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New Shrinkage Parameters for the Generalized Liu-Type Estimator Using Mathematical Programming

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Abstract

Liu-type estimator is a biased estimator introduced as an alternative to the ordinary least squares estimators when multicollinearity exist. It is based on two shrinkage parameters that play basic roles in estimating the regression parameters. This article introduces a new method for estimating shrinkage parameters of generalized Liu-type estimator (GLTE) using mathematical programming techniques. Using a real data set the performance of the proposed method is evaluated based on the estimated mean squared error criterion and the results show that the suggested method outperforms other existing methods mentioned in this article.

Mathematics Subject Classification: 62J07

Keywords: Generalized Liu-type estimator, Shrinkage parameters, Mathematical programming, Multicollinearity

Introduction

Multiple linear regression model is applied in many applications to analyze the relationship between the independent variables $x_1, x_2, \dots, x_p$ and the dependent variable Y. The ordinary least squares (OLS) method is used to estimate the model parameters under some assumptions. One of these assumptions is that $x_1, x_2, \dots, x_p$

are uncorrelated, which usually unsatisfied in real life, this problem is called multicollinearity. It leads to inflated confidence intervals and theoretically important variables become insignificant in testing hypotheses. (Hoerl & Kennard,1970 a,b) introduced the well-known ridge regression (RRE) & generalized ridge regression estimators (GRRE) to handle this problem, many researchers have introduced different methods for estimating ridge parameter (s) for example: (Hocking et al, 1976), (Lawless & Wang, 1976), (Nomura, 1988), (Troshie & Chalton, 1996), (Batach & Gore, 2008), (El-Dash et al, 2011), (Hamed et al, 2012), (Asar et al, 2014), (Kibria & Banik, 2016) , (Bhat & Vidya, 2017), (Fayose & Ayinde, 2019).

In (RRE) and (GRRE) shrinkage parameter (s) are added to main diagonal of the information matrix $(X'X)$ to minimize parameter estimates and its standard errors. Liu-Type estimator is a two-parameter biased estimator, these two shrinkage parameters have different functions: $k\ , k > 0$, is used to reduce the condition index of the information matrix $(X'X)$ to a desired level and $d\ , -\infty < d < \infty$, is used to reduce the mean squared errors of the parameter estimates $MSE(\hat{\beta})$. (Liu, 1993), (Liu, 2003, 2004) (Asar, 2016), (Ebiad et al, 2017) introduced several methods for the estimation of the shrinkage parameters (k, d) separately.

Following (Hoerl & Kennard,1970 a,b) ,(Liu, 1993) and (Liu, 2003, 2004) (Farghali, 2019) introduced generalized Liu-type estimator that accommodates separate shrinkage parameters (k, d) for each independent variable. For estimating these shrinkage parameters separately, she used some methods that were introduced for generalized ridge regression and generalized Liu estimators.

(El-Dash et al, 2011), (Hamed et al, 2012), (El-Hefnawy & Farag, 2014) , (Ebiad et ali, 2017) have used mathematical programming models to obtain the optimal values of shrinkage parameters in ridge regression, generalized ridge regression, and Liu- type estimators.

The purpose of this article is to introduce a new method for estimating the shrinkage parameters used in generalized Liu-type estimator defined by (Farghali, 2019) to combat multicollinearity in linear regression using mathematical programming models. This article is organized as follows. In section 2, the generalized Liu-type estimator is introduced. The suggested model is introduced in section 3. An application of real data is demonstrated in section 4. Finally, a brief summary and conclusion are presented in section 5.

2. Generalized Liu-type estimator

Consider the following relation:

$$Y = X\beta + \epsilon \tag{2.1}$$

where X is a $n \times (p + 1)$ matrix of the independent variables, which is predetermined, Y is a $(n \times 1)$ vector of the dependent variable, ϵ is a $(n \times 1)$ vector of errors with $E(\epsilon) = 0$ and $V(\epsilon) = \sigma^2 I$, by using the ordinary least squares

(OLS) method the fitted linear regression model is as follows:

$$\hat{Y} = X\hat{\beta}_{OLS} \tag{2.2}$$

Here, $\hat{\beta}_{OLS} = (X'X)^{-1}X'Y$, with $V(\hat{\beta}_{OLS}) = \hat{\sigma}^2\ (X'X)^{-1}$, $\hat{\sigma}^2$ is the residual mean square and it is defined as: $\hat{\sigma}^2 = \frac{(Y-X\hat{\beta}_{OLS})'(Y-X\hat{\beta}_{OLS})}{n-p}$. $\hat{\beta}_{OLS}$ are the best unbiased estimators for estimating the linear regression parameters, but if multicollinearity occurs, then their performance deteriorates. As, the information matrix $(X'X)$ becomes ill-conditioned, the values of $(X'X)^{-1}$ become large. $\hat{\beta}_{OLS}$ still unbiased but with large variances, inflated confidence intervals and theoretically important variables are insignificant variables in testing hypotheses. (Belsley, 1991) reviewed different methods for diagnosing multicollinearity in linear egression, such as the condition index $(C.I.)$, it is defined as:

$$C.I. = \sqrt{\frac{\lambda_{max}}{\lambda_{min}}} \tag{2.3}$$

Where, λ_{max} and λ_{min} are the largest and the smallest eigenvalues of $(X'X)$, if $C.I. \leq 10$, then there is no multicollinearity among $x_1, x_2, \ldots, x_p$, if $10 < C.I. < 30$, then the multicollinearity is moderate and may be corrected, but if $C.I. \geq 30$, then it means that there is a severe multicollinearity and corrective actions must be taken.

Different corrective actions were adopted as using regularization methods which penalized the least squares by putting constraints on the matrix $(X'X)$, the new estimates are biased but with lower standard errors. One of these biased estimator is generalized Liu-type estimator $(GLTE)$for linear regression which is defined as:

$$\hat{\beta}_{GLTE} = (X'X + K)^{-1}\ (X'X - D)\hat{\beta}_{OLS} \tag{2.4}$$

$K = diag(k_1, k_2, \ldots, k_p)$, $k_j \geq 0$, $D = diag(d_1, d_2, \ldots, d_p)$, $-\infty < d_j < \infty$, $j = 1,2, \ldots, p$.

With a variance- covariance matrix:

$$Var(\hat{\beta}_{GLTE}) = \hat{\sigma}^2(X'X + K)^{-1}\ (X'X - D)(X'X)^{-1}(X'X - D)(X'X + K)^{-1} \tag{2.5}$$

It represents a general case of other biased estimator such as: generalized Liu estimator (GLE), Liu estimator (LE), generalized ridge regression estimator $(GRRE)$ and ridge regression estimator (RRE). It aims to choose appropriate values of k_j and d_j such that the reduction in the variance term is greater than the increase of the squared bias. Thus, $MSE(\hat{\beta}_{GLTE})$ will be less than $MSE(\hat{\beta}_{OLS})$. Suppose that there exists a matrix V such that:

$$V(X'X)DV' = \Lambda = diag(\lambda_1, \lambda_2, \dots, \lambda_p) \tag{2.6}$$

Where, $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p$, are the ordered eigenvalues of $(X'X)$ and V is a $(p \times p)$ orthogonal matrix whose columns are the corresponding eigenvectors of $\lambda_1, \lambda_2, \dots, \lambda_p$. Rewrite model (2.1) in a canonical form.

$$Y = Z\alpha + \epsilon \tag{2.7}$$

Where, $Z = XV$, $\alpha = V'\beta$, for model (2.12), the ordinary least squares estimator is:

$$\hat{\alpha}_{OLS} = \Lambda^{-1}Z'Y \tag{2.8}$$

The corresponding estimates $\hat{\beta}$ can be obtained as: $\hat{\beta}_{OLS} = V\,\hat{\alpha}_{OLS}$. Generalized Liu-type estimator is defined as:

$$\hat{\alpha}_{GLTE} = (\Lambda + K)^{-1}\,(\Lambda - D)\hat{\alpha}_{OLS} \tag{2.9}$$

Where, K *and* D as defined before, its variance-covariance matrix is defined as:

$$Var(\hat{\alpha}_{GLTE}) = \hat{\sigma}^2(\Lambda + K)^{-1}\,(\Lambda - D)\Lambda^{-1}(\Lambda - D)(\Lambda + K)^{-1} \tag{2.10}$$

$$Var(\hat{\alpha}_{GLTE}) = \hat{\sigma}^2 \sum_{j=1}^{p} \frac{(\lambda_j - d_j)^2}{\lambda_j(\lambda_j + k_j)^2} \tag{2.11}$$

Since that, the generalized Liu-type estimator is a biased estimator, then its bias is:

$$E(\hat{\alpha}_{GLTE}) = (\Lambda + K)^{-1}\,(\Lambda - D)\alpha \tag{2.12}$$

$$bias^2 = (E(\hat{\alpha}_{GLTE}) - \alpha)^2 \tag{2.13}$$

$$bias^2 = \sum_{j=1}^{p} \frac{(d_j + k_j)^2 \alpha_j^2}{(\lambda_j + k_j)^2} \tag{2.14}$$

The shrinkage parameters d_j are chosen to minimize $MSE(\hat{\alpha}_{GLTE})$ as follows:

$$MSE(\hat{\alpha}_{GLTE}) = E[(\hat{\alpha}_{GLTE} - \alpha)'(\hat{\alpha}_{GLTE} - \alpha)] = Var(\hat{\alpha}_{GLTE}) + bias^2 \tag{2.15}$$

Substituting equations (2.11) and (2.14), in equation (2.15), it can be written be as:

$$MSE(\hat{\alpha}_{GLTE}) = \hat{\sigma}^2 \sum_{j=1}^{p} \frac{(\lambda_j - d_j)^2}{\lambda_j(\lambda_j + k_j)^2} + \sum_{j=1}^{p} \frac{\alpha_j^2 (d_j + k_j)^2}{(\lambda_j + k_j)^2} \tag{2.16}$$

Differentiating equation (2.16) with respect to d_j and equating it to zero, then:

$$d_j = \frac{\hat{\sigma}^2 - \alpha_j^2 k_j}{\alpha_j^2 + \frac{\hat{\sigma}^2}{\lambda_j}} \qquad j = 1,2,\dots,p \qquad (2.17)$$

It is clear that, k_j is selected firstly, then the minimum $MSE(\hat{\alpha}_{GLTE})$ and the optimal value of d_j are obtained (i.e. $k_j, d_j\ j = 1,2,\dots,p$ are calculated separately). (Bhat & Vidya, 2017) reviewed several methods introduced for estimating shrinkage parameters k_j , $j = 1,2,\dots,p$ of generalized ridge regression estimator, Some of these methods were used to calculate the shrinkage parameters in the generalized Liu-type estimator. Also, (Fayose & Ayinde, 2019) reviewed some of the existing methods for determining the shrinkage parameters of generalized ridge regression as follows:

$$k_j(Nom) = \frac{\hat{\sigma}^2}{\hat{\alpha}_j^2}\left\{1 + \left[1 + \lambda_j\sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_j^2}}\right]\right\} \quad , \ j = 1,2,\dots,p \qquad (2.18)$$

$$k_j(TC) = \frac{\lambda_j \hat{\sigma}^2}{\lambda_j \hat{\alpha}_j^2 + \hat{\sigma}^2} \quad , \ j = 1,2,\dots,p \qquad (2.19)$$

$$k_j(Fir) = \frac{\lambda_j \hat{\sigma}^2}{\lambda_j \hat{\alpha}_j^2 + (n-p)\hat{\sigma}^2} \quad , \ j = 1,2,\dots,p \qquad (2.20)$$

$$k_j(Dor) = \frac{2\,\hat{\sigma}^2}{\lambda_{Max}\, \hat{\alpha}_j^2} \quad , \ j = 1,2,\dots,p \qquad (2.21)$$

$$k_j(LA) = \frac{\hat{\sigma}^2}{\lambda_j\, \hat{\alpha}_j^2} \quad , \ j = 1,2,\dots,p \qquad (2.22)$$

They proposed new methods for determining $k_j, j = 1,2,\dots,p$ by replacing $\lambda_j, j = 1,2,\dots,p$ in each method (2.18) – (2.22) by its minimum (λ_{min}), maximum (λ_{max}) , midrange (λ_{MR}), arithmetic mean (λ_{AM}), median (λ_{MD}) and the geometric mean (λ_{GM}) and the harmonic mean (λ_{HM}) as follows:

$$\lambda_{min} = minimum(\lambda_j) \quad ,j = 1,2,\dots,p \qquad (2.23)$$

$$\lambda_{max} = maximum(\lambda_j) \quad ,j = 1,2,\dots,p \qquad (2.24)$$

$$\lambda_{MR} = \frac{\lambda_{max}+\lambda_{min}}{2} \qquad (2.25)$$

$$\lambda_{AM} = \frac{1}{p}\sum_{j=1}^{p}\lambda_j \qquad (2.26)$$

$$\lambda_{MD} = median(\lambda_j) \quad ,j = 1,2,\dots,p \qquad (2.27)$$

$$\lambda_{GM} = \sqrt[p]{\prod_{j=1}^{p}\lambda_j} \qquad (2.28)$$

$$\lambda_{HM} = \frac{p}{\sum_{j=1}^{p}\frac{1}{\lambda_j}} \qquad (2.29)$$

Thus, there are 35 method for determining $k_j, j = 1,2,\dots,p$ of the generalized ridge regression estimator. In this article for estimating shrinkage parameters of generalized Liu-type estimator separately, these new 35 methods beside equation (2.17) are used.

3. The suggested nonlinear programming model

In this article, a single-objective nonlinear model is suggested to determine optimal values of the shrinkage parameters (the decision variables) $(k_j, d_j\, j = 1,2, \dots, p)$ simultaneously that minimizes the mean squared errors of the regression estimates $SE(\hat{\beta}_{GLTE})$. Thus, the objective function is:

$$Min\ \ H = \sigma^2 \sum_{j=1}^{p} \frac{(d_j - \lambda_j)^2}{\lambda_j\, (\lambda_j + k_j)^2} + \sum_{j=1}^{p} \frac{(d_j + k_j)^2\, \alpha_j^2}{(\lambda_j + k_j)^2} \tag{3.1}$$

Under the constraints:

1) $MSE(\hat{\beta}_{GLTE}) \ \le\ MSE(\hat{\beta}_{OLSE})$, i.e.

$$\sigma^2 \sum_{j=1}^{p} \frac{(d_j - \lambda_j)^2}{\lambda_j\, (\lambda_j + k_j)^2} + \sum_{j=1}^{p} \frac{(d_j + k_j)^2\, \alpha_j^2}{(\lambda_j + k_j)^2} \le \sigma^2 \sum_{j=1}^{p} \frac{1}{\lambda_j} \tag{3.2}$$

2) The multicollinearity problem is corrected at each independent variable, i.e.

$$\sqrt{\frac{\lambda_{max} + k_j}{\lambda_{min} + k_j}} \quad \le \quad 10 \tag{3.3}$$

The mathematical formulation of the suggested model is:

$$Find \quad k_j\, , d_j \quad , j = 1,2, \dots, p \quad such\ that \tag{3.4}$$

$$Min\ \ H = \sigma^2 \sum_{j=1}^{p} \frac{(d_j - \lambda_j)^2}{\lambda_j\, (\lambda_j + k_j)^2} + \sum_{j=1}^{p} \frac{(d_j + k_j)^2\, \alpha_j^2}{(\lambda_j + k_j)^2} \tag{3.5}$$

$subject\ \ to$

$$\sigma^2 \sum_{j=1}^{p} \frac{(d_j - \lambda_j)^2}{\lambda_j\, (\lambda_j + k_j)^2} + \sum_{j=1}^{p} \frac{(d_j + k_j)^2\, \alpha_j^2}{(\lambda_j + k_j)^2} \quad \le \quad \sigma^2 \sum_{j=1}^{p} \frac{1}{\lambda_j} \tag{3.6}$$

$$\sqrt{\frac{\lambda_{max} + k_j}{\lambda_{min} + k_j}} \quad \le \quad 10 \qquad , j = 1,2, \dots, p \tag{3.7}$$

$$k_j > 0 \quad , \quad d_j \ \ unrestricted \quad , \quad j = 1,2, \dots, p \tag{3.8}$$

Where: $\lambda_j\, , \hat{\alpha}_j^2, j = 1,2, \dots, p$ and $\hat{\sigma}^2$: parameters, as defined before.

$k_j\, , d_j, j = 1,2, \dots, p$: decision variables, shrinkage parameters assigned to the j^{th} independent variable. Model (3.5)-(3.8) has the following properties: 1) it consists of (p+1) constraints and (2p) decision variables. 2) It is a nonlinear programming model as both the objective function and the constraints are nonlinear. 3) It can be solved by the gradient methods (the generalized reduced gradient algorithm) with available software like the general algebraic modeling system (GAMS).

4) If $d_1 = d_2 = \cdots = d_p = 0$, then the suggested model represents a new method for selecting optimal shrinkage parameter of generalized ridge regression in linear regression. 5) If $k_1 = k_2 = \cdots = k_p = 0$, then the suggested model represents a new method for selecting optimal shrinkage parameter of generalized Liu estimator for linear regression.

4. Real data applications

In this section, generalized Liu-type estimator with the new shrinkage parameters is applied to the dataset taken from the Egyptian Annual Bulletin of Medical Ambulance of the year 2015. A multiple linear regression model is set where the dependent variable is: number of ambulances. We try to explain the dependent variable by the following independent variable, X_1 : The number of population per each governorate,X_2 :The number of ambulance centers available in each governorate, X_3 :The number of workers in each ambulance center, X_4 :The number of cars available in each center, X_5 :The number of medical equipment necessary for medical ambulance, X_6 :Per capita emergency medical services. Using these dependent and independent variables after standardization a multiple linear regression model was conducted, the fitted model using OLS estimates and its standard errors are:

$$\hat{y}_i^* = \underset{(0.850)}{0.493} x_{1i}^* - \underset{(0.255)}{0.262} x_{2i}^* + \underset{(0.337)}{0.107} x_{3i}^* + \underset{(0.273)}{0.644} x_{4i}^* - \underset{(1.314)}{0.502} x_{5i}^* + \underset{(0.810)}{0.577} x_{6i}^* \quad , \quad i = 1,2,..,27 \qquad MSE(\hat{\beta}_{OLS}) = 2.4338$$

Although the fitted model is significant (at 5% level of significance), yet all the independent variables are insignificant except (X_5). This indicates the existence of multicollinearity so the bivariate correlation matrix was calculated as follows:

$$\begin{bmatrix} 1 & 0.812 & 0.665 & 0.711 & 0.633 & 0.774 \\ 0.812 & 1 & 0.795 & 0.855 & 0.783 & 0.280 \\ 0.665 & 0.795 & 1 & 0.693 & 0.644 & 0.308 \\ 0.711 & 0.855 & 0.693 & 1 & 0.854 & 0.217 \\ 0.633 & 0.783 & 0.644 & 0.854 & 1 & 0.160 \\ 0.774 & 0.280 & 0.308 & 0.217 & 0.160 & 1 \end{bmatrix}$$

It assures that the multicollinearity problem may exist since that, there are nearly high correlations among the independent variables, to detect multicollinearity the condition index is calculated using equation (2.3), $10 < C.I. = 26.475 < 30$, it indicates that there is a moderate multicollinearity and may be corrected.
The generalized Liu-type estimator is used with both the existing shrinkage parameters estimators and the proposed model for correcting multicollinearty in linear regression. The parameter estimates and its standard errors are shown in tables (4-1) and (4-2) respectively.

Table (4-1): estimated regression parameters and estimated MSE

Estimates	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	MSE(β)
OLS	0.4932	-0.2623	0.1072	0.6448	-0.5024	0.5776	2.4338
GLT_{Nom}	-0.2218	0.4835	-0.0108	0.1675	0.3402	0.0045	0.7533
$GLT_{Nom.min}$	0.0001	-0.0094	0.0079	0.0045	-0.0787	-0.0560	0.5734
$GLT_{Nom.max}$	-0.1111	0.1317	0.0024	-0.1612	0.1163	0.3331	3.3190
$GLT_{Nom.MR}$	-0.0262	-0.7344	0.0001	-0.0031	-0.7178	0.0013	2.7955
$GLT_{Nom.AM}$	0.0232	0.3470	0.1376	0.0561	0.2085	-0.5382	2.2439
$GLT_{Nom.MD}$	0.0079	-0.0001	-0.0583	-0.6002	0.0003	0.0022	3.1053
$GLT_{Nom.GM}$	-0.2218	-0.5877	0.0933	-0.1242	-0.2436	0.0005	2.0430
$GLT_{Nom.HM}$	0.0004	0.0112	-0.6856	0.0002	-0.0240	-0.1880	0.8411
GLT_{TC}	0.3588	-0.6382	-0.1515	0.3618	-0.6387	-0.1232	0.9447
$GLT_{TC.min}$	-0.0118	0.0494	0.6846	-0.0118	0.0636	0.2094	1.0768
$GLT_{TC.max}$	-0.6425	-0.0462	0.3600	-0.6433	-0.0605	0.3668	0.5997
$GLT_{TC.MR}$	0.1057	0.1559	-0.0118	0.1025	0.1601	-0.0118	0.5909
$GLT_{TC.AM}$	-0.1392	0.3673	-0.6399	-0.1142	0.3625	-0.6674	0.5804
$GLT_{TC.MD}$	0.6846	-0.0118	0.0813	0.1951	-0.0118	0.1174	0.5622
$GLT_{TC.GM}$	0.3588	-0.6785	-0.0816	0.3592	-0.6451	-0.1500	0.5909
$GLT_{TC.HM}$	-0.0114	0.1178	0.1688	-0.0117	0.1071	0.4465	0.5552
GLT_{Fir}	0.3645	-0.6485	-0.1582	0.3721	-0.6569	-0.3318	2.6752
$GLT_{Fir.max}$	-0.0225	0.2443	1.2846	-0.0159	0.2722	1.0526	0.6919
$GLT_{Fir.min}$	-0.6853	-0.2863	0.3701	-0.6885	-0.3723	0.3695	2.8155
$GLT_{Fir.MR}$	0.1972	0.2395	-0.0223	0.2089	0.3221	-0.0122	0.7299
$GLT_{Fir.AM}$	-0.2620	0.3679	-0.6694	-0.3613	0.3723	-0.7001	0.8728
$GLT_{Fir.MD}$	1.2846	-0.0119	0.2610	0.8782	-0.0149	0.1297	1.7567
$GLT_{Fir.GM}$	0.3645	-0.6869	-0.4113	0.3674	-0.6937	-0.1859	2.4205
$GLT_{Fir.HM}$	-0.0480	0.1200	0.4799	-0.0320	0.1919	1.9071	7.0153
GLT_{Dor}	0.3760	-0.7227	-0.0012	0.3713	-0.7206	-0.1378	0.6749
$GLT_{Dor.max}$	-0.0027	0.3933	-0.1218	-0.0007	0.3242	1.3089	2.8539
$GLT_{Dor.min}$	-0.7068	-0.6791	0.3744	-0.7036	-0.5646	0.3449	0.7716
$GLT_{Dor.MR}$	0.1028	3.4007	-0.0026	0.1219	3.1174	-0.0052	2.5393
$GLT_{Dor.AM}$	-0.0564	0.2753	-0.7164	-0.1896	0.3697	-0.5649	1.6290
$GLT_{Dor.MD}$	-0.1218	-0.0092	0.2353	1.6547	-0.0052	0.0012	6.0457
$GLT_{Dor.GM}$	-0.0027	0.3933	-0.1218	-0.0075	0.3242	0.1308	0.6749
$GLT_{Dor.HM}$	-0.7068	-0.6791	0.3744	-0.7036	-0.5646	0.3449	0.7716
GLT_{LA}	0.3762	-0.7236	-0.0039	0.3729	-0.7224	-0.2091	0.6391
$GLT_{LA.max}$	-0.0051	0.4077	-0.0931	-0.0014	0.3473	2.0145	31.2370
$GLT_{LA.min}$	-0.7143	-0.7122	0.3750	-0.7125	-0.6180	0.3570	0.6859
$GLT_{LA.AM}$	0.1402	0.3555	-0.0049	0.1600	3.3852	-0.0010	27.9638
$GLT_{LA.MD}$	-0.1039	0.3153	-0.7199	-0.2675	0.3719	-0.6295	22.5927
$GLT_{LA.MR}$	-0.0931	-0.0019	0.2677	2.3305	-0.0010	0.0250	12.3693
$GLT_{LA.GM}$	0.3762	-0.0452	-0.4796	0.3758	-0.7084	-0.0240	27.9637
$GLT_{LA.HA}$	-0.0299	0.0050	3.0770	-0.0123	0.1314	0.0957	9.0481
GLT_{SM}	**0.2411**	**-0.0962**	**0.0363**	**0.1872**	**-0.3862**	**0.2671**	**0.4995**

Table (4-2): the standard errors of the estimated regression parameters

Estimates	$SE(\hat{\beta}_1)$	$SE(\hat{\beta}_1)$	$SE(\hat{\beta}_1)$	$SE(\hat{\beta}_1)$	$SE(\hat{\beta}_1)$	$SE(\hat{\beta}_1)$
OLS	0.7366	0.2198	0.1691	0.2763	1.1201	0.6951
GLT_{Nom}	0.4447	0.1503	0.8040	0.3359	0.8460	0.3376
$GLT_{Nom.max}$	0.1622	0.2215	0.2073	0.1108	0.1847	0.5178
$GLT_{Nom.min}$	0.2027	0.9729	0.6049	0.2401	0.8595	0.6785
$GLT_{Nom.MR}$	0.0615	0.6790	0.3122	0.7293	0.6637	0.0025
$GLT_{Nom.AM}$	0.1718	0.6959	0.3422	0.4151	0.4182	0.1339
$GLT_{Nom.MD}$	0.3173	0.4021	0.1366	0.5549	0.2461	0.5202
$GLT_{Nom.GM}$	0.4447	0.1462	0.6895	0.2491	0.6068	0.3713
$GLT_{Nom.HM}$	0.2288	0.2640	0.6339	0.5042	0.5643	0.3738
GLT_{TC}	0.7195	0.1588	1.1195	0.7254	0.1589	0.9104
$GLT_{TC.max}$	0.2187	0.1601	0.6329	0.2196	0.1791	0.2936
$GLT_{TC.min}$	0.1599	0.3415	0.7218	0.1601	0.4472	0.7355
$GLT_{TC.MR}$	0.2480	0.1441	0.2187	0.2403	0.1980	0.2198
$GLT_{TC.AM}$	1.0286	0.7366	0.1592	0.8438	0.7269	0.3661
$GLT_{TC.MD}$	0.6329	0.2198	0.1906	0.1803	0.2197	0.2754
$GLT_{TC.GM}$	0.7195	0.1688	0.6032	0.7203	0.1605	1.1083
$GLT_{TC.HM}$	0.2128	0.2763	0.1560	0.2168	0.2512	0.4128
GLT_{Fir}	0.7308	0.1614	1.1688	0.7462	0.1635	2.4515
$GLT_{Fir.max}$	0.4174	0.5729	1.1876	0.2958	0.6382	0.9732
$GLT_{Fir.min}$	0.1705	2.1152	0.7421	0.1713	2.7509	0.7409
$GLT_{Fir.MR}$	0.4624	0.2215	0.4131	0.4898	0.2978	0.2268
$GLT_{Fir.AM}$	1.9362	0.7376	0.1666	2.6696	0.7464	0.4742
$GLT_{Fir.MD}$	1.1876	0.2211	0.6121	0.8120	0.2761	0.3041
$GLT_{Fir.GM}$	0.7308	0.1709	3.0390	0.7366	0.1726	1.3738
$GLT_{Fir.HM}$	0.8903	0.2815	0.4436	0.5927	0.4499	1.7632
GLT_{Dor}	0.7539	0.1798	0.2292	0.7444	0.1793	1.0183
$GLT_{Dor.max}$	0.5150	0.9222	0.1926	0.2139	0.7601	1.2101
$GLT_{Dor.min}$	0.1759	5.0172	0.7506	0.1751	4.1713	0.6916
$GLT_{Dor.MR}$	0.2410	3.1440	0.4981	0.2860	2.8821	0.6497
$GLT_{Dor.AM}$	0.4169	0.5519	0.1783	1.4010	0.7413	0.4061
$GLT_{Dor.MD}$	0.1826	0.1734	0.5517	1.5298	0.9600	0.2880
$GLT_{Dor.GM}$	0.7539	0.8052	2.9965	0.7528	0.1733	0.6931
$GLT_{Dor.HM}$	0.3728	0.1651	2.4336	0.1377	0.2215	0.3331
GLT_{LA}	0.7543	0.1800	0.1291	0.7478	0.1797	1.5450
$GLT_{LA.max}$	0.9350	0.9559	0.3861	0.3274	0.8142	1.8624
$GLT_{LA.min}$	0.1777	5.2620	0.7520	0.1773	4.5661	0.7157
$GLT_{LA.AM}$	0.3288	3.2820	0.6907	0.3751	3.1297	0.0020
$GLT_{LA.MD}$	0.7681	0.6322	0.1791	1.9768	0.7457	0.5661
$GLT_{LA.MR}$	0.0861	0.3005	0.6278	2.1546	0.1918	0.5870
$GLT_{LA.GM}$	0.7543	0.1826	3.5433	0.7535	0.1763	0.7751
$GLT_{LA.HA}$	0.5537	0.3117	2.8448	0.2291	0.3082	0.8853
GLT_{SM}	**0.1581**	**0.1350**	**0.1074**	**0.1012**	**0.1593**	**0.2102**

According to tables (4-1) and (4-2), we observe that the standard errors of the generalized Liu-type estimators are quite smaller then OLSE which shows that they are more stable than OLSE. Also, it is clear that all generalized Liu-type estimators have smaller $MSE(\hat{\beta})$ than OLSE but the estimates using the suggested model have the best performance among others since that it have the smallest standard errors and the smallest $MSE(\hat{\beta})$.

5. Summary and Conclusions

In this article, some new shrinkage parameters estimators have been proposed for the generalized Liu-type estimator in linear regression. Also, a new method for estimating these shrinkage parameter simultaneously is proposed using mathematical programming. A real dataset is considered and a comparison is made between the different methods for estimating shrinkage parameters. The standard errors of regression parameters estimates and MSE are used as performance criterions. The results showed that the generalized Liu-type estimators with different shrinkage parameters are better than the OLSE since that it have smaller standard errors and $MSE(\hat{\beta})$.

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References

[1] M.R. Abonazel, and R.A. Farghali, Liu-Type Multinomial Logistic Estimator, *Sankhya B*, (2018) https://doi.org/10.1007/s13571-018-0171-4

[2] Y. Asar, Liu-type logistic estimators with optimal shrinkage parameter, *J. Modern Appl. Statist. Methods,* **15** (2016), 738–751. https://doi.org/10.22237/jmasm/1462077300

[3] Y. Asar, A. Karaibrahimo Glu, and A. Genc¸ Modified ridge regression parameters: A comparative Monte Carlo study, *Hacettepe J. Math. Statist*., **43** (2014), 827–841.

[4] F.S. Batach and S.D. Gore, The efficiency of modified Jackknife and ridge type regression estimators: a comparison, *Surveys in Mathematics and its Applications,* **24** (2008), 157-174.

[5] S. Bhat and R. Vidya, A class of generalized ridge estimator, *Communications in Statistics-Simulation and Computation,* **47** (2017), 1094-1103.

[6] D. Belsley, A guide to using the collinearity diagnostics, *Computer Science in Economics and Management*, **4** (1991), 33-50.

[7] R. Ebiad, R. Farghali and S. Abo-El-Hadid, A mathematical programming approach for Liu-type estimator, *Advances and Applications in Statistics,* **50** (2017), 293-314.

[8] A. El-Dash , A. El-Hefnawy and R. Farghali, Treating multicollinearity problem using goal programming technique, *The Egyptian Statistical Journal,* **55** (2011), 67-92.

[9] A. El-Hefnawy and A. Farag, A combined nonlinear programming model and Kibria method for choosing ridge parameter regression, *Communications in Statistics: Simulation and Computations,* **43** (2014), 1442-1470. https://doi.org/10.1080/03610918.2012.735317

[10] R. Farghali, Generalized Liu-Type estimator for linear regression, *International Journal of Research and Reviews and in Applied Sciences,* **38** (2019), 52-63.

[11] T.S. Fayose and K. Ayinde, Different forms biasing parameters for generalized ridge regression estimator, *International Journal of Computer Applications,* **181** (2019), 21-29. https://doi.org/10.5120/ijca2019918339

[12] R. Hamed, A. El-Hefnawy and A. Farag, Selection of the ridge parameter using mathematical programming, *Communications in Statistics: Simulation and Computations,* **42** (2012), 1409-1432. https://doi.org/10.1080/03610918.2012.659821

[13] E. Hocking, M., Speed and J., Lynn, A class of biased estimators in linear regression, *Technometrics,* **18** (1976), 455. https://doi.org/10.2307/1268658

[14] A. Hoerl and W. Kennard, Ridge regression: biased estimation for nonorthogonal problems, *Technometrics,* **12** (1970), 55-67. https://doi.org/10.1080/00401706.1970.10488634

[15] A. Hoerl and W. Kennard, Ridge regression: applications to nonorthogonal problems, *Technometrics,* **12** (1970), 69-82. https://doi.org/10.2307/1267352

[16] G. Kibria and S. Banik, Some ridge regression estimators and their performance, *Journal of Modern Applied Statistical Methods,* **15** (2016), 206-238. https://doi.org/10.22237/jmasm/1462075860

[17] A.F. Lukman and K. Ayinde, Review and classifications of the ridge parameter estimation techniques, *Haccetteppe Journal of Mathematics and Statistics,* **46** (2017), 953-967. https://doi.org/10.15672/hjms.201815671

[18] F. Lawless and P. Wang, A simulation study of ridge and other regression estimators, *Communications in Statistics-Theory and Methods*, **5** (1976), 307-323. https://doi.org/10.1080/03610927608827353

[19] K. Liu, A new class of biased estimate in linear regression, *Communications in Statistics-Theory and Methods*, **22** (1993), 393-402. https://doi.org/10.1080/03610929308831027

[20] K. Liu, Using Liu-type estimator to combat collinearity, *Communications in Statistics-Theory and Methods*, **32** (2003), 1009-2003. https://doi.org/10.1081/sta-120019959

[21] K. Liu, More on Liu-type estimator in linear regression, *Communications in Statistics-Theory and Methods*, **33** (2004), 2723-2733. https://doi.org/10.1081/sta-200037930

[22] M., Nomura, On the almost unbiased ridge regression estimation, *Communications in Statistics-Simulation and Computation*, **17** (1988), 729-743. https://doi.org/10.1080/03610918808812690

[23] C.G. Troshie and D.O. Chalton, A Bayesian estimate for the constants in ridge regression, *South African Statistical Journal*, **30** (1996), 119-137.

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