_{j=1}^{\infty}\lambda_j y_j$ and all $j\in\mathbb{N}$ we have $p(\lambda_j y_j)\leq q(y)$. And by lemma1, $\sum_{j=1}^{\infty}\lambda_j y_j=\sum_{i=1}^{\infty}\left(\sum_{j=1}^{\infty}a_{ij}\lambda_j x_i\right)$, then the theorem holds. □

corollary 1 *Under the conditions of theorem4, if T is complete, then T is preserving if and only if for every $p\in(\mathcal{P})$ there exists $q\in(\mathcal{P})$ such that for every $\lambda=(\lambda_j)_j\in\omega_T$ there exists $i_0\in\mathbb{N}$ such that $\overline{p}(\lambda)\leq\left|\sum_{j=1}^{\infty}a_{i_0j}\lambda_j\right|q(x_{i_0})$.*

Theorem 5 *Let (X,τ) be a locally K-convex, sequentially complete and barrelled space, then T is preserving if and only if A maps ω_T one-to-one onto Λ.*

Proof. The same proof us in ([7], theorem2.1, p. 548). □

Theorem 6 *Let (X,τ) be a locally K-convex, sequentially complete and barrelled space, then*

(i). A is injective if and only if Φ_T^{-1} is injective,

(ii). T is preserving if and only if A has an inverse $B=(b_{ij})_{i,j}$ verifying

(a). For every $i\geq 1$, $(b_{ij})_j\in\Lambda^{\beta}$ and (b). For every $\lambda\in\Lambda$, $B\lambda\in\omega_T$.

Where $B\lambda=\left(\sum_{j=1}^{\infty}b_{ij}\lambda_j\right)_i$ for every $\lambda=(\lambda_j)_j\in\Lambda$.

Proof. (i). By theorem5.

(ii). Suppose that T is preserving, then $(y_i)_i$ is a Schauder basis of (X,τ), so there exists $(b_{ij})_{i,j}$ such that for all $j \in I\!N$ $x_j = \sum_{i=1}^{\infty} b_{ij} y_i$, thus for every $\lambda = (\lambda_i)_i \in \Lambda$ we have $\sum_j \lambda_j x_j$ converges in X then, by lemma1, $\sum_j b_{ij}\lambda_j$ converges and $\sum_{j=1}^{\infty} \lambda_j x_j = \sum_{i=1}^{\infty} \left(\sum_{j=1}^{\infty} b_{ij}\lambda_j\right) y_i$, therefore$\left(\sum_{j=1}^{\infty} b_{ij}\lambda_j\right)_i \in \omega_T$. Then for all $i \in I\!N$ $(b_{ij})_j \in \Lambda^{\beta}$ and $B(\Lambda) \subset \omega_T$. On the other hand, for all $j \in I\!N$ $x_j = \sum_{i=1}^{\infty} b_{ij} y_i$ so for all $j \in I\!N$ and all $k \in I\!N$ $\sum_i a_{ki} b_{ij}$ converges and for all $j \in I\!N$ $x_j = \sum_{i=1}^{\infty} b_{ij} y_i = \sum_{k=1}^{\infty} \left(\sum_{i=1}^{\infty} a_{ki} b_{ij}\right) x_k$, then $\sum_{k=1}^{\infty} a_{ki} b_{ij} = 1$ if $k = j$ and $\sum_{k=1}^{\infty} a_{ki} b_{ij} = 0$ if $k \neq j$. Then for all $j \in I\!N$ $A(b_{ij})_i = e^{\,j}$.

At the same we show that for all $j \in I\!N$ $B(a_{ij})_i = e^{\,j}$. Then A has an inverse B verifing (a) and (b).

Conversely, suppose that A has an inverse B verifing (a) and (b), then $A(Be^i) = e^i$ and $B(Ae^i) = e^i$ for all $i \in I\!N$, moreover $(e^i)_i$ is a Schauder basis of Λ and E_T. On the other hand, for every $\lambda = (\lambda_j)_j \in \omega_T$ and for every $p \in (\mathcal{P})$ we have $\overline{p}(A\lambda) = p\left(\sum_{i=1}^{\infty} \left(\sum_{j=1}^{\infty} a_{ij}\lambda_j\right) x_i \right) = p\left(\sum_{j=1}^{\infty} \lambda_j y_j\right) \leq \overline{p}(\lambda)$.

Then A is continuous of $(\omega_T, \overline{\tau}_T)$ towards $(\Lambda, \overline{\tau})$. At the same, B is continuous because for all $\lambda = (\lambda_j)_j \in \Lambda$ and all $p \in (\mathcal{P})$ $\overline{p}(B\lambda) = \sup_i p\left(\sum_{j=1}^{\infty} b_{ij}\lambda_j y_i \right)$, but $\overline{\tau}_T = \tau_{/E_T}$ ((X,τ) *is barrelled*), then there exists $q \in (\mathcal{P})$ such that $\sup_i p\left(\sum_{j=1}^{\infty} b_{ij}\lambda_j y_i \right) \leq q\left(\sum_{i=1}^{\infty} \left(\sum_{j=1}^{\infty} b_{ij}\lambda_j\right) y_i\right)$, but $\sum_{i=1}^{\infty} \left(\sum_{j=1}^{\infty} b_{ij}\lambda_j\right) y_i = \sum_{j=1}^{\infty} \lambda_j x_j$ ($A^{-1} = B$), so

$\overline{p}(B\lambda) \leq q\left(\sum_{j=1}^{\infty} \lambda_j x_j\right) \leq \overline{q}(\lambda)$.

Then A maps ω_T one-to-one onto Λ and theorem5 gives the conclusion.□

Theorem 7 *Let (X,τ) be a locally K-convex, sequentially complete and barrelled space, then T is preserving if and only if A has an inverse $B = (b_{ij})_{i,j}$ and (3) : for every $p \in (\mathcal{P})$ there exists $q \in (\mathcal{P})$ such that for all $i,j \in I\!N$ $p(b_{ij} y_j) \leq q(x_i)$ and $p(a_{ij} x_j) \leq q(y_i)$ where $y_i = T(x_i)$ for all $i \geq 1$.*

Proof. Suppose that T is preserving and let $B = (b_{ij})_{i,j}$ the inverse of A, then for all $p \in (\mathcal{P})$ there exists $q \in (\mathcal{P})$ such that for all $i,j \in I\!N$ $p(b_{ij} y_j) \leq q(x_i)$ (theorem4) $\left(x_i = \sum_{j=1}^{\infty} b_{ij} y_j \ for\ all\ i,\ j \in I\!N\right)$. At the same we have the second inequality of (3).

Conversely, suppose that A has an inverse $B = (b_{ij})_{i,j}$ and (3) holds; let $p \in (\mathcal{P})$ so there exists $q \in (\mathcal{P})$ such that for all $i,j \in I\!N$ $p(b_{ij} y_j) \leq q(x_i)$. Let $\lambda = (\lambda_j)_j \in \omega_T$ then $\sum_{j=1}^{\infty} \lambda_j y_j = \sum_{i=1}^{\infty} \left(\sum_{j=1}^{\infty} a_{ij}\lambda_j\right) x_i$ (*lemma*1); for all $i \in I\!N$ let $\mu_i = \sum_{j=1}^{\infty} a_{ij}\lambda_j$, then $\mu = (\mu_i)_i \in \Lambda$ and $\lambda = B\mu$, so for all $i \in I\!N$ $\lambda_i = \sum_{j=1}^{\infty} b_{ij}\mu_j$ and for every $i \in I\!N$ we have

$$
\begin{aligned}
p(\lambda_i y_i) &= p\left(\sum_{j=1}^{\infty} b_{ij}\mu_j y_i\right) \\
&\leq \max_j\ p(b_{ij}\mu_j y_j) \\
&\leq \max_j |\mu_j|\, q(x_i) \quad (\ by\, (3)) \\
&\leq \max_j\ q(\mu_j x_i)
\end{aligned}
$$

$\leq \max_j \ q \left(\sum_{k=1}^{\infty} a_{jk} \lambda_k x_j\right)$.

On the other hand, for all $i \in I\!N \ if \ x_i = \sum_{j=1}^{\infty} b_{ij} y_j$ then $\left(\ x_i\right)_i \subset \ E_T$ and by the same us before we obtain $\lambda = (\lambda_j)_j \in \Lambda \quad p\left(\lambda_i x_i\right) \leq \max_j \ q\left(\sum_{k=1}^{\infty} b_{jk} \lambda_k y_j\right)$,

so $\left(x_i\right)_i$ is an orthogonal basis of E_T (remark3). Then T is complete.

Finally T is copmplete and verifying the conditions of theorem4, then T is preserving (corollary1). □

corollary 2 *Under the conditions of theorem before, T is preserving if and only if A has an inverse $B = (b_{ij})_{i,j}$ and for all $i,j \in I\!N$*

$$|b_{ij}| \leq \inf_p \in (\mathcal{P}) \frac{\lambda_{i,m(p)}}{\mu_{j,p}} \text{ and } |\ a_{ij}| \leq \inf_p \in (\mathcal{P}) \frac{\mu_{i,m(p)}}{\lambda_{j,p}},$$

where $\lambda_{i,p} = p\ (x_i)\,, \ \ \mu_{i,m(p)} = \ m\,(p)\,(y_i)$ for all $\ i \in I\!N$ and all $p \in (\mathcal{P})$.

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