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Chance-Constrained Programming (CCP) with Bivariate Generalized Exponential Distributed Random Parameters

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Abstract

CCP technique is considered an important tool for modeling and solving some decision problems. In this paper, we introduce an equivalent deterministic model of a CCP linear model, assuming that two of the left-hand side (L.H.S) coefficients are independent random parameters and following bivariate generalized exponential distribution with three parameters $\mathrm{GE}(\alpha_j, \lambda_j, \mu_j), j = 1,2$. Firstly, the probability density function (PDF) and cumulative distribution function (CDF) of linear combination of $\mathrm{GE}(\alpha_j, \lambda_j, \mu_j),\ j = 1,2$ are derived. Secondly, an equivalent deterministic model is introduced through theorem (1). Thirdly; some special cases are presented. Finally, a numerical example is introduced to illustrate how to find the PDF, CDF and equivalent deterministic model.

Keywords: Binomial theorem, CCP technique, $\mathrm{GE}(\alpha, \lambda, \mu)$ Distribution, Probabilistic Programming, Stochastic Programming.

1. Introduction

The CCP technique is the most frequent and major technique used to transform a probabilistic programming model into an equivalent deterministic model. It was introduced by Charnes and Cooper (1959). This technique depends on knowing the inverse of CDF of the random parameters to build Chance Constraints (CC's), which are achieved at a specific level of Tolerance Measure. CC's could be restricted with tolerance measures individually or jointly (Prékopa, 1995; El-Dash, 2015).

Since Charnes and Cooper had introduced the CCP technique, many studies have been presented to develop and apply this technique under certain some probability distributions such as; Normal distribution (Charnes and Cooper, 1962; Jagannathan, 1974), Gamma distribution (Lingaraj and Wolfe, 1974; Atalay and Apaydin, 2011), Chi-square distribution (Sengupta, 1972a; El-Dash, 1984), Weibull distribution (Ismail et. al, 2018).

Also, many main researches are presented to convert CC's models into its equivalent deterministic models when random parameters are distributed as exponential distribution with single parameter Exp(λ), two parameters Exp(λ,μ), and three parameters $\mathrm{GE}(\alpha,\lambda,\mu)$. Here, we classify these researches into two categories according to whether random parameters are independent or dependent.

Firstly, as for the assumption of independence of the random parameters, Sengupta (1972a) assumed that L.H.S parameters are random and derived an approximate equivalent deterministic model depending on the non-central chi-square distribution. El-Dash (1984) assumed that L.H.S parameters are random and derived the exact equivalent deterministic model depending on Box's theorem. Biswal et. al (1998) assumed that the L.H.S coefficients are random parameters and derived the exact equivalent deterministic model depending on the mathematical induction method. Hafez et. al (2018) extended the approach presented by Biswal et. al for the case of Exp(λ,μ). Gupta and Kundu (1999) introduced the $\mathrm{GE}(\alpha,\lambda,\mu)$, which got a lot of attention recently, since it is considered in many cases more flexible and applicable in analyzing lifetime data than gamma, Weibull and exponential distributions (Gupta and Kundu, 2001). Examples of studies that used this distribution in CCP, El-Dash (2018), assumed that some parameters in R.H.S and one parameter in L.H.S are random. Then, El-Dash and Hafez (2019), assumed that the R.H.S random parameters follow $\mathrm{GE}(\alpha_i,\lambda_i,\mu_i)$ for joint constraints.

Secondly, under the assumption of dependent random parameters, Hafez et al. (2018) assumed that the L.H.S or R.H.S random parameters are distributed as Downton bivariate exponential distribution. El-Dash (2019), proposed bivariate exponential distribution as an extension of bivariate Freund exponential distribution. Then in (2020), she proposed an extension of Farlie, Gumble and Morgenstern bivariate exponential distribution and presented the equivalent deterministic constraints in the case of joint and individual constraints.

In this paper, the transformation of CCP model into an equivalent deterministic model is presented with random parameters following bivariate $\mathrm{GE}(\alpha_j,\lambda_j,\mu_j), j=1,2$.

2. The proposed equivalent deterministic model

In this section, we present the equivalent deterministic model of the CCP Linear model with two random L.H.S coefficients $\widetilde{a_{i1}}$, $\widetilde{a_{i2}}$ distributed as $\mathrm{GE}(\alpha_{ij}, \lambda_{ij}, \mu_{ij})$, $j = 1,2$. Let:

$$\text{Max. } z = \sum_{j=1}^{n} c_j x_j \tag{2.1}$$

$$S.T \qquad P_r\left(\sum_{j=1}^{2} \widetilde{a_{ij}} x_j \le b_i\right) \ge \gamma_i \quad ; \quad i = 1, 2, \dots, m \tag{2.2}$$

$$x_j \ge 0 \quad ; \quad j = 1, 2, \dots, n$$

Where; x_j, c_j; $j = 1, 2, \dots, n$. are the decision variables and the objective function coefficients respectively, b_i are the R.H.S values, and $\widetilde{a_{ij}}$ are independent Random parameters following $\mathrm{GE}(\alpha_{ij}, \lambda_{ij}, \mu_{ij})$ distribution, and γ_i is the tolerance measure of the i^{th} CC, where $0 \le \gamma_i \le 1$, $i = 1, 2, \dots, m$, which describes the extent to which the i^{th} constraint is satisfied.

For model (2.1)-(2.2), we assume that the $\widetilde{a_{i1}}$, $\widetilde{a_{i2}}$. are independent random parameters and follow the $\mathrm{GE}(\alpha_{ij}, \lambda_{ij}, \mu_{ij}), j = 1,2$. In order to obtain the equivalent deterministic constraints of (2.2), it is required to find at first the probability distribution of the linear combination of $\widetilde{Y_{i1}} = \widetilde{a_{i1}} x_1 + \widetilde{a_{i2}} x_2$ as introduced in theorem (1), where the joint PDF of $\widetilde{a_{i1}}$, $\widetilde{a_{i2}}$ is given by:

$$f(\widetilde{a_{ij}}) = \prod_{j=1}^{2} \alpha_{ij}\lambda_{ij}\left(1 - e^{-\lambda_{ij}(\widetilde{a_{ij}} - \mu_{ij})}\right)^{\alpha_{ij}-1} e^{-\lambda_{ij}(\widetilde{a_{ij}} - \mu_{ij})} \tag{2.3}$$

$$\widetilde{a_{ij}} > \mu_{ij} \ ; \quad \alpha_{ij}, \lambda_{ij}, \mu_{ij} > 0; \ j = 1,2$$

Theorem (1). Consider the CC's in (2.2) and let $\widetilde{a_{ij}}$, $j = 1,2$ be independent random parameters that follow the $\mathrm{GE}(\alpha_{ij}, \lambda_{ij}, \mu_{ij})$ respectively, where α_{ij}, $j = 1,2$ are integer values, and $\lambda_{ij}, \mu_{ij} > 0$; $j = 1,2$ then:

1- the PDF and CDF of the random variable $\widetilde{Y_{i1}} = \sum_{j=1}^{2} \widetilde{a_{ij}} x_j$ are as follows, respectively:

$$f(\widetilde{Y_{i1}}) = \left(\prod_{j=1}^{2} \alpha_j \lambda_j\right) \sum_{l_1=0}^{\alpha_1-1} \sum_{l_2=0}^{\alpha_2-1} \frac{C_{l_1}^{\alpha_1-1} C_{l_2}^{\alpha_2-1}}{x_2\lambda_1(\alpha_1 - l_1) - x_1\lambda_2(\alpha_2 - l_2)} \cdot$$

$$\left[\sum_{j=1}^{2} (-1)^j e^{\frac{\lambda_{ij}(\alpha_{ij} - l_j)}{x_j}\left[\sum_{j=1}^{2} \mu_{ij} x_j - \widetilde{Y_{i1}}\right]}\right]; \widetilde{Y_{i1}} > \sum_{j=1}^{2} \mu_{ij} x_j; \lambda_{ij}, \mu_{ij} > 0 \ ; \alpha_{ij} = 1,2, \dots \tag{2.4}$$

$$F(\widetilde{y_{i1}}) = \left(\prod_{j=1}^{2} \alpha_{ij} \lambda_{ij}\right) \sum_{l_1=0}^{\alpha_{i1}-1} \sum_{l_2=0}^{\alpha_{i2}-1} C_{l_1}^{\alpha_{i1}-1} C_{l_2}^{\alpha_{i2}-1} \cdot$$

$$\left\{\sum_{\substack{j=1 \\ j \ne m}}^{2} \frac{e^{\frac{\lambda_{ij}(\alpha_{ij} - l_j)}{x_j}\left[\sum_{j=1}^{2} \mu_{ij} x_j - \widetilde{y_{i1}}\right]}}{\frac{x_m}{x_j}\left[\lambda_{ij}(\alpha_{ij} - l_j)\right]^2 - \prod_{j=1}^{2} \lambda_{ij}(\alpha_{ij} - l_j)} + \frac{1}{\prod_{j=1}^{2} \lambda_{ij}(\alpha_{ij} - l_j)}\right\} \quad ; \widetilde{Y_{i1}} > \sum_{j=1}^{2} \mu_{ij} x_j \ ;$$

$$\lambda_{ij}, \mu_{ij} > 0 \ ; \quad \alpha_{ij} = 1,2, \dots \ ; \quad j = 1,2 \tag{2.5}$$

2- the equivalent deterministic constraint of the CC (2.2) is as follows:

$$\sum_{l_1=0}^{\alpha_{i1}-1}\sum_{l_2=0}^{\alpha_{i2}-1} C_{l_1}^{\alpha_{i1}-1} C_{l_2}^{\alpha_{i2}-1}\left\{\sum_{\substack{j=1\\ j\neq m}}^{2}\frac{e^{\frac{\lambda_{ij}(\alpha_{ij}-l_j)}{x_j}\left[\sum_{j=1}^{2}\mu_{ij}x_j-b_i\right]}}{\frac{x_m}{x_j}\left[\lambda_{ij}(\alpha_{ij}-l_j)\right]^2-\prod_{j=1}^{2}\lambda_{ij}(\alpha_{ij}-l_j)}\right\}\geq R_i \quad ;\ i=1,2,\dots,m \tag{2.6}$$

where; $R_i=\frac{\gamma_i}{\prod_{j=1}^{2}\alpha_{ij}\lambda_{ij}}-\sum_{l_1=0}^{\alpha_{i1}-1}\sum_{l_2=0}^{\alpha_{i2}-1} C_{l_1}^{\alpha_{i1}-1} C_{l_2}^{\alpha_{i2}-1}\frac{1}{\prod_{j=1}^{2}\lambda_{ij}(\alpha_{ij}-l_j)}$

Proof: The proof of Theorem (1) is based on the Transformation technique. Firstly; Let us define the variable $\widetilde{Y_{\iota 2}}=\widetilde{a_{\iota 1}}x_2$. There is an one-to-one relationship between $\widetilde{Y_{\iota 1}},\widetilde{Y_{\iota 2}}$. Then:

$$\widetilde{a_{\iota 1}}= g_1^{-1}(\widetilde{Y_1},\widetilde{Y_2})=\frac{\widetilde{Y_{\iota 1}}}{x_1}-\frac{\widetilde{Y_{\iota 2}}}{x_1}\quad ;\ \widetilde{a_{\iota 2}}= g_2^{-1}(\widetilde{Y_{\iota 1}},\widetilde{Y_{\iota 2}})=\frac{\widetilde{Y_2}}{x_2} \tag{2.7}$$

The Determinant of the Jacobian for (2.7) is:

$$J=\left|\frac{\partial(\widetilde{a_{\iota 1}},\widetilde{a_{\iota 2}})}{\partial(\widetilde{Y_{\iota 1}},\widetilde{Y_{\iota 2}})}\right|=\begin{vmatrix}\frac{1}{x_1} & \frac{-1}{x_1}\\ 0 & \frac{1}{x_2}\end{vmatrix}=\frac{1}{x_1x_2}$$

Therefore, the Joint probability density function for $\widetilde{Y_{\iota 1}},\widetilde{Y_{\iota 2}}$ is as follows:

$$f(\widetilde{Y_{\iota 1}},\widetilde{Y_{\iota 2}})=\frac{1}{x_1x_2}\alpha_{i1}\lambda_{i1}\left[1-e^{-\lambda_{i1}\left(\frac{\widetilde{Y_{\iota 1}}}{x_1}-\frac{\widetilde{Y_{\iota 2}}}{x_1}-\mu_{i1}\right)}\right]^{\alpha_{i1}-1} e^{-\lambda_{i1}\left(\frac{\widetilde{Y_{\iota 1}}}{x_1}-\frac{\widetilde{Y_{\iota 2}}}{x_1}-\mu_{i1}\right)}\alpha_{i2}\lambda_{i2}\,\cdot \left[1-e^{-\lambda_{i2}\left(\frac{\widetilde{Y_{\iota 2}}}{x_2}-\mu_{i2}\right)}\right]^{\alpha_{i2}-1} e^{-\lambda_{i2}\left(\frac{\widetilde{Y_{\iota 2}}}{x_2}-\mu_{i2}\right)} \tag{2.8}$$

and the marginal probability density function of $\widetilde{Y_{\iota 1}}$ could be derived as follows:

$$f(\widetilde{Y_{\iota 1}})=\frac{\alpha_{i1}\lambda_{i1}\alpha_{i2}\lambda_{i2}}{x_1x_2}\int_{\widetilde{Y_{\iota 2}}=\mu_{i2}x_2}^{\widetilde{Y_{\iota 1}}-\mu_{i1}x_1}\left[1-e^{-\lambda_{i1}\left(\frac{\widetilde{Y_{\iota 1}}}{x_1}-\mu_{i1}\right)}e^{\frac{\lambda_{i1}}{x_1}\widetilde{Y_{\iota 2}}}\right]^{\alpha_{i1}-1}\cdot e^{-\lambda_{i1}\left(\frac{\widetilde{Y_{\iota 1}}}{x_1}-\mu_{i1}\right)}e^{\frac{\lambda_{i1}}{x_1}\widetilde{Y_{\iota 2}}}\left[1-e^{-\frac{\lambda_{i2}}{x_2}\widetilde{Y_{\iota 2}}}e^{\lambda_{i2}\mu_{i2}}\right]^{\alpha_{i2}-1} e^{-\frac{\lambda_{i2}}{x_2}\widetilde{Y_{\iota 2}}}e^{\lambda_{i2}\mu_{i2}}\,d\widetilde{Y_{\iota 2}} \tag{2.9}$$

By using the binomial expansion we find that:

$$\left[1-e^{-\lambda_{i1}\left(\frac{\widetilde{Y_{\iota 1}}}{x_1}-\mu_{i1}\right)}e^{\frac{\lambda_{i1}}{x_1}\widetilde{Y_{\iota 2}}}\right]^{\alpha_{i1}-1}=\sum_{l_1=0}^{\alpha_{i1}-1} C_{l_1}^{\alpha_{i1}-1}\left(-e^{-\lambda_{i1}\left(\frac{\widetilde{Y_{\iota 1}}}{x_1}-\mu_{i1}\right)}e^{\frac{\lambda_{i1}}{x_1}\widetilde{Y_{\iota 2}}}\right)^{\alpha_{i1}-1-l_1} \tag{2.10}$$

Similarly;

$$\left[1-e^{-\frac{\lambda_{i2}}{x_2}\widetilde{Y_{\iota 2}}}e^{\lambda_{i2}\mu_{i2}}\right]^{\alpha_{i2}-1}=\sum_{l_2=0}^{\alpha_{i2}-1} C_{l_2}^{\alpha_{i2}-1}\left(-e^{-\frac{\lambda_{i2}}{x_2}\widetilde{Y_{\iota 2}}}e^{\lambda_{i2}\mu_{i2}}\right)^{\alpha_{i2}-1-l_2} \tag{2.11}$$

Then, by substituting (2.10) and (2.11) in (2.9), we get:

$$f\left(\widetilde{Y_{i1}}\right)=\frac{\alpha_{i1}\lambda_{i1}\alpha_{i2}\lambda_{i2}}{x_1x_2}\int_{\widetilde{Y_{i2}}=\mu_{i2}x_2}^{\widetilde{Y_{i1}}-\mu_{i1}x_1}\sum_{l_1=0}^{\alpha_{i1}-1}C_{l_1}^{\alpha_{i1}-1}\left(-e^{-\lambda_{i1}\left(\frac{\widetilde{Y_{i1}}}{x_1}-\mu_{i1}\right)}e^{\frac{\lambda_{i1}}{x_1}\widetilde{Y_{i2}}}\right)^{\alpha_{i1}-1-l_1}\cdot$$
$$e^{\frac{\lambda_{i1}}{x_1}\widetilde{Y_{i2}}}\,e^{-\lambda_{i1}\left(\frac{\widetilde{Y_{i1}}}{x_1}-\mu_{i1}\right)}\sum_{l_2=0}^{\alpha_{i2}-1}C_{l_2}^{\alpha_{i2}-1}\left(-e^{-\frac{\lambda_{i2}}{x_2}\widetilde{Y_{i2}}}\,e^{\lambda_{i2}\mu_{i2}}\right)^{\alpha_{i2}-1-l_2}\,e^{-\frac{\lambda_{i2}}{x_2}\widetilde{Y_{i2}}}\,e^{\lambda_{i2}\mu_{i2}}\,\mathrm{d}\widetilde{Y_{i2}} \tag{2.12}$$

The function in (2.12) can be rewritten as follows:

$$f\left(\widetilde{Y_{i1}}\right)=\frac{\alpha_{i1}\lambda_{i1}\alpha_{i2}\lambda_{i2}}{x_1x_2}\sum_{l_1=0}^{\alpha_{i1}-1}\sum_{l_2=0}^{\alpha_{i2}-1}C_{l_1}^{\alpha_{i1}-1}C_{l_2}^{\alpha_{i2}-1}\left(e^{-\lambda_{i1}\left(\frac{\widetilde{Y_{i1}}}{x_1}-\mu_{i1}\right)(\alpha_{i1}-l_1)}\right)\cdot$$
$$\left(e^{\lambda_{i2}\mu_{i2}(\alpha_{i2}-l_2)}\right)\int_{\widetilde{Y_{i2}}=\mu_{i2}x_2}^{\widetilde{Y_{i1}}-\mu_{i1}x_1}e^{\widetilde{Y_{i2}}\left(\frac{x_2\lambda_{i1}(\alpha_{i1}-l_1)-x_1\lambda_{i2}(\alpha_{i2}-l_2)}{x_1x_2}\right)}\mathrm{d}\widetilde{Y_{i2}} \tag{2.13}$$

$$f\left(\widetilde{Y_{i1}}\right)=\left(\prod_{j=1}^{2}\alpha_{ij}\lambda_{ij}\right)\sum_{l_1=0}^{\alpha_{i1}-1}\sum_{l_2=0}^{\alpha_{i2}-1}\frac{C_{l_1}^{\alpha_{i1}-1}C_{l_2}^{\alpha_{i2}-1}}{x_2\lambda_{i1}(\alpha_{i1}-l_1)-x_1\lambda_{i2}(\alpha_{i2}-l_2)}\cdot$$
$$\left[e^{-\frac{\lambda_{i2}(\alpha_{i2}-l_2)}{x_2}\widetilde{Y_{i1}}}e^{\frac{\lambda_{i2}(\alpha_{i2}-l_2)\left(\sum_{j=1}^{2}\mu_{ij}x_j\right)}{x_2}}-e^{-\frac{\lambda_{i1}(\alpha_{i1}-l_1)}{x_1}\widetilde{Y_{i1}}}e^{\frac{\lambda_{i1}(\alpha_{i1}-l_1)\left(\sum_{j=1}^{2}\mu_{ij}x_j\right)}{x_1}}\right] \tag{2.14}$$

$$f\left(\widetilde{Y_{i1}}\right)=\left(\prod_{j=1}^{2}\alpha_{ij}\lambda_{ij}\right)\sum_{l_1=0}^{\alpha_{i1}-1}\sum_{l_2=0}^{\alpha_{i2}-1}\frac{C_{l_1}^{\alpha_{i1}-1}C_{l_2}^{\alpha_{i2}-1}}{x_2\lambda_{i1}(\alpha_{i1}-l_1)-x_1\lambda_{i2}(\alpha_{i2}-l_2)}\cdot$$
$$\left[e^{\frac{\lambda_{i2}(\alpha_{i2}-l_2)}{x_2}\left[\sum_{j=1}^{2}\mu_{ij}x_j-\widetilde{Y_{i1}}\right]}-e^{\frac{\lambda_{i1}(\alpha_{i1}-l_1)}{x_1}\left[\sum_{j=1}^{2}\mu_{ij}x_j-\widetilde{Y_{i1}}\right]}\right]\quad;\ \widetilde{Y_{i1}}>\sum_{j=1}^{2}\mu_{ij}x_j\ ;$$
$$\lambda_{ij},\mu_{ij}>0\ ;\ \alpha_{ij}=1,2,\dots;\ \ j=1,2. \tag{2.15}$$

Equation (2.15) is the PDF of $\widetilde{Y_{i1}}=\sum_{j=1}^{2}\widetilde{a_{ij}}x_j$ as indicated in equation (2.4).

Secondly, we can obtain the corresponding CDF as follows:

$$\mathrm{F}(\widetilde{y_{i1}})=\int_{\sum_{j=1}^{2}\mu_{ij}x_j}^{\widetilde{y_{i1}}}f\left(\widetilde{Y_{i1}}\right)d\widetilde{Y_{i1}} \tag{2.16}$$

$$F(\widetilde{y_{i1}})=\int_{\sum_{j=1}^{2}\mu_{ij}x_j}^{\widetilde{y_{i1}}}\left(\prod_{j=1}^{2}\alpha_{ij}\lambda_{ij}\right)\sum_{l_1=0}^{\alpha_{i1}-1}\sum_{l_2=0}^{\alpha_{i2}-1}\frac{C_{l_1}^{\alpha_{i1}-1}C_{l_2}^{\alpha_{i2}-1}}{x_2\lambda_{i1}(\alpha_{i1}-l_1)-x_1\lambda_{i2}(\alpha_{i2}-l_2)}\cdot$$
$$\left[e^{\frac{\lambda_{i2}(\alpha_{i2}-l_2)}{x_2}\left[\sum_{j=1}^{2}\mu_{ij}x_j-\widetilde{Y_{i1}}\right]}-e^{\frac{\lambda_{i1}(\alpha_{i1}-l_1)}{x_1}\left[\sum_{j=1}^{2}\mu_{ij}x_j-\widetilde{Y_{i1}}\right]}\right]d\widetilde{Y_{i1}} \tag{2.17}$$

$$F(\widetilde{y_{i1}})=\left(\prod_{j=1}^{2}\alpha_{ij}\lambda_{ij}\right)\sum_{l_1=0}^{\alpha_{i1}-1}\sum_{l_2=0}^{\alpha_{i2}-1}\frac{C_{l_1}^{\alpha_{i1}-1}C_{l_2}^{\alpha_{i2}-1}}{x_2\lambda_{i1}(\alpha_{i1}-l_1)-x_1\lambda_{i2}(\alpha_{i2}-l_2)}\left\{-\frac{x_2}{\lambda_{i2}(\alpha_{i2}-l_2)}\cdot\right.$$
$$\left.\left[e^{\frac{\lambda_{i2}(\alpha_{i2}-l_2)}{x_2}\left[\sum_{j=1}^{2}\mu_{ij}x_j-\widetilde{y_{i1}}\right]}-1\right]+\frac{x_1}{\lambda_{i1}(\alpha_{i1}-l_1)}\left[e^{\frac{\lambda_{i1}(\alpha_{i1}-l_1)}{x_1}\left[\sum_{j=1}^{2}\mu_{ij}x_j-\widetilde{y_{i1}}\right]}-1\right]\right\} \tag{2.18}$$

$$F(\widetilde{y_{\imath 1}}) = \left(\prod_{j=1}^{2}\alpha_{ij}\lambda_{ij}\right)\sum_{l_1=0}^{\alpha_{i1}-1}\sum_{l_2=0}^{\alpha_{i2}-1}C_{l_1}^{\alpha_{i1}-1}C_{l_2}^{\alpha_{i2}-1}\left\{\frac{e^{\frac{\lambda_{i2}(\alpha_{i2}-l_2)}{x_2}\left[\sum_{j=1}^{2}\mu_{ij}x_j-\widetilde{y_{\imath 1}}\right]}}{\frac{x_1}{x_2}[\lambda_{i2}(\alpha_{i2}-l_2)]^2-\prod_{j=1}^{2}\lambda_{ij}(\alpha_{ij}-l_j)}+\frac{e^{\frac{\lambda_{i1}(\alpha_{i1}-l_1)}{x_1}\left[\sum_{j=1}^{2}\mu_{ij}x_j-\widetilde{y_{\imath 1}}\right]}}{\frac{x_2}{x_1}[\lambda_{i1}(\alpha_{i1}-l_1)]^2-\prod_{j=1}^{2}\lambda_{ij}(\alpha_{ij}-l_j)}+\frac{1}{\prod_{j=1}^{2}\lambda_{ij}(\alpha_{ij}-l_j)}\right\} \quad (2.19)$$

$$F(\widetilde{y_{\imath 1}}) = \left(\prod_{j=1}^{2}\alpha_{ij}\lambda_{ij}\right)\sum_{l_1=0}^{\alpha_{i1}-1}\sum_{l_2=0}^{\alpha_{i2}-1}C_{l_1}^{\alpha_{i1}-1}C_{l_2}^{\alpha_{i2}-1}\cdot\left\{\sum_{\substack{j=1\\ j\neq m}}^{2}\frac{e^{\frac{\lambda_{ij}(\alpha_{ij}-l_j)}{x_j}\left[\sum_{j=1}^{2}\mu_{ij}x_j-\widetilde{y_{\imath 1}}\right]}}{\frac{x_m}{x_j}\left[\lambda_{ij}(\alpha_{ij}-l_j)\right]^2-\prod_{j=1}^{2}\lambda_{ij}(\alpha_{ij}-l_j)}+\frac{1}{\prod_{j=1}^{2}\lambda_{ij}(\alpha_{ij}-l_j)}\right\} \quad (2.20)$$

The above function is the same as (2.5), which represents the CDF of $\widetilde{Y_{\imath 1}}=\sum_{j=1}^{2}\widetilde{a_{\imath\jmath}}x_j$. Finally, by using a CCP technique, the equivalent deterministic constraint of the CC's in (2.2) is as follows: $F(b_i) \geq \gamma_i \quad ; \quad i = 1, 2, \ldots, m$ (2.21)
Here, $F(\cdot)$ represents the CDF of the random variable $\widetilde{y_{\imath 1}} = \sum_{j=1}^{2}\widetilde{a_{\imath\jmath}}x_j$. Hence, by substituting (2.5) in constraint (2.21), we obtain the equivalent deterministic constraint of the CC's (2.2) as given in (2.6).

3. Special cases

1) If $\alpha_{i1} = \alpha_{i2} = 1$ in (2.6), we obtain:

$$\frac{x_2\lambda_{i1}}{x_2\lambda_{i1}-x_1\lambda_{i2}}e^{-\frac{\lambda_{i2}}{x_2}b_i+\frac{\lambda_{i2}\left(\sum_{j=1}^{2}\mu_{ij}x_j\right)}{x_2}}-\frac{x_1\lambda_{i2}}{x_2\lambda_{i1}-x_1\lambda_{i2}}e^{-\frac{\lambda_{i1}}{x_1}b_i+\frac{\lambda_{i1}\left(\sum_{j=1}^{2}\mu_{ij}x_j\right)}{x_1}} \leq 1-\gamma_i \quad ; i = 1, 2, \ldots, m. \quad (2.22)$$

This result is equivalent to results provided by El-Dash, (1984) and Hafez et al. (2018a).
(2) If $\alpha_{i1} = \alpha_{i2} = 1$, $\mu_{i1} = \mu_{i2} = 0$ in (2.6), we obtain:

$$\frac{x_2\lambda_{i1}}{x_2\lambda_{i1}-x_1\lambda_{i2}}e^{-\frac{\lambda_{i2}}{x_2}b_i}-\frac{x_1\lambda_{i2}}{x_2\lambda_{i1}-x_1\lambda_{i2}}e^{-\frac{\lambda_{i1}}{x_1}b_i} \leq 1-\gamma_i \quad ; i = 1, 2, \ldots, m \quad (2.23)$$

This result is equivalent to results provided by Biswal et al. (1998).

4. Numerical example

In this section, a numerical example is presented to illustrate the procedure of the transformation from probabilistic programming model into an equivalent deterministic model based on the assumption of GE distributed random parameters. Consider the following CCP model:

$$\text{Max}\,.\,\text{Z} = 3\text{x}_1 + 5\text{x}_2 \tag{4.1}$$

$$S.T: \quad P_r(\widetilde{a_{i1}}\text{x}_1 + \widetilde{a_{i2}}\text{x}_2 \leq 100) \geq 0.9 \qquad ,\text{x}_1,\text{x}_2 \geq 0 \tag{4.2}$$

Here; x_1, x_2 are decision variables, $\widetilde{a_{i1}}, \widetilde{a_{i2}}$ are independent Random parameters following GE distributions, such that $\widetilde{a_{i1}} \sim \text{GE}(\alpha_{i1} = 2, \lambda_{i1} = 1, \mu_{i1} = 2)$ and $\widetilde{a_{i2}} \sim \text{GE}(\alpha_{i2} = 2, \lambda_{i2} = 2, \mu_{i2} = 3)$.

Depending on Theorem (1) and by substituting values of the parameters in (2.6), the equivalent deterministic model of model (4.1)- (4.2) becomes as follows:

$$\text{Max}\,.\,\text{Z} = 3\text{x}_1 + 5\text{x}_2 \tag{4.3}$$

$$\text{S.T} \quad \frac{(2x_2)e^{-\frac{400}{x_2}+\frac{8x_1+12x_2}{x_2}}}{2x_2-4x_1} + \frac{(4x_1)e^{-\frac{200}{x_1}+\frac{4x_1+6x_2}{x_1}}}{4x_1-2x_2} + \frac{(4x_2)e^{-\frac{200}{x_2}+\frac{4x_1+6x_2}{x_2}}}{2x_2-2x_1} + \frac{(2\lambda_2 x_1)e^{-\frac{200}{x_1}+\frac{4x_1+6x_2}{x_1}}}{2x_1-2x_2} + \frac{(2x_2)e^{-\frac{400}{x_2}+\frac{8x_1+12x_2}{x_2}}}{x_2-4x_1} + \frac{(8x_1)e^{-\frac{100}{x_1}+\frac{2x_1+3x_2}{x_1}}}{4x_1-x_2} + \frac{(4x_2)e^{-\frac{200}{x_2}+\frac{4x_1+6x_2}{x_2}}}{x_2-2x_1} + \frac{(8x_1)e^{-\frac{100}{x_1}+\frac{2x_1+3x_2}{x_1}}}{2x_1-x_2} \leq 8.1 \qquad ,\text{x}_1,\text{x}_2 \geq 0 \tag{4.4}$$

This model is nonlinear programming model and can approximated to linear programming model.

5. Conclusion

In this paper, we introduced the PDF and CDF of a linear combination of two random parameters, which follow $\text{GE}(\alpha_{ij}, \lambda_{ij}, \mu_{ij})$, $j = 1{,}2$. Then; by using the CCP technique, the equivalent deterministic model under the assumption that two L.H.S parameters are random and follow bivariate $\text{GE}(\alpha_{ij}, \lambda_{ij}, \mu_{ij})$ is presented. Also, some special cases are introduced, this cases satisfy the result provided by Biswal et al. (1998), El-Dash, (1984) and Hafez et al. (2018a). Hence, this paper is considered a generalization to cases of random parameters following single or two-parameter exponential distribution.

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