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Ranks in Elliptic Curves $y^2 = x^3 \pm Ax$

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Abstract

We appoint that E_{-2p} and E_{-4p} are elliptic curves $y^2 = x^3 - 2px$ and $y^2 = x^3 - 4px$ with prime p then, we will investigate the ranks of these curves and suggest examples of it.

Mathematics Subject Classification: 11A41, 11G05

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1 Introduction

Usually, generalized rank 1 is induced often in both E_{-2p} and E_{-4p} when prime is the form $p = Hu^4 + Iu^2v^2 + Kv^4$ but there also educed rank 1 in other forms. In this article, we shall calculate the ranks of E_{-2p} and E_{-4p} with other form $p(\neq Hu^4 + Iu^2v^2 + Kv^4)$.

First of all, it is necessary to see some notations in [11].

Assume that E is an elliptic curve of the form $y^2 = x^3 + ax^2 + bx$ and Γ is the set of rational points on E.

From $Mordell's$ Theorem Γ is a finitely generated abelian group.

In addition, we gain $\Gamma \cong E(Q)_{tors} \oplus Z^r$ with torsion subgroup $E(Q)_{tors}$ and Mordell-Weil rank r.

Let $Q^\times$ be the multiplicative group which is comprised of non-zero rational numbers.

Take $Q^{\times 2}$ as the subgroup of squares of elements of $Q^\times$.

Assign α as a homomorphism $\alpha: \Gamma \to Q^\times / Q^{\times 2}$ that satisfies

$$\alpha(O) = 1(mod\ Q^{\times 2}) \text{ and}$$

$$\alpha(0, 0) = b(mod\ Q^{\times 2}) \text{ and}$$

$$\alpha(x, y) = x(mod\ Q^{\times 2}).$$

Here O is point at infinity and x is non-zero.
Let $\bar{E}$ be the curve

$$y^2 = x(x^2 - 2ax + a^2 - 4b) \text{ and}$$

$\bar{\Gamma}$ be the set of rational points on curve $\bar{E}$.
Set $\bar{\alpha}$ is a homomorphism such that $\bar{\alpha}: \bar{\Gamma} \to Q^{\times}/Q^{\times 2}$ with

$$\bar{\alpha}(O) = 1(mod\ Q^{\times 2}) \text{ and}$$

$$\bar{\alpha}(0, 0) = a^2 - 4b(mod\ Q^{\times 2}) \text{ and}$$

$$\bar{\alpha}(x, y) = x(mod\ Q^{\times 2}).$$

Here, O denotes point at infinity and $x \neq 0$.
Define

$$N^2 = b_1 M^4 + aM^2 e^2 + b_2 e^4$$

as relating equation for Γ where b_1 and b_2 are divisors of b as $b = b_1 b_2$ with $b_1 \not\equiv 1, b(mod\ Q^{\times 2})$.
Assume that

$$N^2 = b_1 M^4 - 2aM^2 e^2 + b_2 e^4$$

is relating equation for $\bar{\Gamma}$ where b_1 and b_2 are divisors of $a^2 - 4b$ as $b_1 b_2 = a^2 - 4b$ with $b_1 \not\equiv 1, a^2 - 4b(mod\ Q^{\times 2})$.
Denote the triple $(M, e\ N)$ as an integral solution of above equations with $1 = (M, N) = (M, e) = (N, e) = (b_1, e) = (b_2, M)$ and $M \neq 0, e \neq 0$.
We confront to

$$2^r = \frac{\#\alpha(\Gamma)\#\bar{\alpha}(\Gamma)}{4} \text{ with rank } r \text{ of } E.$$

Next, we define the notations as follows:

$w.i.t.u.1$: with integers t and u and $(t, u) = 1$.

$$r4.2\colon 2^r = \frac{4\cdot 2}{4}([4]).$$

2 Consideration in Some Results

In section 2, we will submit several results of ranks in curves $y^2 = x^3 \pm Ax$.

Lemma 2.1. Let p and q be two distinct odd primes as $p = 30u^4 - 10u^2v^2 + 9v^4$ and $q = 30u^4 - 10u^2v^2 + 11v^4\ w.i.u.v.1$ and $p \equiv 13(mod\ 16)$ and $q \equiv 15(mod\ 16)$ in elliptic curve $y^2 = x^3 + pqx$ then, we conclude that

$$rank(E_{(30u^4-10u^2v^2+9v^4)(30u^4-10u^2v^2+11v^4)}(Q)) = 1.$$

Proo$f.$ See [5]. □

Lemma 2.2. If E_{pq} is gotten as an elliptic curve $y^2 = x^3 + pqx$ where different odd primes p and q are $p = u^4 + 12u^2v^2 - 40v^4$ and $q = u^4 + 12u^2v^2 - 38v^4\ w.i.u.v.1$ and $p \equiv 5(mod\ 16)$ and $q \equiv 7(mod\ 16)$ then, we obtain that

$$rank(E_{(u^4+12u^2v^2-40v^4)(u^4+12u^2v^2-38v^4)}(Q)) = 1.$$

Proo$f.$ See [5]. □

Lemma 2.3. Define E_{-pq} as an elliptic curve $y^2 = x^3 - pqx$ where p and q are distinct odd primes such that $p = 4u^4 - 20u^2v^2 + 27v^4$ and $q = 4u^4 - 20u^2v^2 + 23v^4\ w.i.u.v.1$ and $p \equiv 11(mod\ 16)$ and $q \equiv 7(mod\ 16)$ then, we gain

$$rank(E_{-(4u^4-20u^2v^2+27v^4)(4u^4-20u^2v^2+23v^4)}(Q)) = 2.$$

Proo$f.$ See [5]. □

Lemma 2.4. Assign distinct odd primes p and q as $p = 50u^4 + 20u^2v^2 + v^4$ and $q = 50u^4 + 20u^2v^2 + 3v^4\ w.i.u.v.1$, $p \equiv 1(mod\ 8)$ and $q \equiv 3(mod\ 16)$ in $y^2 = x^3 + pqx$ then,

$$rank(E_{(50u^4+20u^2v^2+v^4)(50u^4+20u^2v^2+3v^4)}(Q)) \geq 2 \text{ is given.}$$

Proo$f.$ See [5]. □

Lemma 2.5. Take distinct odd primes p and q as $p = u^4 - 10u^2v^2 + 12v^4$ and $q = u^4 - 10u^2v^2 + 14v^4\ w.i.u.v.1$, $p \equiv 3(mod\ 16)$ and $q \equiv 5(mod\ 16)$ in $y^2 = x^3 + pqx$ then,

$$rank(E_{(u^4-10u^2v^2+12v^4)(u^4-10u^2v^2+14v^4)}(Q)) = 1 \text{ is deduced.}$$

Proo$f$. See [4]. □

Lemma 2.6. Let p and q be distinct odd primes as $p \equiv 5(mod\ 16)$ and $q \equiv 5(mod\ 16)$ in$y^2 = x^3 + 2pqx$ then,

$$rank(E_{2(16k+5)(16k'+5)}(Q)) = 0 \text{ is educed.}$$

Proo$f$. See [6]. □

3 Forms E_{-2p} and E_{-4p}

In E_{-2p}, there deduced much results of generalized rank 1. The most case is $p = Hu^4 + Iu^2v^2 + Kv^4 \cdots\cdots (GG)$ but rank 1 is also gotten in other forms. And in E_{-4p}, it is similar tendency as in E_{-2p}. In any form(that induces rank 1) the primes p are the forms $p \equiv 5, 3, 11, 13(mod\ 16) \cdots\cdots (S)$ in curve E_{-2p}. In E_{-2p}, except (GG), there exist various forms p which yields rank 1. In some cases , 2 variables and 5 terms and there is case 3 variables and 6 terms and there is case that has more than 4 variables and 8 terms. Now we shall numerate ranks of E_{-2p} and E_{-4p} where p is the form ($\neq Hu^4 + Iu^2v^2 + Kv^4$).

Theorem 3.1. (1). Set E_{-2p} as an elliptic curve $y^2 = x^3 - 2px$ where an odd prime$p$ is the form$p = 6t^4 - 8t^3u - 8tu^3 + 16t^2u^2 + 3u^4\ w.i.t.u.1$ and $p \equiv 3(mod\ 16)$ then, we are faced with

$$rank(E_{(30u^4-10u^2v^2+9v^4)(30u^4-10u^2v^2+11v^4)}(Q))$$

$$= rank(E_{-2(6t^4-8t^3u-8tu^3+16t^2u^2+3u^4)}(Q)).$$

(2). If an odd prime p is assigned as $p = 18t^4 - 8t^3u - 8tu^3 + 36t^2u^2 + 11u^4\ w.i.t.u.1$ and $p \equiv 11(mod\ 16)$ in E_{-2p} then, we are confronted with

$$rank(E_{(u^4+12u^2v^2-40v^4)(u^4+12u^2v^2-38v^4)}(Q))$$

$$= rank(E_{-2(18t^4-8t^3u-8tu^3+36t^2u^2+11u^4)}(Q)).$$

(3). We suppose that prime p is $p = 18t^4 - 8t^3u - 8tu^3 + 84t^2u^2 + 83u^4\ w.i.t.u.1$ and $p \equiv 3(mod\ 16)$ in E_{-2p} then, we gain

$$rank(E_{-(4u^4-20u^2v^2+27v^4)(4u^4-20u^2v^2+23v^4)}(Q))$$

$$> rank(E_{-2(18t^4-8t^3u-8tu^3+84t^2u^2+83u^4)}(Q)).$$

(4). Denote prime p as $p = 6t^4 + 8t^3u + 8tu^3 + 56t^2u^2 + 123u^4\ w.i.t.u.1$ and $p \equiv 11(mod\ 16)$ in E_{-2p} then, we conclude that

$$rank(E_{(50u^4+20u^2v^2+v^4)(50u^4+20u^2v^2+3v^4)}(Q))$$

$$> rank(E_{-2(6t^4+8t^3u+8tu^3+56t^2u^2+123u^4)}(Q)).$$

(5). Let prime p be $p = 578t^4 - 8t^3u - 8tu^3 + 156t^2u^2 + 11u^4\ w.i.t.u.1$ and $p \equiv 11(mod\ 16)$ in E_{-2p} then, we have that

$$rank(E_{(u^4-10u^2v^2+12v^4)(u^4-10u^2v^2+14v^4)}(Q))$$

$$= rank(E_{-2(578t^4-8t^3u-8tu^3+156t^2u^2+11u^4)}(Q)).$$

(6). Assume that odd prime p is $p = 1060t^4 - 1024t^3u - 64tu^3 + 420t^2u^2 + 13u^4\ w.i.t.u.1$ and $p \equiv 13(mod\ 16)$ in E_{-4p} then, we say that

$$rank(E_{2(16k+5)(16k'+5)}(Q))$$

$$< rank(E_{-4(1060t^4-1024t^3u-64tu^3+420t^2u^2+13u^4)}(Q)).$$

(7). We appoint that an odd prime p is $p = 65t^4 - 128t^3u - 32tu^3 + 140t^2u^2 + 488u^4\ w.i.t.u.1$ and $p \equiv 5(mod\ 16)$ in E_{-4p} then, we acquire that

$$rank(E_{2(16k+5)(16k'+5)}(Q))$$

$$< rank(E_{-4(65t^4-128t^3u-32tu^3+140t^2u^2+488u^4)}(Q)).$$

Proo$f$. (1). There is remained only equation

$$4) N^2 = -2M^4 + (6t^4 - 8t^3u - 8tu^3 + 16t^2u^2 + 3u^4)e^4 \text{ for } \Gamma$$

which is necessary to find the solution from [3].
In numerical values

$$-2M^4 + 6t^4e^4 \text{ and } -2M^4 + 3u^4e^4$$

we take e as 1.
For getting the squares t^4 and u^4 the integer M must have the variables t and u.
Thus, we select the value M as $t - u$.
Now we note the terms

$$-8t^3u \text{ and } -8tu^3.$$

To obtain square in resultant, these should be eliminated.
From $-2(t-u)^4$ we attain that

$$8t^3u + 8tu^3 - 8t^3u - 8tu^3.$$

Whence, the terms for t^3u and tu^3 are eliminated.
Next, we got the squares $4t^4$ and u^4.
Finally, we have that

$$-12t^2u^2 + 16t^2u^2 = 4t^2u^2.$$

Consequently, there are left the terms

$$4t^4 \text{ and } u^4 \text{ and } 4t^2u^2.$$

So we get the resultant

$$4t^4 + 4t^2u^2 + u^4.$$

Wherefore, the value N is gotten as

$$N = 2t^2 + u^2.$$

Whence, the triple

$$(t-u, 1, 2t^2 + u^2)$$

is deduced as the solution of equation.
Thereby, we take conclusions as

$$\#\alpha(\Gamma) = 4 \text{ and } r4.2.$$

And it induces the result

$$rank(E_{-2(6t^4-8t^3u-8tu^3+16t^2u^2+3u^4)}(Q)) = 1.$$

Now due to lemma 2.1 the proof is done.
(2). If we find the solution of equation

$$4) N^2 = -2M^4 + (18t^4 - 8t^3u - 8tu^3 + 36t^2u^2 + 11u^4)e^4 \text{ for } \Gamma$$

then, computing the rank is sufficient owing to [3].
In arithmetical values

$$-2M^4 + 18t^4e^4 \text{ and } -2M^4 + 11u^4e^4$$

we assume that $e = 1$.
To attain the squares

$$16t^4 \text{ and } 9u^4$$

the value M should possess the variables t and u.
Accordingly, we set M as $t - u$.
From $-2(t-u)^4$ we get that

$$8t^3u + 8tu^3 - 8t^3u - 8tu^3.$$

So the terms for t^3u and tu^3 are crossed out.
And we took two squares $16t^4$ and $9u^4$.
Now we confront to

$$-12t^2u^2 + 36t^2u^2 = 24t^2u^2.$$

Resultantly, the terms

$$16t^4 \text{ and } 9u^4 \text{ and } 24t^2u^2$$

are remained.
Hence, we gain

$$16t^4 + 24t^2u^2 + 9u^4.$$

So it gives that

$$N = 4t^2 + 3u^2.$$

Therefore, we acquire the triple

$$(t-u, 1, 4t^2 + 3u^2)$$

as the solution of equation.
Eventually, we reach that

$$\#\alpha(\Gamma) = 4 \text{ and } r4.2.$$

Consequentially, we arrive at the result

$$rank(E_{-2(18t^4-8t^3u-8tu^3+36t^2u^2+11u^4)}(Q)) = 1.$$

On this account, we achieved the proof by lemma 2.2.
(3). There is only left

$$4)N^2 = -2M^4 + (18t^4 - 8t^3u - 8tu^3 + 84t^2u^2 + 83u^4)e^4 \text{ for } \Gamma$$

that requires to search the solution because of [3].
Replace $t-u$ and 1 into M and e respectively then, we are faced with

$$-2(t-u)^4 + 18t^4 - 8t^3u - 8tu^3 + 84t^2u^2 + 83u^4.$$

$$= -2t^4 + 8t^3u + 8tu^3 - 12t^2u^2 - 2u^4$$

$$+18t^4 - 8t^3u - 8tu^3 + 84t^2u^2 + 83u^4$$

$$= 16t^4 + 72t^2u^2 + 81u^4$$

Henceforth, there comes that $N = 4t^2 + 9u^2$.
Whence, we say that $\#\alpha(\Gamma) = 4$.
So the result

$$rank(E_{-2(18t^4-8t^3u-8tu^3+84t^2u^2+83u^4)}(Q)) = 1$$

is derived on account of $r4.2$.
For this reason, we complete the proof by lemma 2.3.
(4). Due to [3] it is sufficient that we only look for the solution of next equation for Γ:

$$4)N^2 = -2M^4 + (6t^4 + 8t^3u + 8tu^3 + 56t^2u^2 + 123u^4)e^4.$$

Substitute $t+u$ and 1 into M and e then, we take that

$$-2(t+u)^4 + 6t^4 + 8t^3u + 8tu^3 + 56t^2u^2 + 123u^4.$$

$$= -2t^4 - 8t^3u - 8tu^3 - 12t^2u^2 - 2u^4$$

$$+6t^4 + 8t^3u + 8tu^3 + 56t^2u^2 + 123u^4$$

$$= 4t^4 + 44t^2u^2 + 121u^4.$$

It shows that $N = 2t^2 + 11u^2$.
So we take the conclusion $\#\alpha(\Gamma) = 4$.
On this account, we face that

$$rank(E_{-2(6t^4+8t^3u+8tu^3+56t^2u^2+123u^4)}(Q)) = 1$$

owing to $r4.2$.
Consequently, lemma 2.4 finishes the proof.
(5). It is enough that we only search the solution of equation

$$4) N^2 = -2M^4 + (578t^4 - 8t^3u - 8tu^3 + 156t^2u^2 + 11u^4)e^4$$

for Γ owing to [3].
Assign M and e as $t - u$ and 1 then, we acquire the computation

$$-2(t-u)^4 + 578t^4 - 8t^3u - 8tu^3 + 156t^2u^2 + 11u^4.$$

$$= -2t^4 + 8t^3u + 8tu^3 - 12t^2u^2 - 2u^4$$

$$+578t^4 - 8t^3u - 8tu^3 + 156t^2u^2 + 11u^4$$

$$= 576t^4 + 144t^2u^2 + 9u^4$$

Henceforth, the integer N is derived as $24t^2 + 3u^2$.
It yields that $\#\alpha(\Gamma) = 4$, thus we attain the result

$$rank(E_{-2(578t^4-8t^3u-8tu^3+156t^2u^2+11u^4)}(Q)) = 1$$

since we have $r4.2$.
On that account, lemma 2.5 accomplishes the proof.
(6). The only equation is

$$6) N^2 = -4M^4 + (1060t^4 - 1024t^3u - 64tu^3 + 420t^2u^2 + 13u^4)e^4 \text{ for } \Gamma$$

that is needed to find the solution because of [8].
First, we ought to consider two arithmetical values

$$-4M^4 + 1060t^4e^4 \text{ and } -4M^4 + 13u^4e^4.$$

If M has variable u and e is gotten as 1 then, we can obtain a square $9u^4$ in above second arithmetical value.

Whereas first one is different.

It should be needed to treat more.

1056 is not a square, hence we should regard other value except t.

Because there is coefficient -4 of M^4, if $2t$ comprises the component of integer M then, we are faced with

$$-4 \cdot 16t^4 + 1060t^4 = 996t^4.$$

This is not a square, thus we consider other value.

If we assume that $3t$ is component of M then, we are confronted with

$$-4 \cdot 81t^4 + 1060t^4 = 736t^4.$$

It is not also a square, therefore we treat other value.

Suppose that $4t$ consists of the integer M then, there comes

$$-4 \cdot 256t^4 + 1060t^4 = 36t^4.$$

We took the square, thus we appoint that M and e as $4t - u$ and 1 respectively.

Next, from the calculation

$$1024t^3u + 64tu^3 - 1024t^3u - 64tu^3$$

the terms for t^3u and tu^3 are eliminated.

And there is remained numerical value

$$420t^2u^2 - 384t^2u^2.$$

Wherefore, we got the computation

$$36t^4 + 36t^2u^2 + 9u^4.$$

Thereby, we reach that

$$N = 6t^2 + 3u^2.$$

As a result, the solution is induced as

$$(4t - u, 1, 6t^2 + 3u^2).$$

So we arrive at that $\#\alpha(\Gamma) = 4$.

Resultantly it gives that

$$rank(E_{-4(1060t^4-1024t^3u-64tu^3+420t^2u^2+13u^4)}(Q)) = 1$$

from $r4.2$

Finally, lemma 2.6 completes the proof.

(7). If we look for the solution of relating equation

$$6) N^2 = -4M^4 + (65t^4 - 128t^3u - 32tu^3 + 140t^2u^2 + 488u^4)e^4 \text{ for } \Gamma$$

then, calculating the rank is done owing to [2].

Take M and e as $2t - u$ and 1 then, we acquire that

$$-4(2t-u)^4 + 65t^4 - 128t^3u - 32tu^3 + 140t^2u^2 + 488u^4.$$

$$= -64t^4 + 128t^3u + 32tu^3 - 96t^2u^2 - 4u^4$$

$$+65t^4 - 128t^3u - 32tu^3 + 140t^2u^2 + 488u^4$$

$$= t^4 + 44t^2u^2 + 484u^4.$$

It shows that $N = t^2 + 22u^2$.

Accordingly, there comes that $\#\alpha(\Gamma) = 4$.

And it suggests that

$$rank(E_{-4(65t^4-128t^3u-32tu^3+140t^2u^2+488u^4)}(Q)) = 1.$$

Now the proof is completed by lemma 2.6. □

When treating the rank of curves E_{-2p}, E_{-4p} with the prime as the form $p = At^4 + Bt^3u + Ctu^3 + Dt^2u^2 + Fu^4$, usually the value of M which is a part of solution is gotten as $t \pm u$. Theorem 3.1(1), (2), (3), (4), (5) are up to this case. We can also confront to $4t \pm u$ in several cases. Theorem 3.1(6) is that case. In relative the case that $M = 2t - u$ is emerged less than $M = t \pm u$, $4t \pm u$. This is treatment of calculation.

Remark 3.2. In above, the coefficients of components of prime p are numbers of places from 1 to 4. In many results of generalized rank 1 in curves E_{-2p} or E_{-4p} numbers of places are exist in this range. But there is the prime whose coefficient of components has 5 numbers of places.

Remark 3.3. In [7], the author showed that ranks of curves $E_{3p}: y^2 = x^3 + 3px$, $E_{-2p}: y^2 = x^3 - 2px$, $E_{-pq}: y^2 = x^3 - pqx$, $E_{2p}: y^2 = x^3 + 2px$. Preceding three curves take rank at least 2 whereas last curve has rank 2. There is possibility that rank became 3 in curves E_{3p}, E_{-2p} and for curve E_{-pq} rank can become until 4. But E_{2p} has rank 2.

Remark 3.4. In [9], the author treated rank of curve $E_{-2p}: y^2 = x^3 - 2px$ where primes is consisted of 14 variables and 105 terms$\cdots\cdots(YT)$. The composition of prime is complex but the rank is also 1. If we force to mention the form of it then, it is the prime that are composed of square terms. In [10], the author considered the rank of $y^2 = x^3 - 2px$ where prime is the form $p = At^4 + Bt^3u + Ctu^3 + Du^4 \cdots\cdots (SW)$. This form is different from general form of prime $p = Hu^4 + Iu^2v^2 + Kv^4$ or $p = Hu^8 + Iu^4v^4 + Kv^8$ that is emerged often in generalized rank 1 in elliptic curve $y^2 = x^3 - 2px$. The form (YT) is a kind of prime which were comprised of squares meanwhile for form (SW) it is different. Even if there are square terms we confront to non-square terms Bt^3u and Ctu^3, thus if we should conclude then, (SW) is a prime consisted of non-squares. In elliptic curve of the form $E_{-2p}: y^2 = x^3 - 2px$ there derived various forms of primes that induces rank 1.

4 Examples

In section 4, we suggest examples of preceding theorems.

We can check the primality in [1].

Examples (p, t, u) from theorem 3.1(1) to (7) are the followings:

(83, 2, 1), (211571, 14, 1);

(22859, 6, 1), (364523, 12, 1);

(24659, 6, 1), (136163, 6, 5);

(523, 2, 1), (32443, 8, 1);

(9803, 2, 1), (2373323, 8, 1);

(5101, 2, 3), (10333, 2, 1);

(3461, 3, 1), (344693, 9, 1), (798373, 11, 1).

In above, ; differentiate the theorems.

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