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Estimation for a Simple Step–Stress Model with Type–II Hybrid Censored Data from the Exponentiated Rayleigh Distribution

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Abstract

In this article, the problem of simple step–stress accelerated life tests when the life time follows the exponentiated Rayleigh distribution is considered. Based on type–II hybrid censoring scheme, the maximum Likelihood and Bayes methods of estimation are used for estimating the distribution parameters and acceleration factor. A Monte Carlo simulation study is carried out to examine the performance of obtained estimates.

Keywords: Simple step–stress accelerated life tests; Bayes estimation; Exponentiated Rayleigh distribution; Maximum likelihood estimation

(1) Introduction

The recent reliability levels attained by many electromechanically materials and items make it infeasible to test their failure times under normal use operating conditions science items tend to have a long life and lengthy applied tests tend to be far too expensive. For this reason, accelerated life tests (*ALTs*) are performed to be used in manufacturing industries to obtain enough failure data, in a short period of time; necessary to make inferences regarding its relationship with external variables. In *ALTs*, the test items are tested only at accelerated condition, via; higher than normal level of stress, to induce early failures. Data collected at such accelerated conditions are then extrapolated through a physically appropriate statistical model to estimate the life time distribution at a normal use conditions.
Some key references in the area of accelerated testing included [Wang and Balakrishnan (2008)], [Pascual (2008)]. A special class of the ALT is called the step–stress (SS) testing which allows the experiments to choose one or more stress

factors in a life–testing experiment. Stress factors can include humidity, temperature, vibration, voltage load or any other factor that directly affects the life of the products.

We consider here a Simple Step–Stress (*SSS*) model with only two stress levels. This model has been extensively in the literature. In 1980 Nelson proposed the Cumulative Exposure (*CE*) model, while Miller and Nelson (1983) and Baiet, et. al. (1989) discussed the determination of optimal time at which to change the stress level from S_0 to S_2. Balakrishnan and Xie (2007) derived the exact inference for a simple step–stress model with type–II hybrid censored data from the exponential distribution. Type–II *HCS* was discussed by more recent research on ALTs, see Chils et. al. (2003) and Chandrasekar et. al. (2004). Exponentiated Rayleigh (*ER*) distribution as a special case from Kumaraswamy Weibull distribution, is one of the most popular models, it has been extensively used for modeling data in reliability, engineering and biological studies.

In this paper, *SSS* is applied to *ER* distribution. The cumulative distribution function (*CDF*) and the probability density function (*pdf*) of *ER* distribution are obtained as follows;

$$f(x;\lambda,\theta)=2\theta\lambda.\,x\,e^{-\lambda x^2}.\left(1-e^{-\lambda x^2}\right)^{\theta-1} \qquad x>0 \;,\; \lambda,\theta>0 \tag{1}$$

and,

$$F(x;\lambda,\theta)=\left(1-e^{-\lambda x^2}\right)^{\theta} \qquad x>0 \;,\; \lambda,\theta>0 \tag{2}$$

Where θ is a shape parameter and λ is a scale parameter. The reliability (*RF*) and hazard rate function (*hrf*) of *ER* distribution are;

$$R(x;\lambda,\theta)=\left[1-\left(1-e^{-\lambda x^2}\right)^{\theta}\right] \tag{3}$$

and,

$$h(x;\lambda,\theta)=\frac{2\theta\lambda.\,x\,e^{-\lambda x^2}.\left(1-e^{-\lambda x^2}\right)^{\theta-1}}{\left[1-\left(1-e^{-\lambda x^2}\right)^{\theta}\right]} \tag{4}$$

Here λ is the scale parameter and θ is the shape parameter. The behavior of the distribution or its failure rate function depends on the shape parameter θ. For any λ the *ER* distribution has an increasing *hrf* if $(\theta>1)$, it has a decreasing *hrf* if $(\theta<1)$ and if $(\theta=1)$ *hrf* is constant.

In this paper, we consider a *SSS* model in which the life testing experiment gets terminated either at a pre fixed time (say, τ_{m+1}) or at a random time ensuring at least a specified number of failure (say, r out of n). Under this model in which the data obtained are type–II *HCS*, we consider the case of two stress levels with underlying life–times being *ER* distribution.

The model considered here is discussed in section (2). Due to the form of time constraint, the *MLEs* of the unknown parameters are discussed in section (3).

In section (4), we discussed the Bayesian estimation under Gamma prior distribution and using square error loss function (*SEL*), weighted loss function (*WL*), linear exponential loss function (*LINEX*) and general entropy loss function (*GEL*). Monte Carlo simulation results are presented in section (5). Finally conclusion is presented in section (6).

(2) Model Description

During the simple step stress (SSS) Alts, units are subjected to successively high levels of stress. After a units is used to normal levels of stress S_0, it is subjected to an initial level of stress S_1 for a predetermined time τ_1 at the first step in the test. If it does not fail, it is subjected to a higher level stress S_2 for a predetermined τ_2 at the next step. In analogy, it is repeatedly subjected to higher levels of stress until it fails. The other units are tested similarly. The pattern of stress levels and time intervals is the all units. The model assumptions for SSSALTs procedures will be described as follows;

Based on type–II *HCS SSALTs* has the following assumptions;

1. There are two levels of stress S_1 and S_2 where, $S_1 < S_2$, are applied such that each units is initially put under stress S_1.

2. The experiment begins with n identical units under an initial stress S_1. The stress level is raised to S_2 at time τ_1, and the life testing is terminated at a random time τ_2^*. Here $\tau_2^* = \max(x_r, \tau_2)$, where;

(i) $r(\leq n)$ and $0 < \tau_1 < \tau_2 < \infty$ are fixed in advance,

(ii) $x_1 < x_2 < ... < x_n$ denote the order failure times of n units under test,

(iii) τ_1 denotes a fixed time at which the stress level is changed from S_1 to S_2,

(iv) x_r denotes the time when the r^{th} failure occurs,

(v) τ_2 denotes a fixed time before which if the r^{th} failure occurs the experiment is terminated at time τ_2.

(vi) τ_2^* denotes the random time when the life–testing experiment is terminated. when, $n_1 =$ number of units that fail before time τ_1, $n_2 =$ number of units that fail before time τ_2 at stress level S_1 and $n_2^* =$ number of units that fail before time τ_2^* at stress level S_2. Then, it is evident that;

$$n_2^* = \begin{cases} r - n_1 & , \quad if \quad x_{r,n} > \tau_2 \\ \\ n_2 & , \quad if \quad x_{r,n} \leq \tau_2 \end{cases} \quad \Leftrightarrow \quad \tau_2^* = \begin{cases} x_r & , \quad if \quad x_r > \tau_2 \\ \\ \tau_2 & , \quad if \quad x_r \leq \tau_2 \end{cases}$$

3. The *ER* scale parameters $\lambda_j, j = 1,2$ of the underlying life time distribution is assumed to have an inverse power function of stress level i. e.;

$\lambda_j = c\,S_j^{\,p}$, $j = 1,2$, $c, p > 0$, $S_j = \dfrac{v^*}{v_j}$,

$v^* = \prod_{j=1}^{2} v_j^{\,b_j}$, $b_j = \dfrac{n_j}{\sum_{j=1}^{2} n_j}$, c is the constant power function and p is the power of applied stress.

To analyze the data from *SSSALTs*, a model in needed to relate the distribution under *SS* to the distribution under constant stress. The most commonly used model is cumulative exposure (*CE*) model proposed by Nelson (1980). The basic idea of the *CE* model starts from the fact that, a *SSSALTs* model must explain the cumulative effect of the applied stresses. The *CE* model assumed that the remaining test units are failed according to the *CDF* of current stress levels. According to Nelson (1990), the *CE* model $G(x)$ with k *SSSALTs* is given as;

$$G(x) = F_j(x^*; c, p, \theta) \quad , \quad j = 1,2,\ldots,k \tag{5}$$

where $x_{ij}^* = \left[\left(x_{ij} - \tau_{j-1}\right) + U_{j-1}\right]$ for $j = 1,2,\ldots,k$, $i = 1,2,\ldots,n_j$ and $F_j(x_{ij}^*) = \left(1 - e^{-c S_j^p x_{ij}^{*2}}\right)^{\theta}$ the *CDF* of the failure at stress S_j , u_{j-1} is the solution of the equation $F_j\left(U_{j-1}, S_j\right) = F_{j-1}\left(\tau_{j-1}^*, S_{j-1}\right)$. Therefore the general form solution is given as $U_{j-1} = \tau_j^* \sqrt{\dfrac{S_{j-1}}{S_j}}$. Note that, $U_0 = 0$, $\tau_j^* = (\tau_j - \tau_{j-1}) + U_{j-1}$, and $\tau_0 = 0$ where, τ_j is the time of changing stress. Also $F_j(\cdot)$ is as given in (2). Thus the corresponding *pdf* will be as follows;

$$g(x) = f_j(x^*) \quad , \quad j = 1,2,\ldots,k \quad , \quad \tau_{j-1} \le x_{ij} < \tau_j \tag{6}$$

(3) Maximum Likelihood Estimation:

The likelihood function based on type–II *HSC* is then given by;
The likelihood function based on type–II *HSC* is then given by;

$$L(c, p, \theta; x) = \frac{n!}{(n - r^*)!} \prod_{j=1}^{k} \prod_{i=1}^{r^*} g_j(x_{ij}^*) \left[1 - G_k(\tau_k^*)\right]^{n - r^*} \tag{7}$$

where $(n - r^*)$ is the number of surviving units $r^* = n_1 + n_2^*$; from *CED* in (5) and corresponding *pdf* in (6), we obtain the likelihood function for 2–parameter *ER* distribution for 2–*SSALT* with type–II *HCS*, as follows;

$$L(c,p,\theta;x)=(c\,\theta)^{r^*}\sum_{j=1}^{2}S_j^{n_j\,p}\left\{\sum_{j=1}^{2}\sum_{i=1}^{n_j}\left(x_{ij}^{*}.e^{-c.S_j^p.x_{ij}^{*2}}\right).\left(D_{n_j}^{\theta-1}(c,p)\right)\right\}.$$

$$.\left(1-D_k^{\theta}(c,p)\right)^{n-r^*} \tag{8}$$

where $D_{n_j}(c,p)=\left(1-e^{-c.S_j^p.x_{ij}^{*2}}\right)$ and $D_k=\left(1-e^{-c.S_k^p.\tau_k^{*2}}\right)$ for $j=1,2,...,k$. The MLEs of the unknown parameters are obtained by maximizing the logarithm of the likelihood function expressed in the following form;

$$\ell=r^*\left[\ell n(c)+\ell n(\theta)\right]+p\sum_{j=1}^{2}n_j\;\ell n(S_j)+\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n(x_{ij}^{*})-c\sum_{j=1}^{2}\sum_{i=1}^{n_j}S_j^p.x_{ij}^{*2}$$

$$+(\theta-1)\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\left(D_{n_j}(c,p)\right)+(n-r^*)\ell n\left(1-D_k^{\theta}(c,p)\right) \tag{9}$$

The first partial derivatives of the likelihood function with respect to the parameters c,θ and p respectively, will be as follows;

$$\frac{\partial\ell}{\partial\theta}=\frac{r^*}{\theta}+\sum_{j=1}^{2}\sum_{i=1}^{n_j}D_{n_j}(c,p) \tag{10},$$

$$\frac{\partial\ell}{\partial c}=\frac{r^*}{c}-\sum_{j=1}^{2}\sum_{i=1}^{n_j}S_j^p.x_{ij}^{*2}+(\theta-1).\sum_{j=1}^{2}\sum_{i=1}^{n_j}\left(\frac{h_1(\square)}{D_{n_j}(c,p)}\right)-(n-r^*)\left(\frac{h_2(\square)}{1-D_k^{\theta}(c,p)}\right)=0 \tag{11},$$

and,

$$\frac{\partial\ell}{\partial p}=\sum_{j=1}^{2}n_j\;\ell n(S_j)-c\sum_{j=1}^{2}\sum_{i=1}^{n_j}S_j^p.x_{ij}^{*2}\,\ell n(S_j)+(\theta-1).\sum_{j=1}^{2}\sum_{i=1}^{n_j}\left(\frac{c\;\ell n(S_j)h_1(\square)}{D_{n_j}(c,p)}\right)$$

$$-(n-r^*)\left(\frac{c\;\ell n(S_j)h_2(\square)}{1-D_k^{\theta}(c,p)}\right)=0 \tag{12}$$

where,

$h_1(\square)=S_j^p.x_{ij}^{*2}.e^{-c.S_j^p.x_{ij}^{*2}}$, $j=1,2,...,k$, $i=1,2,...,n_j$ and $h_2(\square)=S_k^p.\tau_k^{*2}.e^{-c.S_k^p.\tau_k^{*2}}$

Thus the likelihood equations (10), (11) and (12) are reduced to a system of two nonlinear equations by substituting from (10) to (11) and (12) which could be solved numerically with respect to θ,c and p to get the *MLEs* of θ,c and p by using equations (11) and (12).

In addition, estimates value $\hat{\lambda}_j$ for each stress will be obtained by substituting the estimates value of $\hat{c}$ and $\hat{p}$ in the inverse power law relationship $\lambda_j=c.S_j^p$.

(4) Bayesian Estimation

In this section, we assume that the three parameters c,θ and p are unknown. It is assumed that c,θ and p are independent, where c,θ and p have the following prior density distributions, respectively;

$$\begin{aligned}
&\pi(c)=b_1^{a_1}.c^{a_1-1}.e^{-b_1 c}\big/\Gamma a_1 \quad &; \quad c>0 \quad , \quad (a_1,b_1>0)\\
&\pi(p)=b_2^{a_2}.p^{a_2-1}.e^{-b_2 p}\big/\Gamma a_2 \quad &; \quad p>0 \quad , \quad (a_2,b_2>0) \qquad (13)\\
&\pi(\theta)=b_3^{a_3}.\theta^{a_3-1}.e^{-b_3 c}\big/\Gamma a_3 \quad &; \quad \theta>0 \quad , \quad (a_3,b_3>0)
\end{aligned}$$

As in Gupt and Kundu (2001) the joint density functions of c,θ and p is obtained from (13) and written as;

$$\pi(c,p,\theta)=\frac{b_1^{a_1}.b_2^{a_2}.b_3^{a_3}}{\Gamma a_1\,\Gamma a_2\,\Gamma a_3}.c^{a_1-1}.\theta^{a_3-1}.p^{a_2-1}.e^{-b_1 c-b_2 p-b_3\theta} \qquad (14)$$

since, the parameters are unknown, the likelihood function can be written as;

$$\begin{aligned}
\ell(\theta,c,p/x)\,\alpha\,(c\theta)^{r^*}.\exp\Bigg\{&\sum_{j=1}^{2}\sum_{i=1}^{n_j}x_{ij}^{*}-c\sum_{j=1}^{2}\sum_{i=1}^{n_j}S_j^{p}.x_{ij}^{*2}+(\theta-1)\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\left(D_{n_j}(c,p)\right)\\
&+p\sum_{j=1}^{2}n_j\,\ell n(S_j)+(n-r^*)\ell n\left(1-D_k^{\theta}(c,p)\right)\Bigg\} \qquad (15)
\end{aligned}$$

The joint posterior distribution of the parameters θ,c and p respectively, is given by;

$$\begin{aligned}
\pi(\theta,c,p/x)=J\,c^{r^*+a_1-1}.\theta^{r^*+a_3-1}.p^{a_2-1}\exp\Bigg\{&\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\,x_{ij}^{*}-c\left(b_1+\sum_{j=1}^{2}\sum_{i=1}^{n_j}S_j^{p}.x_{ij}^{*2}\right)\\
&-\theta\left(b_3\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\left(D_{n_j}(c,p)\right)\right)-p\left(b_2-\sum_{j=1}^{2}n_j\,\ell n\,S_j\right)\\
&-\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\left(D_{n_j}(c,p)\right)+(n-r^*)\ell n\left(1-D_k^{\theta}(c,p)\right)\Bigg\} \qquad (16)
\end{aligned}$$

where,

$$J^{-1}=\int_1^{\infty}\int_0^{\infty}\int_0^{\infty}\pi(\theta,c,p/x)\,d\theta\,dc\,dp$$

(4.1) Bayesian Estimators Using Squares Error Loss Function (SEL):

The Bayes estimators of c,θ and p, say $u(c,\theta,p)$ under SEL is given by the following form;

$$\hat{u}_{BS}(c,\theta,p) = E\left[u(c,\theta,p/x)\right] = \int_0^\infty\int_0^\infty\int_0^\infty u(c,\theta,p)\ \pi(c,\theta,p/x)\,dc\,d\theta\,dp \qquad (17)$$

From (17) the Bayes estimators of the parameters c,θ and p cannot be obtained in a simple closed form, in this case as Jaheen, et. al (2014) Monte Carlo Integration (*MCI*) may be used. The original MC approach method was developed by Physicists to use random number generation to compute integrals. Suppose that, we wish to compute a complex integral, $\int_a^b h(x)\,dx$.

If we can decompose $h(x)$ into the production of a function $f(x)$ and a pdf $P(x)$ defined over the interval (a,b), then note that; $\int_a^b h(x)\,dx = \int_a^b f(x)P(x)\,dx = E\left(f(x)\right)$.So that; the integral can be expressed as an expectation of $f(x)$ over the density $P(x)$. Thus, if we draw a large number $x_1, x_2, ..., x_M$ of random variables from the density $P(x)$, then; $\int_a^b h(x)\,dx = E\left(f(x)\right) \cong \frac{1}{M}\sum_{k=1}^{M} f(x_k)$. This is referred to as *MCI*.

Now if we want to find Bayes estimators for the function $u(c,\theta,p)$ based on SEL, we may use the following formula;

$$\hat{u}_{BS}(c,\theta,p) = E\left[u(c,\theta,p)\right] = \frac{\sum_{k=1}^{M} u(c^k,\theta^k,p^k).\,L(c^k,\theta^k,p^k/x)}{\sum_{k=1}^{M} L(c^k,\theta^k,p^k/x)} \qquad (18)$$

Under a *SEL* the Bayes estimation of c,θ and p are obtained from equations (15) and (18) as follows;

(i) Bayes estimation of c: If $u(c,\theta,p) = c$ in (18), the Bayes estimate of c is then given by;

$$\hat{c}_{BS} = \frac{w_2}{w_1} \qquad (19)$$

where,

$$w_1 = \sum_{k=1}^{M} c_k^{r^*}.\theta_k^{r^*}.\exp\left\{\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\, x_{ij}^{*} - c_k\sum_{j=1}^{2}\sum_{i=1}^{n_j} S_j^{p_k}.x_{ij}^{*2}\right.$$
$$\left. + (\theta_k - 1)\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\left(D_{n_j}(c,p)\right) + p_k\sum_{j=1}^{2} n_j.\ell n\, S_j + (n - r^*)\ell n\left(1 - D_k^{\theta}(c,p)\right)\right\}$$

and,

$$w_2=\sum_{k=1}^{M} c_k^{r^*+1}.\theta_k^{r^*}.\exp\left\{\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\, x_{ij}^{*}-c_k\sum_{j=1}^{2}\sum_{i=1}^{n_j} S_j^{p_k}.x_{ij}^{*^2}\right.$$
$$\left.+(\theta_k-1)\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\left(D_{n_j}(c,p)\right)+p_k\sum_{j=1}^{2} n_j.\ell n\, S_j+(n-r^*)\ell n\left(1-D_k^{\theta}(c,p)\right)\right\}$$

(ii) Bayes estimation of θ: If $u(c,\theta,p)=\theta$ in (18), the Bayes estimate of θ is then given by;

$$\hat{\hat{\theta}}_{BS}=\frac{w_3}{w_1} \qquad (20)$$

where,

$$w_3=\sum_{k=1}^{M} c_k^{r^*}.\theta_k^{r^*+1}.\exp\left\{\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\, x_{ij}^{*}-c_k\sum_{j=1}^{2}\sum_{i=1}^{n_j} S_j^{p_k}.x_{ij}^{*^2}\right.$$
$$\left.+(\theta_k-1)\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\left(D_{n_j}(c,p)\right)+p_k\sum_{j=1}^{2} n_j.\ell n\, S_j+(n-r^*)\ell n\left(1-D_k^{\theta}(c,p)\right)\right\}$$

(iii) Bayes estimation of p: If $u(c,\theta,p)=p$ in (18), the Bayes estimate of p is then given by;

$$\hat{\hat{p}}_{BS}=\frac{w_4}{w_1} \qquad (21)$$

where,

$$w_4=\sum_{k=1}^{M} (c_k\ \theta_k)^{r^*}.p_k.\exp\left\{\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\, x_{ij}^{*}-c_k\sum_{j=1}^{2}\sum_{i=1}^{n_j} S_j^{p_k}.x_{ij}^{*^2}\right.$$
$$\left.+(\theta_k-1)\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\left(D_{n_j}(c,p)\right)+p_k\sum_{j=1}^{2} n_j.\ell n\, S_j+(n-r^*)\ell n\left(1-D_k^{\theta}(c,p)\right)\right\}$$

(4.2) Bayesian Estimators Using Weighted Loss Function (WL):

The Bayes estimators of c,θ and p, say $u(c,\theta,p)$ under WL is given by the following form;

$$\tilde{\hat{u}}_{BS}(c,\theta,p)=\frac{1}{E\left[u(c,\theta,p)^{-1}\right]}=\frac{\sum_{k=1}^{M} 1/u(c^k,\theta^k,p^k)^{-1}.\,L(c^k,\theta^k,p^k/x)}{\sum_{k=1}^{M} L(c^k,\theta^k,p^k/x)} \qquad (22)$$

Under a *WL* the Bayes estimation of c,θ and p are obtained from equations (22) and (15) as follows;

(i) Bayes estimation of c: If $u(c,\theta,p)=c$ in (22), the Bayes estimate of c is then given by;

$$\tilde{\hat{c}}_{BS}=\frac{w_5}{w_1} \qquad (23)$$

where,

$$w_5=\sum_{k=1}^{M}\left(1/\theta_k^{r^*}.c_k^{r^*-1}\right).\exp\left\{\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\,x_{ij}^{*}-c_k\sum_{j=1}^{2}\sum_{i=1}^{n_j}S_j^{p_k}.x_{ij}^{*2}\right.$$
$$\left.+(\theta_k-1)\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\left(D_{n_j}(c,p)\right)+p_k\sum_{j=1}^{2}n_j.\ell n\,S_j+(n-r^*)\ell n\left(1-D_k^{\theta}(c,p)\right)\right\}$$

(ii) Bayes estimation of θ: If $u(c,\theta,p)=\theta$ in (22), the Bayes estimate of θ is then given by;

$$\tilde{\hat{\theta}}_{BS}=\frac{w_6}{w_1} \qquad (24)$$

where,

$$w_6=\sum_{k=1}^{M}\left(\frac{1}{c_k^{r^*}.\theta_k^{r^*-1}}\right).\exp\left\{\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\,x_{ij}^{*}-c_k\sum_{j=1}^{2}\sum_{i=1}^{n_j}S_j^{p_k}.x_{ij}^{*2}\right.$$
$$\left.+(\theta_k-1)\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\left(D_{n_j}(c,p)\right)+p_k\sum_{j=1}^{2}n_j.\ell n\,S_j+(n-r^*)\ell n\left(1-D_k^{\theta}(c,p)\right)\right\}$$

(iii) Bayes estimation of p: If $u(c,\theta,p)=p$ in (22), the Bayes estimate of p is then given by;

$$\tilde{\hat{p}}_{BS}=\frac{w_7}{w_1} \qquad (25)$$

where,

$$w_7=\sum_{k=1}^{M}\frac{1}{(c_k.\theta_k)^{r*}}.p_k^{-1}.\exp\left\{\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\,x_{ij}^{*}-c_k\sum_{j=1}^{2}\sum_{i=1}^{n_j}S_j^{p_k}.x_{ij}^{*2}\right.$$
$$\left.+(\theta_k-1)\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\left(D_{n_j}(c,p)\right)+p_k\sum_{j=1}^{2}n_j.\ell n\,S_j+(n-r^*)\ell n\left(1-D_k^{\theta}(c,p)\right)\right\}$$

(4.3) Bayesian Estimators Using Non-Linear Exponential Loss Function (LINEX):

The Bayes estimators of c,θ and p, say $u(c,\theta,p)$ under LINEX is given by the following form;

$$\bar{\hat{u}}_{BS}(c,\theta,p)=-\frac{1}{\varepsilon}\ell n\left[E_u\left(e^{-\varepsilon[u(c,\theta,p)]}\right)\right]=$$
$$-\frac{1}{\varepsilon}\ell n\left\{\frac{\sum_{k=1}^{M}e^{-\varepsilon\left[u(c^k,\theta^k,p^k)\right]}.L(c^k,\theta^k,p^k/x)}{\sum_{k=1}^{M}L(c^k,\theta^k,p^k/x)}\right\} \qquad (26)$$

Under a LINEX the Bayes estimation of c,θ and p are obtained from equations (26) and (15) as follows;

(i) Bayes estimation of c: If $u(c,\theta,p)=c$ in (26), the Bayes estimate of c is then given by;

$$\bar{\hat{c}}_{BS} = -\frac{1}{\varepsilon}\ell og\left(\frac{w_8}{w_1}\right) \tag{27}$$

where,

$$w_8 = \sum_{k=1}^{M}\theta_k^{r^*}.c_k^{r^*}.\exp\left\{-c_k\left(\sum_{j=1}^{2}\sum_{i=1}^{n_j} S_j^{p_k}.x_{ij}^{*^2}+\varepsilon\right)+\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\, x_{ij}^{*}\right.$$
$$\left.+(\theta_k-1)\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\left(D_{n_j}(c,p)\right)+p_k\sum_{j=1}^{2}n_j.\ell n\, S_j+(n-r^*)\ell n\left(1-D_k^{\theta}(c,p)\right)\right\}$$

(ii) Bayes estimation of θ: If $u(c,\theta,p)=\theta$ in (26), the Bayes estimate of θ is then given by;

$$\bar{\hat{\theta}}_{BS} = -\frac{1}{\varepsilon}\ell og\left(\frac{w_9}{w_1}\right) \tag{28}$$

where,

$$w_9 = \sum_{k=1}^{M}(\theta_k.c_k)^{r^*}.\exp\left\{\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\, x_{ij}^{*} - c_k.\sum_{j=1}^{2}\sum_{i=1}^{n_j} S_j^{p_k}.x_{ij}^{*^2}\right.$$
$$-\theta_k\left(\varepsilon-\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\left(D_{n_j}(c,p)\right)\right)-\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\left(D_{n_j}(c,p)\right)+p_k\sum_{j=1}^{2}n_j.\ell n\, S_j$$
$$\left.+(n-r^*)\ell n\left(1-D_k^{\theta}(c,p)\right)\right\}$$

(iii) Bayes estimation of p: If $u(c,\theta,p)=p$ in (26), the Bayes estimate of p is then given by;

$$\bar{\hat{p}}_{BS} = \frac{w_{10}}{w_1} \tag{29}$$

where,

$$w_{10} = \sum_{k=1}^{M}(\theta_k.c_k)^{r^*}.\exp\left\{\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\, x_{ij}^{*} - c_k.\sum_{j=1}^{2}\sum_{i=1}^{n_j} S_j^{p_k}.x_{ij}^{*^2}\right.$$
$$\left.+(\theta_k-1).\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\left(D_{n_j}(c,p)\right)-p_k\left(\varepsilon-\sum_{j=1}^{2}n_j.\ell n\, S_j\right)+(n-r^*)\ell n\left(1-D_k^{\theta}(c,p)\right)\right\}$$

(4.4) Bayesian Estimators Using General Entropy Loss Function (GEL):

The Bayes estimators of c,θ and p, say $u(c,\theta,p)$ under LINEX is given by the following form;

$$\breve{u}_{BS}(c,\theta,p)=\left[E\left(u(c,\theta,p)^{-V}\right)\right]^{-\frac{1}{V}}=$$

$$=\left[\frac{\sum_{k=1}^{M}\left(u(c^k,\theta^k,p^k)^{-V}\right).\,L(c^k,\theta^k,p^k/x)}{\sum_{k=1}^{M}L(c^k,\theta^k,p^k/x)}\right]^{-\frac{1}{V}} \tag{30}$$

Under a GEL the Bayes estimation of c,θ and p are obtained from equations (30) and (15) as follows;

(i) Bayes estimation of c: If $u(c,\theta,p)=c$ in (30), the Bayes estimate of c is then given by;

$$\breve{\hat{c}}_{BS}=\left[\frac{w_{11}}{w_1}\right]^{-\frac{1}{V}} \tag{31}$$

where,

$$w_{11}=\sum_{k=1}^{M}\theta_k^{r^*}.c_k^{r^*-V}.\exp\left\{\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\,x_{ij}^{*}-c_k\sum_{j=1}^{2}\sum_{i=1}^{n_j}S_j^{p_k}.x_{ij}^{*2}\right.$$
$$\left.+(\theta_k-1)\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\left(D_{n_j}(c,p)\right)+p_k\sum_{j=1}^{2}n_j.\ell n\,S_j+(n-r^*)\ell n\left(1-D_k^{\theta}(c,p)\right)\right\}$$

(ii) Bayes estimation of θ: If $u(c,\theta,p)=\theta$ in (30), the Bayes estimate of θ is then given by;

$$\breve{\hat{\theta}}_{BS}=\left[\frac{w_{12}}{w_1}\right]^{-\frac{1}{V}} \tag{32}$$

where,

$$w_{12}=\sum_{k=1}^{M}\theta_k^{r^*-V}.c_k^{r^*}.\exp\left\{\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\,x_{ij}^{*}-c_k\sum_{j=1}^{2}\sum_{i=1}^{n_j}S_j^{p_k}.x_{ij}^{*2}\right.$$
$$\left.+(\theta_k-1)\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\left(D_{n_j}(c,p)\right)+p_k\sum_{j=1}^{2}n_j.\ell n\,S_j+(n-r^*)\ell n\left(1-D_k^{\theta}(c,p)\right)\right\}$$

(iii) Bayes estimation of p: If $u(c,\theta,p)=p$ in (30), the Bayes estimate of p is then given by;

$$\breve{\hat{p}}_{BS}=\left[\frac{w_{13}}{w_1}\right]^{-\frac{1}{V}} \tag{33}$$

where,

$$w_{13}=\sum_{k=1}^{M}(\theta_k .c_k)^{r^*}.p^{-V}.\exp\left\{\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\, x_{ij}^{*}-c_k\sum_{j=1}^{2}\sum_{i=1}^{n_j}S_j^{p_k}.x_{ij}^{*^2}\right.$$
$$\left.+(\theta_k-1)\sum_{j=1}^{2}\sum_{i=1}^{n_j}\ell n\left(D_{n_j}(c,p)\right)+p_k\sum_{j=1}^{2}n_j.\ell n S_j+(n-r^*)\ell n\left(1-D_k^{\theta}(c,p)\right)\right\}$$

(5) Simulation Study

In order to obtain the *MLEs* and *BSEs* of the unknown parameters (c,p,θ) and the properties of their estimates through the *RMSE*, several sample sizes $(N=25,75,125,175)$ are generated from two parameter *ER* distribution of a simple step–stress *ALT* data. The simulation procedures are described through the following steps;

(i) For a given values of parameters $(c=1.5,\ p=0.5,\ \theta=1.5)$ and selected values at stress $S_1=1$ and $S_2=2$ calculate $\lambda_j=c.S_j^{p}$ for each stress level, where $(k=2)$.

(ii) Generate random samples of size $(N=25,75,125,175)$ from uniform $(c=1.5,\ p=0.5,\ \theta=1.5)$ given values of parameters (0,1) distribution and obtain the order statistics $(U_{1;N},...,U_{N;N})$.

(iii) For a given value of the first stress change $\tau_1=1$, find n_1 such that:

$$U_{1;N}\le\left(1-e^{-c.S_1^{p}.x^2}\right)^2<U_{n_1+1;N}$$

(iv) For a given value of the second stress change $\tau_2=1.5$, find n_2 such that:

$$U_{n_2;N-n_1}\le\left(1-e^{-c.S_2^{p}.\left(x-\tau_1+\tau_1\sqrt{\left(\frac{S_1}{S_2}\right)^2}\right)^2}\right)^{\theta}<U_{n_2+1;N-n_1}$$

(v) Then the order observations $x_1\le...\le x_{n_1}\le x_{n_1+1}\le\tau$ are calculated from (2).

(vi) Based on n_1,n_2,τ_1,τ_2 and the several observations;

– The *MLEs* $(\hat{c},\hat{p},\hat{\theta})$ of the parameters (c,p,θ) and *RMSEs* for the model parameters over 1000 samples are obtained, respectively, by solving the three non – linear equations from (10) to (12).

– The *BSEs* $(\hat{c},\hat{p},\hat{\theta})$ of the parameters (c,p,θ) and *RMSEs* for the model parameters are obtained by computing summations under different cases:-

– Under SEL in (19), (22) and (24).

– Under WL in (27), (29) and (31).

– Under LINEX in (34), (36) and (38).

– Under GEL in (41), (43) and (45).

(vii) Once the values of $\hat{c}$, $\hat{p}$ and $\hat{\theta}$ are obtained.

The estimates are used to obtain depending inverse power law $\lambda_u = c.S_u^p$, and the design stress, $S_u = 0.5$, the scale parameter under this stress, λ_u is estimated as $\hat{\lambda}_u = \hat{c}.S_u^{\hat{p}}$. Also, the *rf* and *hrf* at different values of mission times under usual conditions using (3) and (4).

A Monte Carlo simulation study is carried out in order to calculated the *MLEs*, *RMSEs* (*ML*), *BSEs* and *RMSE* (*BS*) of the model parameters, based on Replicated = 1000 Monte Carlo Simulation.

Simulations results; based on *SSS ALT* for *ER* distribution under type–II *HSC* with $(k = 2)$ are Summarized in;

- Table (1) present the *MLEs*, *RMSE* (*ML*), *BSEs* and *RMSEs* (*BS*) under *SEL*, *WL*, *LINEX* and *GEL* functions, respectively.
- Table (2) and (3) show the *rf* and *hrf* under different mission times.

Table (1): MLEs and BSs of Unknown Parameters c, p and θ, *RMSE* with different Censoring Scheme $c = 1.5, p = 0.5$ and $\theta = 1.5$

n	*r = n%*	*Parameters*	*MLE*		*Baysian SEL*		*Baysian LINEX*		*Baysian GEL*		*Baysian WL*	
25	0.4	c	1.8953	(1.9291)	1.5689	(0.2325)	1.5543	(0.2376)	1.5574	(0.2379)	1.5155	(0.2582)
		p	0.9593	(1.2747)	0.911	(0.4304)	0.9048	(0.4245)	0.9036	(0.4239)	0.8783	(0.4024)
		θ	1.566	(1.0079)	1.3476	(0.6959)	1.3295	(0.7057)	1.3365	(0.7026)	1.3073	(0.721)
	0.8	c	1.8965	(2.052)	1.5873	(0.2649)	1.5764	(0.2508)	1.5814	(0.2576)	1.5647	(0.2356)
		p	0.938	(1.2263)	1.2706	(0.7758)	1.2691	(0.7742)	1.2694	(0.7746)	1.2659	(0.7707)
		θ	1.5678	(0.9869)	1.5942	(0.7575)	1.5326	(0.733)	1.5658	(0.7488)	1.4895	(0.723)
75	0.4	c	1.5039	(1.0987)	1.858	(0.386)	1.8529	(0.3816)	1.8551	(0.3837)	1.846	(0.3765)
		p	0.819	(1.0145)	0.7866	(0.3051)	0.7846	(0.3016)	0.7847	(0.3019)	0.7796	(0.2936)
		θ	1.3881	(0.8866)	1.8427	(0.3375)	1.824	(0.3397)	1.833	(0.3392)	1.8048	(0.3444)
	0.8	c	1.5338	(1.1126)	1.8597	(0.3875)	1.8545	(0.383)	1.8568	(0.3851)	1.8475	(0.3779)
		p	0.8467	(1.0565)	0.7871	(0.3055)	0.785	(0.302)	0.7851	(0.3023)	0.7801	(0.2939)
		θ	1.4198	(0.8628)	1.8463	(0.3346)	1.8273	(0.3369)	1.8365	(0.3363)	1.8077	(0.3419)
125	0.4	c	1.5344	(1.0273)	1.8446	(0.3668)	1.8416	(0.3638)	1.843	(0.3652)	1.8382	(0.3604)
		p	0.8826	(1.0776)	0.7713	(0.287)	0.7705	(0.2855)	0.7705	(0.2856)	0.7683	(0.2814)
		θ	1.4019	(0.8602)	1.77	(0.3709)	1.7573	(0.3744)	1.7633	(0.3731)	1.744	(0.379)
	0.8	c	1.4872	(1.0042)	1.8452	(0.3673)	1.8422	(0.3642)	1.8436	(0.3657)	1.8388	(0.3608)
		p	0.8227	(1.0227)	0.7714	(0.2872)	0.7707	(0.2856)	0.7707	(0.2858)	0.7684	(0.2816)
		θ	1.3816	(0.8652)	1.7714	(0.3695)	1.7585	(0.3731)	1.7645	(0.3718)	1.7449	(0.378)
175	0.4	c	1.4341	(0.9362)	1.8438	(0.3651)	1.8416	(0.3626)	1.8427	(0.3638)	1.8392	(0.36)
		p	0.7871	(0.9836)	0.7586	(0.261)	0.7583	(0.2607)	0.7583	(0.2607)	0.7575	(0.2598)
		θ	1.3421	(0.8725)	1.7386	(0.3709)	1.7286	(0.3742)	1.7333	(0.3728)	1.7176	(0.3782)
	0.8	c	1.4556	(0.9439)	1.8444	(0.3656)	1.8421	(0.3631)	1.8432	(0.3643)	1.8397	(0.3604)
		p	0.8128	(1.0133)	0.7587	(0.2612)	0.7585	(0.2609)	0.7584	(0.2608)	0.7577	(0.2599)
		θ	1.3645	(0.8545)	1.7398	(0.3699)	1.7296	(0.3733)	1.7344	(0.3719)	1.7186	(0.3774)

From tables (1) the following conclusions can be observed:

- It is clear that the *MLEs* and *BEs* are very close to the initial value of the parameters as the sample size increases.
- As shown in the numerical results the *RMSE* (*MLE* & *BS*) are decreasing when the sample size is in increasing.
- Also shown that *RMSE* (*BS*) better than *RMSE* (*MLE*) for all sample sizes.
- Finally for all sample sizes we note that, $\hat{c}$ performs better than other estimates and $\hat{\theta}$ performs better than $\hat{p}$.

Tables (2) and (3): indicate that the reliability decreases when the mission time τ_0 increases. The results get better in the sense that the aim of an *ALT* experiments is to get large number of failures (reduce of their reliability) of the device with higher reliability.

Table (2): The Reliability Function and the Hazard Rate Function with Different Censoring Scheme

n	***r* = *n*%**	***t***	***MLE***		***Baysian SEL***		***Baysian LINEX***		***Baysian GEL***		***Baysian WL***	
			Rf	***HRf***	***Rf***	***HRf***	***Rf***	***HRf***	***Rf***	***HRf***	***Rf***	***HRf***
25		**0.25**	0.9996	0.0682	0.9986	0.2203	0.9985	0.2319	0.9985	0.2294	0.9982	0.2659
		0.50	0.9951	0.2322	0.9877	0.4836	0.9872	0.4993	0.9873	0.496	0.9857	0.5437
	0.4	**0.75**	0.9787	0.4715	0.9587	0.7733	0.9574	0.7901	0.9577	0.7865	0.954	0.8366
		1.00	0.9427	0.7815	0.9062	1.1011	0.9042	1.1177	0.9046	1.1142	0.8984	1.163
		1.50	0.7978	1.6751	0.7337	1.9856	0.7305	2.0004	0.7311	1.9973	0.7215	2.0404
		0.25	0.9996	0.0679	0.9987	0.2064	0.9986	0.2145	0.9986	0.2108	0.9985	0.2236
		0.50	0.9951	0.2316	0.9883	0.4644	0.988	0.4757	0.9881	0.4705	0.9876	0.4881
	0.8	**0.75**	0.9788	0.4706	0.9602	0.7524	0.9593	0.7647	0.9597	0.7591	0.9583	0.778
		1.00	0.9428	0.7805	0.9088	1.0804	0.9073	1.0927	0.908	1.0871	0.9056	1.1058
		1.50	0.798	1.674	0.7379	1.9669	0.7354	1.978	0.7365	1.9729	0.7328	1.9898
75		**0.25**	0.9981	0.2769	0.9996	0.0781	0.9996	0.0796	0.9996	0.0789	0.9995	0.0816
		0.50	0.9853	0.5576	0.9945	0.2529	0.9945	0.2558	0.9945	0.2545	0.9943	0.2599
	0.4	**0.75**	0.9529	0.8509	0.977	0.4994	0.9768	0.5034	0.9769	0.5017	0.9765	0.5088
		1.00	0.8966	1.1767	0.9394	0.8133	0.9389	0.8177	0.9391	0.8158	0.9383	0.8238
		1.50	0.7188	2.0525	0.7914	1.7084	0.7905	1.713	0.7909	1.711	0.7892	1.7193
		0.25	0.9983	0.2493	0.9996	0.0777	0.9996	0.0791	0.9996	0.0785	0.9995	0.0812
		0.50	0.9864	0.5223	0.9946	0.2519	0.9945	0.2549	0.9945	0.2536	0.9944	0.259
	0.8	**0.75**	0.9556	0.8144	0.9771	0.4982	0.9769	0.5021	0.977	0.5004	0.9765	0.5076
		1.00	0.9011	1.1414	0.9395	0.8118	0.9391	0.8163	0.9393	0.8144	0.9384	0.8224
		1.50	0.7257	2.0215	0.7917	1.7069	0.7908	1.7116	0.7911	1.7095	0.7895	1.7179
125		**0.25**	0.9983	0.2488	0.9995	0.082	0.9995	0.0829	0.9995	0.0825	0.9995	0.0839
		0.50	0.9865	0.5217	0.9943	0.2608	0.9943	0.2625	0.9943	0.2617	0.9942	0.2645
	0.4	**0.75**	0.9557	0.8137	0.9764	0.5099	0.9763	0.5122	0.9763	0.5111	0.9761	0.5149
		1.00	0.9012	1.1407	0.9381	0.825	0.9379	0.8277	0.938	0.8264	0.9375	0.8306
		1.50	0.7259	2.0209	0.789	1.7206	0.7885	1.7233	0.7887	1.722	0.7879	1.7263
		0.25	0.998	0.2937	0.9995	0.0819	0.9995	0.0827	0.9995	0.0823	0.9995	0.0838
		0.50	0.9845	0.5783	0.9943	0.2604	0.9943	0.2622	0.9943	0.2613	0.9942	0.2642
	0.8	**0.75**	0.9512	0.8721	0.9764	0.5094	0.9763	0.5118	0.9764	0.5107	0.9761	0.5145
		1.00	0.8939	1.1969	0.9382	0.8245	0.9379	0.8271	0.938	0.8259	0.9376	0.8302
		1.50	0.7148	2.07	0.7891	1.72	0.7886	1.7227	0.7888	1.7215	0.788	1.7258

Table (2) (continued): The Reliability Function and the Hazard Rate Function with Different Censoring

175	0.4	**0.25**	0.9975	0.3535	0.9995	0.0823	0.9995	0.0829	0.9995	0.0826	0.9995	0.0836
		0.50	0.9821	0.6488	0.9943	0.2612	0.9943	0.2625	0.9943	0.2619	0.9942	0.2639
		0.75	0.9457	0.9422	0.9764	0.5105	0.9763	0.5122	0.9763	0.5114	0.9761	0.5141
		1.00	0.8851	1.263	0.9381	0.8257	0.9379	0.8276	0.938	0.8267	0.9376	0.8298
		1.50	0.7017	2.1264	0.7889	1.7213	0.7885	1.7233	0.7886	1.7223	0.788	1.7254
	0.8	**0.25**	0.9977	0.328	0.9995	0.0821	0.9995	0.0828	0.9995	0.0824	0.9995	0.0835
		0.50	0.9831	0.6193	0.9943	0.2609	0.9943	0.2622	0.9943	0.2616	0.9942	0.2637
		0.75	0.948	0.9132	0.9764	0.5101	0.9763	0.5118	0.9763	0.511	0.9762	0.5137
		1.00	0.8888	1.2359	0.9381	0.8252	0.9379	0.8272	0.938	0.8263	0.9377	0.8293
		1.50	0.707	2.1034	0.7889	1.7208	0.7885	1.7228	0.7887	1.7218	0.7881	1.725

(6) Conclusion

In this article, *ML* and *BS* methods for estimating the unknown parameters with type–II HCS are obtained. The data failure times for *SSSALT* are assumed to follow the two parameters *ER* distribution at each stress level with scale parameter which is an inverse Power Law function of the stress. The performance of the estimate parameters is evaluated using *RMSEs* criteria. In addition, the *rf* and *hrf* obtained with different mission times.

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