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An Efficient Hybrid Simplex Bat Algorithm (HSBA) for Solving the General Nonlinear Global Optimization Problem

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Abstract

In this paper, we propose an efficient hybrid meta-heuristic algorithm combining the Nelder–Mead (NM) simplex search method with the Bat Algorithm (BA), termed as the Hybrid Simplex-Bat Algorithm (HSBA). The proposed method aims to leverage the local search strength of NM and the global exploration ability of BA to solve complex nonlinear global optimization problems. HSBA is tested on a suite of standard benchmark functions and compared with existing meta-heuristic and hybrid algorithms. Experimental results demonstrate that HSBA outperforms the standalone BA and many other existing hybrid approaches in terms of solution quality, convergence speed, and robustness.

Keywords: Meta-heuristic algorithm, Bat Algorithm, Nelder–Mead method, Hybrid algorithm, Global optimization

1. Introduction

Global optimization is essential in solving complex, high-dimensional problems across science and engineering. Traditional optimization techniques, such as gradient-based methods, are often limited by their tendency to converge to local optima, especially when the objective function is non-differentiable or highly multimodal. To address these limitations, meta-heuristic algorithms have emerged as robust alternatives due to their stochastic nature and global search capabilities.

Among these, the Bat Algorithm (BA), inspired by the echolocation behavior of bats, has shown significant promise. However, it suffers from issues such as premature convergence and loss of diversity. To overcome these shortcomings, this paper proposes a hybrid approach that combines BA with the Nelder–Mead (NM) simplex method, resulting in the Hybrid Simplex-Bat Algorithm (HSBA). The proposed algorithm enhances both exploration and exploitation by integrating local search from NM within the global framework of BA.

2. Background and Related Work

The Nelder–Mead simplex method is a direct search technique that operates on a simplex of $n + 1$ vertices in an n-dimensional space. It is particularly effective for unconstrained optimization problems where derivative information is unavailable. The method uses geometric transformations, reflection, expansion, contraction, and shrinkage to iteratively explore the search space.

On the other hand, the Bat Algorithm (BA) models the foraging behavior of bats using frequency tuning, loudness, and pulse emission rates. While BA performs well in diverse optimization scenarios, its reliance on current global best solutions can lead to stagnation in local minima.

Previous hybrid strategies have attempted to combine meta-heuristics with local search methods. For example, NM has been paired with Particle Swarm Optimization (PSO), Genetic Algorithms (GA), and Simulated Annealing (SA). This work continues in that direction by embedding NM into BA’s position update mechanism, with the goal of improving solution quality and robustness.

3. The Proposed HSBA Algorithm

The main idea behind the Hybrid Simplex-Bat Algorithm (HSBA) proposed in this work is to enhance the original Bat Algorithm (BA) by incorporating search updates using reflection, expansion, or contraction points from the Nelder-Mead (NM) algorithm. This modification increases the likelihood of discovering improved solutions at each iteration. The NM algorithm is used as a complementary component to promote the convergence of the BA.

Upon examining the trajectories of bats, we observed that many of them become trapped in local optima due to insufficient diversity. Additionally, some bats are solely influenced by the local optimum. If the best-performing bat is trapped in a local optimum, it may mislead the others. To address this limitation of the BA, we introduced several enhancements, including incorporating quantum behavior to improve diversity among bats. The steps of the proposed algorithm are detailed below.
The Hybrid Simplex-Bat Algorithm (HSBA) proceeds as follows:

Setting Parameters
The following parameters are initialized at the beginning of the algorithm:
- Set the standard parameters for the NM algorithm: $\delta^r, \delta^e, \delta^{ic}, and\ \delta^{oc}$
$\delta^r = 1$, $\delta^e = 2$, and $\delta^{ic} = \delta^{oc} = 1/2$.
- Set the BA parameters: $\beta \in [0,1]$, $f_{\min} = 0$, $f_{\max} = 2$, $A_0 = 0.7$, $r_0 = 0.1$, $\alpha = 0.9$, and $\gamma = 0.9$.

Initialization
Generate n random bat positions, where n represents the problem's dimension:
$x_i = lb_i + rand[ub_i - lb_i]$, $i = 1, 2, 3..., n$
Where lb and ub are lower bound and upper bound vectors of the test problem.
Assign initial velocities v_i and randomly set the initial frequencies f_i . Set The inertia weights to $\omega_{\min} = 0.4$ and $\omega_{\max} = 0.9$

Main Algorithm
Repeat the following steps until a stopping condition is met. The stopping criteria are based on a maximum number of iterations, function evaluations, or CPU time.
1. Perform the BA update steps with enhancements:
a. Quantum Behavior Update:
According to the uncertainty principle, a particle's position and velocity cannot be determined simultaneously. Following J. Sun, B. Feng, and W. Xu (2004), we update the bat's position as:

$$\begin{cases} x_i^k = x^* + rand.\left|M^k{}_{best} - x_i^{k-1}\right| \ln(\frac{1}{u}), u < 0.5 \\ x_i^k = x^* - rand.\left|M^k{}_{best} - x_i^{k-1}\right| \ln(\frac{1}{u}), u \geq 0.5 \end{cases}$$

where u is a uniformly distributed random number, and $M_{best}{}^k = \frac{1}{m}\sum_1^m x^k{}_i$ is a control parameter.

This step significantly enhances bat diversity, improving search efficiency and accelerating convergence.

b. Frequency Update:

$f_i = f_{\min} + (f_{\max} - f_{\min})\beta$, where $\beta \in [0,1]$

c. Inertia Weight Update:

$$\omega^k = \omega_{\min} + (\omega_{\max} - \omega_{\min})(\frac{k_{\max} - k}{k_{\max}})$$

d. Velocity Update:

$$v_i^k = \omega^k \times v_i^{k-1} + (x_i^k - x^*) f_i$$

The inertia weight term was introduced by J.C. Bansal et al. (2011) to balance exploration and exploitation. It controls the influence of the previous velocity on the current one.

e. Position Update:

$$x_i^k = x_i^{k-1} + v_i^k.$$

2. Local Search Update:

$$x_i^k = x^*(1 + randn.\left|A_i - \bar{A}\right|).$$

where $\bar{A}$ is the average loudness, and $randn$ is a standard normal random number.

3. At each iteration, evaluate the fitness and update the best solution found.

4. Once the best solution from the BA is found, refine it using the NM simplex method. This involves sorting out the fitness values and selecting the best bat positions.

Note: The number of bats $n+1$ must be greater than the problem dimension n by one. Then, the NM simplex algorithm is applied as outlined in the following steps.

<u>Pseudo-code of HSBA</u>

1: **Initialize** x (position), v (velocity), other parameters for the bat and NM algorithms.

2: **Set** the frequency f_i, loudness A_i and pulse rate r_i for each bat.

3: **While** $k < \max iter$ **do**,

4: **For** $i = 1 : m$, **do**

5: Generate new positions to obtain new solutions. Update positions using

$$\begin{cases} x_i^k = x^* + rand.\left|M^k{}_{best} - x_i^{k-1}\right| \ln(\frac{1}{u}), u < 0.5 \\ x_i^k = x^* - rand.\left|M^k{}_{best} - x_i^{k-1}\right| \ln(\frac{1}{u}), u \geq 0.5 \end{cases}$$

6: Randomize the frequency for each bat using $f_i = f_{\min} + (f_{\max} - f_{\min})\beta$

7: Update the velocity using $\omega^k = \omega_{\min} + (\omega_{\max} - \omega_{\min})(\frac{k_{\max} - k}{k_{\max}})$

$v_i^k = \omega^k v_i^{k-1} + (x_i^k - x^*)f_i$

8: Update the position using $x_i^k = x_i^{k-1} + v_i^k$

9: **If** $rand > r_i^k$,**then**

Update position using $x_i^k = x^*(1 + randn|A_i - \bar{A}|)$,

10: **end if**

11: Calculate new fitness.

12: Update if the solution is improved and not too loud.

13: **If** $rand < A_i^k$ & $f(x_i^k) < f(x^*)$,**then**

14: Replace the position with the new one.

15: Update the r_i^k and A_i^k using $r_i^{k+1} = r_i^0(1 - e^{-\delta k})$ and $A_i^{k+1} = \alpha A_i^k$ respectively.

16: **end if**

17: **end for**

18: call simple NM algorithm

19: **If** $f(x_{NM}) \le f(x_{best})$,**then**

20: $x_{best} = x_{NM}$

21: **end if**

22: **end while**

4. Benchmark Problems and Experimental Setup

To evaluate HSBA, we employed 30 benchmark test functions sourced from Jamil and Yang (2013). These include unimodal, multimodal, separable, non-separable, and discontinuous functions, designed to test various aspects of optimization algorithms. The test suite includes classical functions such as Rastrigin, Griewank, and Rosenbrock.

Each experiment was conducted with 25 independent runs, using consistent initial conditions and stopping criteria. We recorded metrics such as best, worst, median, mean, standard deviation, CPU time, and number of function evaluations (NFE).

Table 1: Test problems for unconstrained optimization

TP	Function name	n	optimal	TP	Function name	n	optimal
TP1	Rastargin	10,30	0	TP16	Beale	2	0
TP2	Ackley	30	0	TP17	Styblinski-Tank	10,30	-39.16599n
TP3	Rosenbrock	30	0	TP18	Xin-She Yang N. 3	30	-1
TP4	Schaffer.2	2	0	TP19	Michalewicz	2	-1.8013
TP5	Schaffer.4	2	0.292579	TP20	Easom	2	-1
TP6	Holder Table	2	-19.2085	TP21	Exponential	30	-1
TP7	Sum of different power	30	0	TP22	Schwefel	30	0
TP8	Rotated Hyber Ellipsoid	30	0	TP23	Branin	2	0.3979
TP9	Sphere	30	0	TP24	Shekel(m=5)	4	-10.1532
TP10	Penalized 1,2	30	0	TP25	Hartmann	3	-3.8628
TP11	Goldstein-Price	2	3	TP26	Shubert	2	-186.7309
TP12	Griewank	10,30	0	TP27	De jong.5(Foxholes)	2	0.998004
TP13	Xin-She Yang N.2	10,30	0	TP28	Perm	2	0
TP14	Zakharov	10,30	0	TP29	Happy cat	10	0
TP15	Quartic	30	0	TP30	Himmelblau	2	0

5. Results and Discussion

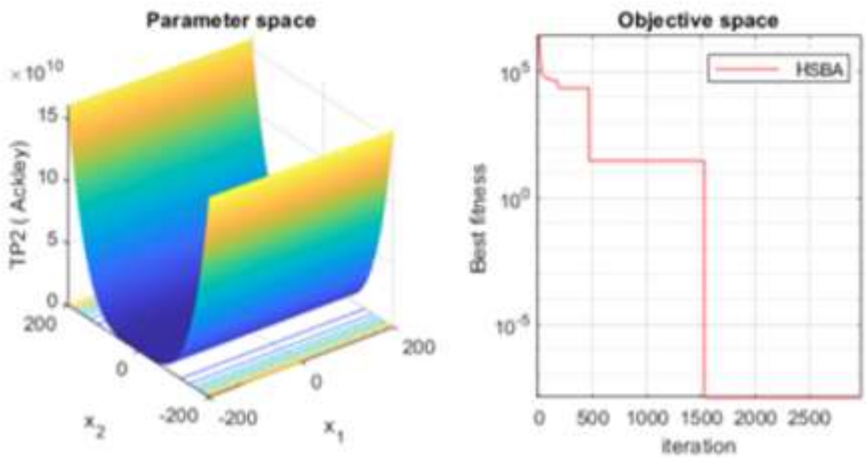
The results show that HSBA consistently outperforms BA and other comparative algorithms such as PSO, DE, and FA across most test problems. HSBA achieves lower average errors and fewer function evaluations, particularly on multimodal and high-dimensional problems. The embedded NM operations clearly enhance local refinement, resulting in faster convergence.

For example, on the 30-dimensional Rastrigin function, HSBA achieved an average value significantly closer to the global optimum compared to BA. This demonstrates the effectiveness of hybridization in balancing exploration and exploitation.

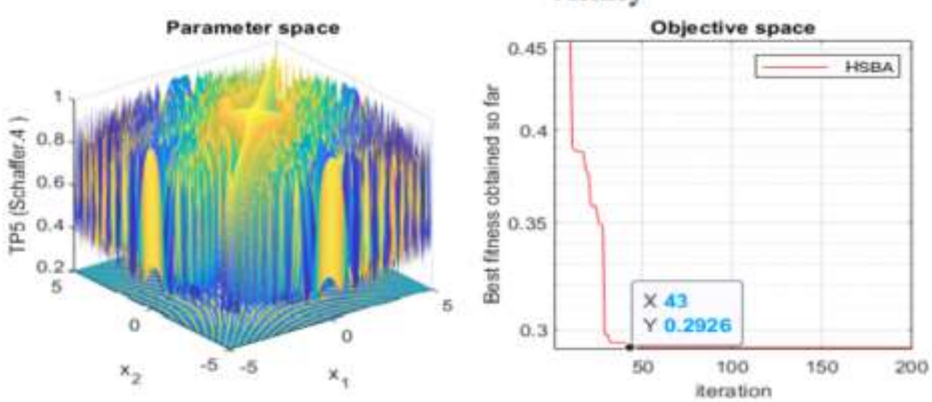
Result of HSBA on 30 unconstrained test functions

TPs	Best	Mean	Std
TP1	9.94E-07, 9.60E-05	2.282E-3, 3.31E-04	6.62E-2, 1.18E-04
TP2	2.87E-05	1.10E-03	8.98E-04
TP3	1.25E-08	6.94E-05	8.10E-05
TP4	0.00E+00	1.02E-08	3.04E-07
TP5	0.292579	2.93E-01	1.93E-04
TP6	-1.92E+01	-1.92E+01	2.01E-05
TP7	6.00E-13	3.22E-11	3.00E-11
TP8	1.95E-05	2.16E-04	1.10E-04
TP9	0.00E+00	4.12E-297	0.00E+00
TP10	7.91E-28 , 1.88E-26	1.40E-14,3.42E-03	8.10E-07,5.09E-2
TP11	3.00E+00	3.00E+00	1.50E-04
TP12	3.02E-10	4.08E-02	4.77E-02
TP13	5.66E-04, 3.51E-12	5.6608E-04, 3.57E-12	1.133E-08, 1.39E-13
TP14	1.339E-07, 2.03E-04	2.4475E-05, 1.45E-03	2.5523E-05, 1.20E-03
TP15	9.17E-07	7.3199E-06	4.865E-06
TP16	6.41E-07	8.14E-05	1.01E-04
TP17	-391.6617, -1.17E+03	-391.5538, -1.17E+03	1.191E-2, 4.78E-02
TP18	1.00E+00	-1.00E+00	5.94E-05
TP19	-1.80E+00	-1.80E+00	7.85E-04
TP20	-1.00E+00	-1.00E+00	3.07E-05
TP21	-1.00E+00	-1.00E+00	2.54E-05
TP22	2.64E-05	1.57E-04	1.11E-04
TP23	3.98E-01	3.98E-01	1.00E-04
TP24	-1.02E+01	-1.02E+01	6.21E-06
TP25	-3.86E+00	-3.86E+00	7.50E-05
TP26	-1.87E+02	-1.87E+02	4.50E-03
TP27	9.98E-01	1.68E+00	6.2e-02
TP28	3.83E-04	8.10E-03	5.70E-03
TP29	8.28E-05	1.60E-03	1.00E-04
TP30	7.50E-06	2.20E-03	2.4E-3

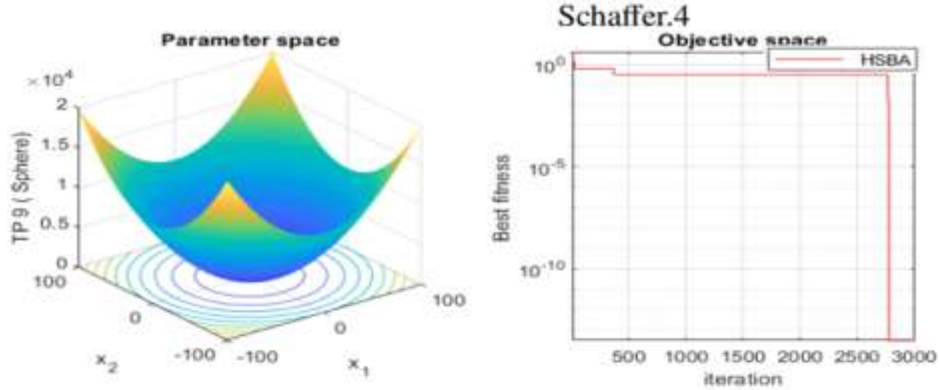
The following figures illustrate the convergence of the HSBA algorithm toward the global optimal solution on various test problems



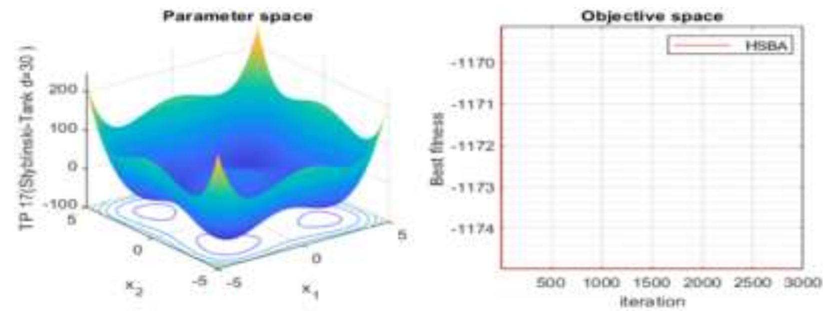
Ackley



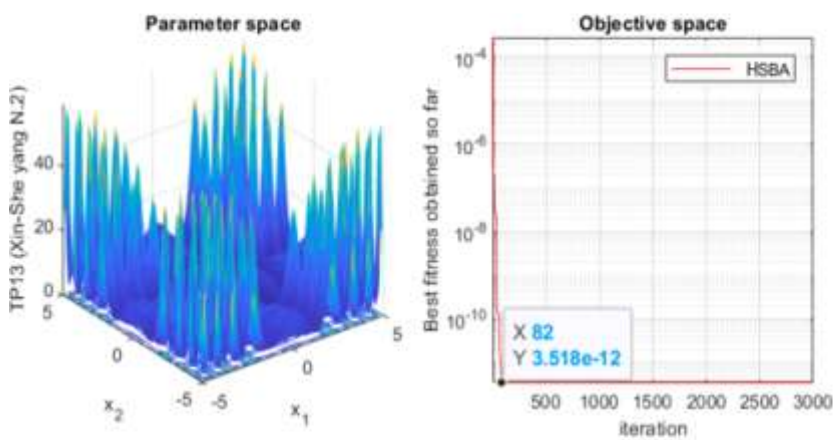
Schaffer.4



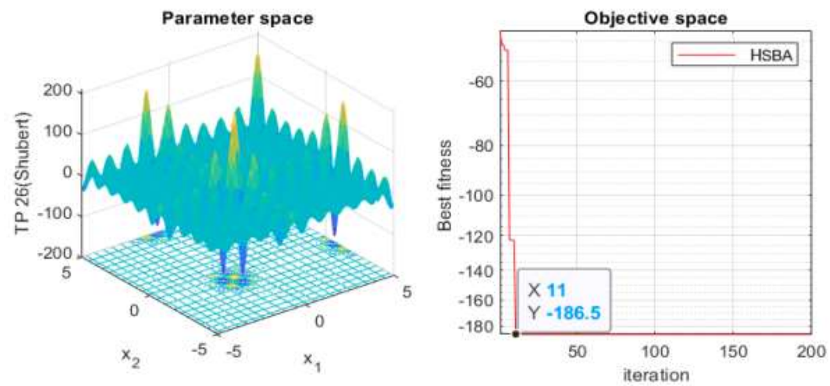
Sphere



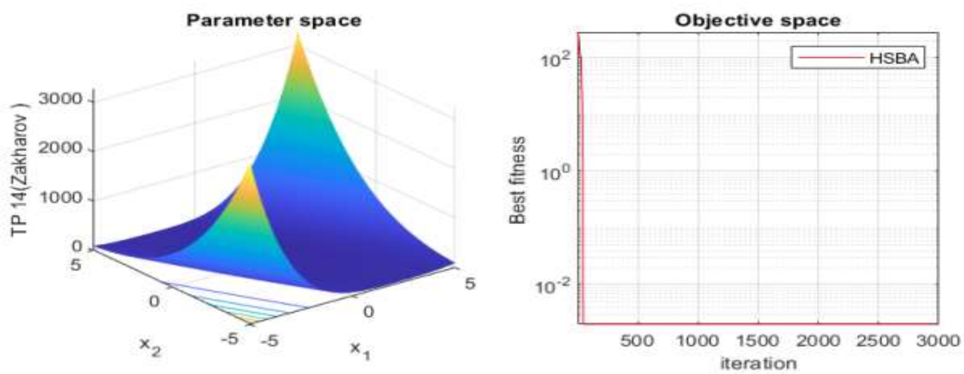
Styblinski-Tank



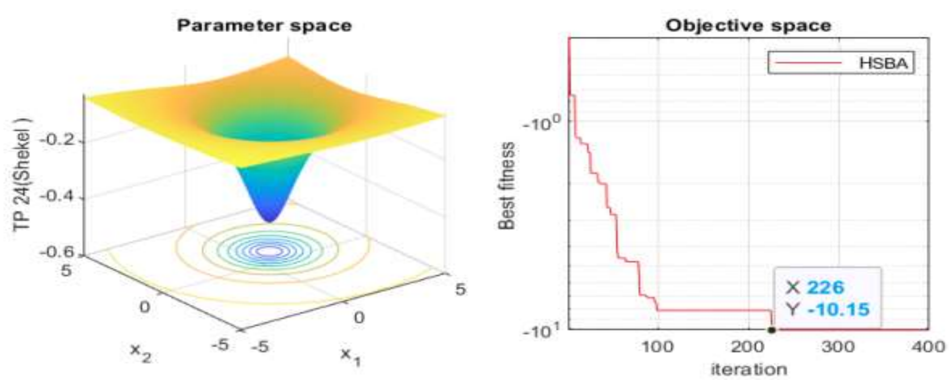
Xin-she yang N.2



Shubert



Zakharov



Shekel

We ran 30 unconstrained test optimization problems to compare our proposed HSBA algorithm against five meta-heuristic algorithms. The following two tables present these algorithms along with the parameters selected for their implementation.

No.	Name	Abbreviation	References
1	Standard Bat Algorithm	BA	X.-S. Yang. A new metaheuristic bat-inspired algorithm. In Nature inspired cooperative strategies for optimization (NICSO 2010), pages 65–74. Springer, 2010.
2	Standard Particle Swarm Optimization Algorithm	PSO	J. Kennedy and R. Eberhart. Particle swarm optimization. In Proceedings of ICNN'95-International Conference on Neural Networks, volume 4, pages 1942–1948. IEEE, 1995
3	Firefly Algorithm	FA	X.-S. Yang. Firefly algorithm, stochastic test functions and design optimization. International journal of bio-inspired computation, 2(2):78–84, 2010
4	Differential Evolution Algorithm	DE	R. Storn and K. Price. Differential evolution-a simple and efficient heuristic for global optimization over continuous spaces. Journal of global optimization.11(4):341-359, 1997
5	Artificial Bee Colony Algorithm	ABC	B. Yuce, M. S. Packianather, E. Mastrocinque, D. T. Pham, and A. Lambiase. Honey bees inspired optimization method: the bees algorithm. Insects, 4(4):646–662, 2013

Meta-heuristic Algorithms used in the comparison

parameter settings	
Population size	m = 40
Maximum number of iterations	$iter_{max} = 100 * dim$
Loudness	A = 0.95
Pulse emission rate	$r = 0.85$
Maximum frequency	$f_{max} = 2$
Minimum frequency	$f_{min} = 0$
Factor updating loudness	$\alpha = 0.9$
Factor updating pulse emission rate	$\gamma = 0.9$

These algorithms are tested on nine benchmark problems: TP1, TP2, TP3, TP9, TP12, TP13, TP14, TP20, and TP22, with a problem dimension of $n = 30$

TPs	BA		PSO		FA		DE		ABC		HSBA	
	mean	Std	mean	Std	mean	Std	mean	Std	mean	Std	mean	Std
TP1	9.2800E+02	8.9000E+02	3.3290E+01	1.0592E+01	2.4400E+02	2.3500E+02	2.2800E+02	1.3300E+01	7.3300E+01	2.2400E+01	**3.31E-04**	1.1800E-04
TP2	2.0000E+01	2.0000E-02	6.0100E-02	6.3200E-02	2.1100E+01	2.1100E+01	1.7700E+00	3.1700E-01	7.1700E+00	1.0300E+00	**1.1E-03**	8.9810E-04
TP3	2.8400E+06	2.9500E+06	1.9670E+01	1.2110E+01	1.1200E+02	1.0100E+02	4.5700E+02	2.2700E+02	5.1800E+02	4.7200E+02	**6.9429E-05**	8.0996E-05
TP9	5.8700E-02	6.5300E-05	1.9000E-03	1.8000E-03	5.1900E+00	5.1400E+00	1.7700E+02	7.1200E+01	1.6300E+02	1.9600E+02	**00E+00**	0.0000E+00
TP12	1.1600E+00	1.1500E+00	4.6800E-01	2.1200E-01	6.6500E-01	6.4100E-01	1.0500E+00	**2.12E-02**	1.0900E+00	1.2300E-01	4.0800E-02	4.7700E-02
TP13	1.5700E-11	1.3100E-11	7.0012E-05	2.3500E-07	1.7000E-04	4.7200E-05	2.4600E-11	1.2000E-12	1.1000E-11	1.9100E-12	**3.5698E-12**	1.3896E-13
TP14	2.7600E+02	2.8200E+02	1.3970E+03	4.3000E+03	1.3200E+04	1.3200E+04	3.7800E+01	8.7400E+00	2.5300E+02	3.1500E+01	**7.0012E-05**	1.2000E-03
TP20	0.0000E+00	0.0000E+00	-4.8313E-29	-4.8313E-25	-3.8100E-30	-3.7300E-31	-2.7600E-175	0.0000E+00	-1.7600E-136	8.7900E-136	00E+00	0.0000E+00
TP22	9.4500E+03	9.5200E+03	1.8420E+03	1.4050E+03	6.7800E+03	6.7500E+03	7.5700E+03	4.4000E+02	2.6400E+03	3.3000E+02	**1.5688E-04**	1.1062E-04

We also compared the performance of the HSBA algorithm against several hybrid metaheuristic algorithms

No.	Name	Abbreviation	References
1	Modified Bat Algorithm	MBA	S. Yılmaz, E. U. Kucuksille, and Y. Cengiz. Modified bat algorithm. Elektronika ir Elektrotechnika, 20(2):71–78, 2014
2	Modified Bat Algorithm with Differential Evolution	MBADE	G. Yildizdan and Ö. K. Baykan. A novel modified bat algorithm hybridizing by differential evolution algorithm. Expert Systems with Applications, 141:112949, 2020.
3	A new Hybrid BA-ABC algorithm	BA-ABC	G. Yildizdan and Ö. K. Baykan. A new hybrid BA-ABC algorithm for global optimization problems. Mathematics, 8(10):1749, 2020
4	Guided Particle Swarm Optimization	GPSO	A. Abdelhalim, K. Nakata, M. El-Alem, and A. Eltawil. Guided particle swarm optimization method to solve general nonlinear optimization problems. Engineering Optimization, 50(4):568–583, 2018

TPs		BA-ABC	HSBA
TP1	best	**0**	9.94e-07
	mean	**0**	2.28e-03
	std	0	6.62e-04
TP2	best	1.2e+02	**4.7855e-05**
	mean	1.4e+02	**1.3e-03**
	std	1.12e-04	7.8766e-04
TP3	best	1.23e-02	**5.766e-08**
	mean	3.37e-02	**2.1e-03**
	std	5.13e-01	3e-03
TP9	best	**0.98e-17**	1.4746e-07
	mean	**2.86e-17**	3.9397e-04
	std	1.73e-17	3.1533e-04
TP12	best	2.21e-03	**3.0205E-10**
	mean	8.80e-03	**4.08e-02**
	std	8.16e-03	4.77e-02
TP13	best	5.61e-04	**5.6607e-04**
	mean	5.66e-04	**5.5201e-04**
	std	1.07e-16	2.3173e-04
TP14	best	1.20e-04	**7.4077e-06**
	mean	1.41e-04	**5.5201e-04**
	std	8.68e-05	2.3173e-04
TP20	best	-1.00e+00	-1.00e+00
	mean	-1.00e+00	-1.00e+00
	std	1.05e-04	1.534e-06
TP22	best	1.11e-04	**2.6393e-05**
	mean	1.27e-04	**1.5688e-04**
	std	8.86e-05	1.1062e-04

HSBA and MBA on 7 test problems (n = 10)

TPs		MBA	HSBA
TP1	best	1.46e+01	**3.8249e-05**
	mean	2.49e+02	2.6e-03
	std	4.35e-00	2.7e-03
TP2	best	3.61e-02	**4.7855e-05**
	mean	1.67e-01	1.3e-03
	std	3.60e-02	7.8766e-04
TP3	best	7.4e-00	**5.766e-08**
	mean	1.03e+01	2.1e-03
	std	1.94e-00	3e-03
TP9	best	3.73e-03	**1.4746e-07**
	mean	8.8e-03	3.9397e-04
	std	3.34e-03	3.1533e-04
TP12	best	2.05e-00	**3.0205e-10**
	mean	8.12e-00	4.08e-02
	std	5.39e-00	4.77e-02
TP20	best	-9.92e-01	**-1.00e+00**
	mean	-9.8e-01	-1.00e+00
	std	7.64e-03	1.534e-06
TP22	best	7.01e+02	**2.6393e-05**
	mean	1.01e+03	1.5688e-04
	std	2.83e+02	1.1062e-04

HSBA, MBADE, and BA on 9 test problems (n = 10)

Comparison of GPSO and HSBA

TPs	GPSO	HSBA
TP1	0.68	**9.6e-05**
TP2	**7.99e-15**	2.87e-05
TP3	6.31	**1.2458e-08**
TP4	0	0
TP9	0	0
TP12	0	3.02e-10
TP16	0	6.408e-07

Average error Comparison of GPSO and HSBA

Average error	TP3	TP11	TP14	TP23	TP25	TP26
GPSO	0.0009	5e-05	0.0003	0.0012	0.0012	7.68e-07
HSBA	**1.45e-07**	**6.4e-07**	**0**	**0**	2e-05	**1.339e-7**

6. Conclusion and Future Work

This study introduces HSBA, a novel hybrid optimization algorithm that combines the global search capabilities of the Bat Algorithm with the local refinement strengths of the Nelder–Mead simplex method. Comprehensive testing on standard benchmark functions confirms the superiority of HSBA in terms of convergence rate, solution quality, and computational efficiency.
Future work will explore extending HSBA to multi-objective optimization problems, integrating constraint-handling techniques, and conducting theoretical convergence analysis.

References

[1] A. Abdelhalim, K. Nakata, M. El-Alem, and A. Eltawil, Guided particle swarm optimization method to solve general nonlinear optimization problems, *Engineering Optimization*, **50** (4) (2018), 568–583.
https://doi.org/10.1080/0305215x.2017.1340945

[2] J. A. Nelder and R. Mead, A simplex method for function minimization, *The Computer Journal*, **7** (4) (1965), 308–313. https://doi.org/10.1093/comjnl/7.4.308

[3] R. Storn and K. Price, Differential evolution-a simple and efficient heuristic for global optimization over continuous spaces, *Journal of Global Optimization*, **11** (4) (1997), 341-359. https://doi.org/10.1023/a:1008202821328

[4] X.-S. Yang, Firefly algorithm, stochastic test functions and design optimization, *International Journal of Bio-Inspired Computation*, **2** (2) (2010), 78–84.
https://doi.org/10.1504/ijbic.2010.032124

[5] X.-S. Yang, A new metaheuristic bat-inspired algorithm, in: *Nature inspired cooperative strategies for optimization* (NICSO 2010), pages 65–74. Springer, 2010.
https://doi.org/10.1007/978-3-642-12538-6_6

[6] S. Yılmaz, E. U. Kucuksille, and Y. Cengiz, Modified bat algorithm, *Elektronika i Elektrotechnika*, **20** (2) (2014), 71–78. https://doi.org/10.5755/j01.eee.20.2.4762

[7] G. Yildizdan and Ö. K. Baykan, A novel modified bat algorithm hybridizing by differential evolution algorithm, *Expert Systems with Applications*, **141** (2020), 112949. https://doi.org/10.1016/j.eswa.2019.112949

[8] G. Yildizdan and Ö. K. Baykan, A new hybrid BA-ABC algorithm for global optimization problems, *Mathematics*, **8** (10) (2020), 1749. https://doi.org/10.3390/math8101749

[9] B. Yuce, M. S. Packianather, E. Mastrocinque, D. T. Pham, and A. Lambiase, Honey bees inspired optimization method: the bees algorithm, *Insects*, **4** (4) (2013), 646–662. https://doi.org/10.3390/insects4040646

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