International Journal of Contemporary Mathematical Sciences
Vol. 16, 2021, no. 4, 157 - 160
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/ijcms.2021.91612

Playing the Standard Backgammon with Entangled Numbers of Two Dice

Elham Mehdi-Nezhad

Department of Mathematics and Applied Mathematics
North-West University, Mafikeng Campus, Private Bag X 2046
Mmabatho 2735, South Africa

Amir M. Rahimi*

School of Mathematics, Institute for Research in Fundamental Sciences (IPM)
P.O. Box 19395-5746, Tehran, Iran

Abstract

We introduce the notion of a backgammon variant and call it EN-gammon which follows exactly all the rules of the standard backgammon with the exception that both players using the entangled numbers of the dice for their turns, respectively. That is, the player who rolls the six-sided dice, plays with the face-up numbers showing on the dice and the opponent plays with the face-down numbers of the same rolling of those dice at his/her turn, respectively. In this game, only one of the players (by convention) rolls the dice till end of the game. We will also discuss some probabilistic aspects related to the numbers that occur on the dice in comparison with rolling the dice by each player separately turn by turn.

Mathematics Subject Classification: 90C02, 68W02, 68T02, 68N02, 60G02, 91A02

*The research of the second author was in part supported by grant no. 1400130011 from IPM.

Keywords: Entangled numbers of an n-sided die, six-sided dice, standard backgammon, EN-gammon

Introduction and Main Results

The main purpose of this note is to introduce the notion of a *backgammon variant* and call it *EN-gammon* which follows exactly all *the rules of the standard backgammon* with the exception that both players using the *entangled numbers* of the dice for their turns, respectively. That is, the player who rolls the six-sided dice, plays with the *face-up numbers* showing on the dice and the opponent plays with the *face-down numbers* of the same rolling of those dice at his/her turn, respectively.

Definition. For any fixed even integer $n \geq 4$, the entangled numbers of an n-sided die are defined to be the pairs

$$(1, n), (2, n-1), (3, n-2), \ldots, (n/2, n-(n/2)+1).$$

Hereafter, we focus only on *six-sided dice* and assume that the reader is familiar with the rules of the standard backgammon (for *the rules of the game and other backgammon-related articles and information,* see, for example, Backgammon Galore and [2]). Thus, according to the above definition, the *entangled numbers on a six-sided die* are as follows:

$$(1, 6), (2, 5), \text{and} (3, 4).$$

Consequently, rolling two dice, if the face-up numbers are (a, b), then the *corresponding entangled numbers* (face-down numbers) are (c, d), where $c = 7 - a$ and $d = 7 - b$ with $1 \leq a, b \leq 6$.

We now explain the process of playing the EN-gammon and discuss some differences between the standard backgammon and EN-gammon.

The game is between Alice and Bob with all the rules and moves similar to the standard backgammon. By *convention*, only one of the players rolling the dice until end of the game. Suppose that Alice rolls the dice and start the game. In this case, she moves her checkers according to the numbers (a, b) that showing on the dice. Then Bob moves his checkers according to the entangled numbers of a and b which are $7-a$ and $7-b$, respectively. Now, again Alice rolls the dice and moves her checkers according to the face-up numbers that showing on the dice and then Bob moves his checkers according to the corresponding entangled face-down numbers. The game continues this way until end of the game according to all the rules of the standard backgammon.

Remark. In [1], Rahimi (the second author) introduced the notion of the "hyper dice backgammon of finite size"" (or HD-gammon for short) as a generalize form of the standard backgammon. Clearly, from the above definition, it is natural to extend the notion of the EN-gammon to playing the HD-gammon with entangled numbers of finitely many finite-sided dice (or hyper EN-gammon of finite size for short) which would be challenging and attractive to the interested researchers such as mathematicians, computer scientists, artificial intelligence (AI) professionals, and game theorists.

We now discuss some specific differences related to the numbers that occur on the dice with one rolling or two separate consecutive rollings.

1. When dice rolling a *double*, then its entangled pair is obviously a double. For example, $(6,6)$, or $(5,5)$, or $(4,4)$ for Alice provides the entangled pair $(1,1)$, or $(2,2)$, or $(3,3)$, respectively, for Bob simultaneously. Similarly, $(1,1)$, or $(2,2)$, or $(3,3)$ for Alice provides the entangled pair $(6,6)$, or $(5,5)$, or $(4,4)$, respectively, for Bob simultaneously. Note that this only happens in EN-gammon to have a double face-up and face-down numbers at the same time with a single rolling with probability $(6/36) = (1/6)$ for both players. Actually this might happen in separate consecutive dice rollings for playing the standard backgammon with probability

$$(6/36) \times (6/36) = (1/6) \times (1/6) = (1/36)$$

for both players. Of course, the double numbers in separate dice rolling for playing the standard backgammon might be equal for both players, which is not possible in the EN-gammon.

2. Another difference is when both face-up numbers are entangled numbers. That is, when the dice are showing $(1,6)$, or $(2,5)$, or $(3,4)$ for Alice; then Bob will have $(6,1)$, or $(5,2)$, or $(4,3)$, respectively, which means that they are equal up to the order since the *order of the numbers on the dice* is irrelevant for moving the checkers which depends only on the decision of the player. In this case, both players having the same entangled pair of numbers up to order and this happens with probability $(6/36) = (1/6)$ for EN-gammon, which can only happen for the standard backgammon (i.e., players rolling the dice separately at each turn) with probability

$$(6/36) \times (2/36) = (1/6) \times (1/18) = (1/108)$$

for both players having the same entangled pair of numbers up to the order with two consecutive rollings of dice.

3. Obviously, the number of rolling the dice in EN-gammon is (almost) half of the number of rolling the dice in the standard backgammon with separate rollings of dice by each player turn by turn. The reason we are using the word "almost" is because there are some cases, in playing with two separate rollings, that one of the players does not roll since no moving checker may land on a point that has two or more opponent checkers.

Finally, from the above evaluation, we will have the following conjecture by intuition.

Conjecture: The duration of playing EN-gammon is less than the duration of playing the standard backgammon. Further, the probability of ending the standard backgammon is less than or equal to the probability of ending the EN-gammon in a long sequence of trials (plays).

References

[1] Amir M. Rahimi, Hyper dice backgammon of finite size, *Missouri Journal of Mathematical Sciences*, **30** (2018), no. 2, 132-139. https://doi.org/10.35834/mjms/1544151690

[2] B. Robertie, *Advanced Backgammon*, The Gammon Press, Arlington, MA, (Vols. 1 and 2), 1991.

Received: October 1, 2021; Published: October 15, 2021