International Journal of Contemporary Mathematical Sciences
Vol. 14, 2019, no. 1, 31 - 42
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/ijcms.2019.914

Optimum Value of Poverty Measure Using Inverse Optimization Programming Problem

Nagwa Mohammed Albehery

Department of Mathematics and Applied Statistics, Faculty of Commerce,
Helwan University, Ain Helwan, Cairo, Egypt

Abstract

The problem of poverty is one of the most important problems faced by many countries in the world, especially developing countries. Decision-makers in many countries and international organizations are developing programs and plans to reduce or eliminate the poverty in the countries that suffer from. In this paper, the new model will be introduced to minimize the poverty level in a population using the inverse optimization programming problem. This model will help the decision maker to determine the optimal distribution of income aids that can be given to the poor people in the poorest areas or groups in a population to decrease the poverty. For illustration, our model will be applied using real data sets collected in Egypt in 2014/2015.

Keywords: Poverty measures, poverty lines, distribution of income aids, mathematical programming problems, dual programming problem, and inverse optimization programming problem

1. Introduction

To alleviate poverty, many programs are developed by decision makers such as increasing the income of poor people to decrease the poverty levels in a population. (B. Bidani and M. Ravallion, 1990) introduced nonlinear programming model to determine the optimal distribution of income aids that give to the poor people to minimize the poverty level and applied the study in Indonesia. (N. M. Albehery, 2003) [2] suggested the deterministic nonlinear goal programming model and the probabilistic nonlinear goal programming model, the two models are introduced to find the optimal distribution of income aids given to the poor people to minimize the poverty and applied the models to Egypt data.

Inverse optimization problem is one of the most important problems that are relatively new area of research and have wide applications in industrial areas and economics. (D. Burton and P. L. Toint, 1992) [5] introduced the inverse optimization. In general, the optimization problem is to find $x^* \epsilon x$ such that the objective function p(x, c) is optimal at x^*. The parameter value of the objective function or the right hand side of the problem can be adjusted as little as possible, so that the given feasible solution becomes optimal solution. (J. Zhang and Z. Liu, 1996) [16] introduced some inverse linear programming problem. (R. Ahuja and J.B. Orlin, 1998) [1] studied the inverse linear programming problem under the L and L norm and provided the general form of the inverse optimization for the linear programming problem. (C. Yang and J. Zhang, 1999) introduced a uniform linear programming problem to formulate a group of inverse optimization problems and provided two computation methods. (C. Heuberger, 2004) [9] presented the various methods for solving the inverse constraint and unconstrained optimization problems. (N. Mohamed, 2006) [14] presented a comparative study for single and multiple objective of the inverse optimization problems. (J. Zhang and C. Xu, 2010) [15] provided the inverse optimization for the linearly constrained convex separable problems and constructed the necessary and sufficient conditions to make the feasible solution is optimal solution of the problem. (Y. Jiang, X. Xian, L. Zhang and J. Zhang, 2011) [11] formulated the inverse linear programming problem as a linear complementarily constrained minimization problem and proposed the perturbation approach to solve the problem. (S. Jain and N. Arya, 2013) [10] introduced the inverse optimization for transportation problems. (H. A. Mohammed and N. M. Albehery, 2016) [13] introduced the algorithm to solve the inverse nonlinear convex problems and presented two applied examples.

In this paper a brief review of poverty measures is presented in section 2. In section 3, mathematical programming to determine the optimal distribution of income aids in the previous studies is presented. In section 4, the definition of the inverse optimization problem is introduced. In section 5, the inverse optimization problem will be used to find the optimal distribution of income aids that make the feasible minimum value of poverty measure is optimum. Applied example will be presented using real data sets collected in Egypt in 2014/2015 in section 6. The conclusion is presented in section 7.

2. A brief review of poverty measures

(J. Y. Duclos and A. Arrar, 2006) [6] defined a poverty measure as the function of random variable X (income or expenditure) and a poverty line z that is the border line between the poor and the non-poor people, denoted by P(x, z), $x \in [0, z)$ and $z \in (0, \infty)$. (J. Foster, J. Greer and E. Thorbock, 1984) [8] introduced the most popular index in this class is called Foster's index that is defined as:

$$\mathbf{P_{\alpha}(x, z) = \frac{1}{n}\sum_{i=1}^{q}\left(\frac{z-x_i}{z}\right)^{\alpha}, \alpha \geq 0} \qquad (1)$$

Where: n is the population size, q is the number of poor people in a population, z is the poverty line. When α = 0, the Foster's index is called the headcount ratio, denoted by H. When α = 1, it is called the poverty gap ratio, denoted by $P_1(x, z)$When α = 2, it is called the severity of poverty or the distribution of income among poor, denoted by $P_2(x, z)$. The foster index can be expressed as a weighted sum of foster indices in m categories or groups of a population as the following:

$$\mathbf{P_\alpha(x, z) = \sum_{j=1}^{m} \left(\frac{n_j}{n}\right) P_\alpha(x^j, z), \alpha \geq 0,} \tag{2}$$

$$\mathbf{P_\alpha(x^j, z) = \frac{1}{n_j} \sum_{i=1}^{q_j} \left(\frac{z - x_i^j}{z}\right)^\alpha, \alpha \geq 0} \tag{3}$$

Where: m is the number of categories (areas or groups of people) in a population, n_j is a number of people in category j, and q_j is a number of poor people in category j. In this paper, the foster poverty index will be used as the measure of poverty level in a population.

3. Mathematical programming to determine the optimal distribution of income aids

(T. Besely and R. Kanbur, 1988) [4] presented a study about the food aids that given to the poorest people in a population as the increasing of their income to decrease the poverty levels. The study used the foster index as followed:

$$\mathbf{P_\alpha(x, z) = \int_0^z \left(1 - \frac{x+y}{z}\right)^\alpha f(x) dx, \ \alpha \geq 0} \tag{4}$$

Where y is the amount of income aid as the increasing of the poor's income. The study derived the change of the poverty measure $P_\alpha(x, z)$ when y changes as the following:

$$\mathbf{\frac{\partial P_\alpha}{\partial y} = -\frac{\alpha}{z} P_{\alpha - 1}, \quad \alpha \geq 0} \tag{5}$$

(N. Kakwani, 1990) [12] calculated the percentage change of poverty measures in each area in a population respect to the income mean and the income inequality as the following:

$$\mathbf{\frac{\partial P_i}{P_i} = \eta_{P_i} \frac{\partial \mu_i}{\mu_i} + \xi_{P_i} \frac{\partial G_i}{G_i} \quad , i = 1, 2, \dots, m} \tag{6}$$

Where: P_i is the poverty measure in area number i, $\mathbf{i = 1, 2, \dots, m}$, m is the number of areas in a population, μ_i is the mean income in area i, is the poverty measure, η_{P_i} is the elasticity of prverty measure respect to the mean income in area i, G_i is the gini index in area i, ξ_{P_i}, market price of the firm at risk equals $100000.

4. The inverse optimization problem

4.1. The mathematical form of programming problem

Consider the following programming problem:

$$\left.\begin{array}{ll}\mathbf{Min\ Z} = \sum_{j=1}^{n} \mathbf{c_j\, f(x_j)} & ,\mathbf{j} = \mathbf{1, 2, \dots, n} \\ \mathbf{S.T} & \\ \sum_{j=1}^{n} \mathbf{a_{ij}\, f(x_j)} \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} \mathbf{b_i} & ,\mathbf{i} = \mathbf{1, 2, \dots, m} \\ \mathbf{x_j} \geq \mathbf{0} & ,\mathbf{j} = \mathbf{1, 2, \dots, n} \end{array}\right\} \quad (7)$$

Where: n is number of decision variables, m is number of constraints,

$f(x_j)$ is the objective function of decision variable x_j j , $j = \mathbf{1, 2}, \dots, n$, c_j is the parameter of decision variables in the objective function, a_{ij} is the parameter of X_j in the constraint i where $i = \mathbf{1, 2}, \dots, m$ and $j = \mathbf{1, 2}, \dots, n$ and b_j is the constant parameters in the right hand side of the constraint i, $i = \mathbf{1, 2}, \dots, m$.

4.2. The mathematical form of the dual programming problem

Since the dual programming problem is important to find the solution of the inverse optimization problem, the mathematical form of the dual programming problem will be defined. The dual programming problem for the linear programming problem defined in (7) takes the following form:

$$\left.\begin{array}{ll}\mathbf{Max\ L} = \sum_{i=1}^{m} \mathbf{b_i f(y_i)} & ,\mathbf{i} = \mathbf{1, 2, \dots, m} \\ \mathbf{S.T} & \\ \sum_{i=1}^{m} \mathbf{a_{ij}\, f(y_i)} \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} \mathbf{c_j} & ,\mathbf{j} = \mathbf{1, 2, \dots, n} \\ \mathbf{y_i} \geq \mathbf{0} & ,\mathbf{i} = \mathbf{1, 2, \dots, m} \end{array}\right\} \quad (8)$$

Where: m is number of dual decision variables, n is number of constraints, y_i is the dual decision variable i , $i = \mathbf{1, 2}, \dots, m$, b_i is the parameter of decision variable in the objective function, a_{ij} is the parameter of Y_i in the constraint j where $j = \mathbf{1, 2}, \dots, n$ and $i = \mathbf{1, 2}, \dots, m$ and c_j is the constant parameters in the right hand side of the constraint j where $j = \mathbf{1, 2}, \dots, n$.

4.3. The mathematical form of the inverse optimization problem

If the optimal solution for the programming problem in (7) is (x^0, z^0)

Where: x^0 and z^0 are the optimal value of decision variable and the objective function respectively. Assume there is a feasible possible solution x^* for the problem in (7) and the decision maker is willing to make (x^*, z^0) is the optimal solution for the problem. The decision maker wants to make x^* is the optimal value for the decision variable and keeps the optimal value of the objective function z^0 for the problem as the same value. Then the inverse optimization for the programming problem in (7) determinate by adjust the coefficient parameters c_j of the decision variable in the objective function in problem (7). The inverse optimization problem for the programming problem can be defined by finding the new coefficient d_j , $j = 1, 2, \dots, n$ of the objective function to make (x^*, z^0) is the optimal solution for the problem as follows:

$$\left.\begin{array}{ll} \mathbf{Min\ Z_{inv}} = \|\mathbf{c}-\mathbf{d}\|_{\mathbf{p}} & \\ \mathbf{S.T} & \\ \sum_{j\in\bar{J}} \mathbf{a_{ji}\, f(y_j)} \geq \mathbf{d_i} & , \mathbf{i} = \mathbf{1, 2, \dots, m} \\ \sum_{j\in J} \mathbf{a_{ji}\, f(y_j)} = \mathbf{d_i} & , \mathbf{i} = \mathbf{1, 2, \dots, m} \\ \mathbf{y_j} \geq \mathbf{0} & , \mathbf{j} = \mathbf{1, 2, \dots, n} \end{array}\right\} \quad (9)$$

Where: y_i is the dual decision variable i , $i = 1, 2, \dots, m$,

$J = \{j \mid X_j^* = 0\}$, $\quad \bar{J} = \{j \mid X_j^* > 0\}$,

$\|c-d\|_p$ presents the vector norm of degree p. When $p = 1$, the objective function is linear function but if $p \geq 2$, the objective function is nonlin ear.

To find the solution of the inverse problem defined in (9) when p = 1, the following equation is used to find the optimal value of d_i^* :

$$\mathbf{d_i^*} = \left[\begin{array}{lll} \mathbf{c_j} - |\mathbf{c_j^*}| & \mathbf{if} & \mathbf{c_j^*} > \mathbf{0},\ \mathbf{X_j^*} > \mathbf{0}, \mathbf{i} = \mathbf{1, 2, \dots, m} \\ \mathbf{c_j} + |\mathbf{c_j^*}| & \mathbf{if} & \mathbf{c_j^*} < 0,\ \mathbf{X_j^*} > \mathbf{0}, \mathbf{j} = \mathbf{1, 2, \dots, n} \\ \mathbf{c_j} & \mathbf{O.W}, & \end{array}\right] \quad (10)$$

$$\mathbf{c_j^*} = \mathbf{c_j} - \sum_{j\in J} \mathbf{a_{ji}\, f(y_j^*)} \quad , \mathbf{i} = \mathbf{1, 2, \dots, m},\ \boldsymbol{j} = \mathbf{1, 2, \dots}, \boldsymbol{n} \quad (11)$$

Where: y_j^* is the optimal value of the dual decision variable j , $j = 1, 2, \dots, n$.

5. The suggested inverse linear programming problem to find the optimal distribution of income aids

Assume the decision maker is willing to decrease the poverty level in a population by increasing the income of the poor people. The decision maker wants to determine the optimal distribution of income aids that can be given to the poor people in a population. As we introduced in section 3, (T. Besely and R. Kanbur, 1988) [4] presented a study about the food aids that given to the poorest people in a population by the increasing of their income to decrease the poverty levels see equation (4). In this paper, the inverse optimization programming problem will be used to make the alternative distribution of income aids is the optimal distribution and make the poverty level in this population is minimum value as the following:

First: The basic objective function of our model is minimizing the poverty measure (Foster poverty index) "$P_\alpha(x, z)$" which is defined in (2) by increasing the income of poor people in a population as the following:

$$\mathbf{Min}\ P_\alpha(x, S, z) = \sum_{j=1}^{m} \left(\frac{n_j}{n}\right) P_{\alpha j}\left(x_j, s_j, z_j\right), \qquad j=1, 2, \ldots, m, \tag{12}$$

$$P_{\alpha j}\left(x_j, s_j, z_j\right) = \sum_{i=1}^{q_j} \left(1 - \frac{x_{ji}+s_{ji}}{z_j}\right)^{\alpha}, \quad \alpha \geq 0, \tag{13}$$

m is the number of areas or groups in a population, j= 1, 2, …, m, n is the population size, x_j is the mean income of poor people in area j, s_j is the amount of income aid that the decision maker needs to give to area j, z_j is the poverty line which is the border line between poor and non-poor people in a population, q_j is the number of poor people in area or group j.To minimize the poverty measure in (12), some constraints will be defined as the following:

$$\sum_{j=1}^{m} a_{ij}\, s_j = S \qquad , i = 1, 2, \ldots, n$$

$$s_j \geq 0 \qquad , j = 1, 2, \ldots, m$$

$$s_j = \sum_{i=1}^{q_j} s_{ji}, \qquad j=1,2,\ldots,m \text{ and } i=1,2,\ldots,q_j$$

Where: S is the total amount of income aids that is fixed by the decision maker to give to the poor people in a population, s_j is the amount of income aids that the decision maker needs to determine to give to the poor people in area or group j, j=1, 2, …, m, a_{ij} is the parameter of the decision variable s_j , j=1,2,…,m in constraint i, i=1, 2, …, n, this parameter is related by the characteristics of the poor people in each area or group. a_j can be defined as the percentage of poor people, the percentage of the illiteracy, the percentage of unemployment, and the percentage of dropout from education in area j, and n is the number of constraints in the problem. Then the programming model will be in the following form:

$$\left.\begin{array}{ll} \mathbf{Min\ P_{\alpha}(x,S,z)} = \sum_{j=1}^{m}\left(\frac{n_j}{n}\right)\sum_{i=1}^{q_j}\left(1-\frac{x_{ji}+s_{ji}}{z_j}\right)^{\alpha} & ,\alpha \geq 0,\ \ j=1,2,\dots,m \\ \mathbf{S.T} & \\ \sum_{j=1}^{m} a_{ij}\, s_j = S & ,i=1,2,\dots,n \\ s_j \geq 0 & ,j=1,2,\dots,m \end{array}\right\} \quad (14)$$

When $\alpha \leq 1$ the objective function is linear function and when $\alpha = 2$ the objective function is nonlinear function.

<u>Second:</u> Assume $\left(P_{\alpha}^{*}, S_{j}^{*}\right), j = 1,2,\dots,m$ is the optimal solution for the model defined in (14). If the decision maker has a feasible possible solution for the decision variables $\left(S_{j}^{0}\right), j = 1,2,\dots,m$.

And he wants to make $\left(P_{\alpha}^{*}, S_{j}^{0}\right), j = 1,2,\dots,m$ is the optimal solution for the problem defined in (14) and keeps the minimum value of the optimal objective function P_{α}^{*} as the same value. Then the inverse optimization programming problem defined in (14) will be defined by adjusting the coefficients in the objective function $\left(c_j\right), j = 1,2,\dots,m$ by detriment the new coefficient parameter $\left(d_j^{*}\right), j = 1,2,\dots,m$ to make $\left(P_{\alpha}^{*}, S_{j}^{0}\right), j = 1,2,\dots,m$ is the optimal solution for the programming problem as the following:

$$\left.\begin{array}{ll} \mathbf{Min\ P_{\alpha_{inv}}} = \|\mathbf{c}-\mathbf{d}\|_{\mathbf{p}} & \\ \mathbf{S.T} & \\ \sum_{j\in\bar{J}} a_{ji}\, Y_j \geq d_i & ,i=1,2,\dots,n \\ \sum_{j\in J} a_{ji}\, Y_j = d_i & ,i=1,2,\dots,n \\ Y_j \geq 0 & ,j=1,2,\dots,m \end{array}\right\} \quad (15)$$

Where: Y_j is the optimal dual decision variable

$j, j = 1,2,\dots,m, J = \left\{j \,\middle|\, s_j^0 = 0\right\}, \quad \bar{J} = \left\{j \,\middle|\, s_j^0 > 0\right\}, \|c-d\|_p$ is the vector norm of degree p, when $p = 1$, the objective function is linear function but if $p \geq 2$, the objective function is nonlinear. Let p = 1, the new value of $d_i^{*}, i = 1,2,\dots,m$ can be calculated by the following form:

$$d_i^* = \begin{cases} \mathbf{c_j} - |\mathbf{c_j^*}| & \textbf{if} \quad \mathbf{c_j^*} > 0, \ \mathbf{s_j^0} > 0, i = 1,2,\dots,m \\ \mathbf{c_j} + |\mathbf{c_j^*}| & \textbf{if} \quad \mathbf{c_j^*} < 0, \ \mathbf{s_j^0} > 0, j = 1,2,\dots,n \\ \mathbf{c_j} & \text{O.W}, \end{cases} \quad (16)$$

$$\mathbf{c_j^*} = \mathbf{c_j} - \sum_{j \in J} \mathbf{a_{ji}} \mathbf{Y_j^*} \quad , \mathbf{i} = \mathbf{1}, \mathbf{2}, \dots, \mathbf{m}, \ \boldsymbol{j} = \mathbf{1}, \mathbf{2}, \dots, \boldsymbol{n} \quad (17)$$

6. Applied example

In this paper we will use data from 2014/2015 that is collected from the Household income, expenditure and consumption survey (HIECS) for the round 2014/2015.This survey was presented by "Central Agency for Public Mobilization and Statistics (CAPMAS)" the government agency to collect data in Egypt. Assume Egypt divided into urban area and rural area, and the decision maker wants to determine the optimal values of the income aid will be distributed to the poor people in these areas to decrease the foster poverty measure $P_\alpha(x, S, z)$, when $\alpha = 1$. The household expenditure is used as indicator of welfare to estimate the poverty line. The relative poverty line will be used as the border line between poor and non poor people in a population and it is defined as a percentage of median of expenditure. (N. M. Albehery and T. Wang, 2011) [3] estimate the lower and the upper limits of the relative poverty line respectively as $Z_- = 1/3$ of the median of annual household expenditure and $Z_+ = 2/3$ of the median of annual household expenditure. The upper limit will be used as the poverty line in this paper and assume the decision maker fix the total amount of income aids to distribute to the urban and rural Egypt areas by S =100 million Egyptian pounds. The following table contains the data used to construct the mathematical model in (14):

Indicators	Urban Egypt (1)	Rural Egypt (2)
Sample size = n	10970	13009
Percentage of poor people	22.334 %	20.078 %
Percentage of illiteracy	13.5 %	25.5 %
Percentage of dropout from education	15 %	35 %
Upper limit of relative poverty line = z_+	22570.17	18856.19

Where:

1- The sample size of people in Urban and Rural Egypt, the percentage of illiteracy, and the percentage of dropout from education are found from "Central Agency for Public Mobilization and Statistics (CAPMAS).

2- The upper limit of relative poverty line z and the percentage of poor people in urban and rural Egypt are calculated using the data from “ Household income, expenditure and consumption survey (HIECS) for the year 2014/2015” which is presented by Central Agency for Public Mobilization and Statistics (CAPMAS).

Then the mathematical model defined in (14) will be the linear programming model as the following:

$\text{Min } P_1(x, S, z) = 0.004431\ S_1 + 0.005303\ S_2$

S.T

$22.334\ S_1 + 20.078\ S_2 = 100$

$13.5\ S_1 + 25.2\ S_2 = 100$

$15\ S_1 + 35\ S_2 = 100$

$s_j \geq 0 \quad , j = 1, 2$

By solving the above model using Excel solver, the optimal solution will be $\boldsymbol{S_1^* = 3.11\ million\ EG, S_2^* = 1.53\ million\ EG\ and\ Min\ P_1^* = 0.022}$. The dual programming problem for the above linear programming problem using the binding constraints can be defined as follow:

$\text{Max } L = 100\ Y_1 + 100\ Y_2$

S.T

$22.334\ Y_1 + 15\ Y_2 = 0.004431$

$20.078\ Y_1 + 35\ Y_2 = 0.005303$

$Y_j \geq 0 \quad , j = 1, 2$

By solving the above dual problem using Excel solver, the optimal solution will be $\boldsymbol{Y_1^* = 0.000157, Y_2^* = 0.0000613\ and\ Max\ L^* = 0.022}$.

Assume the decision maker is willing to make alternative possible distribution of the income aids where the feasible possible amount of income aids that are given to urban and rural Egypt by $\boldsymbol{S_1^0 = 2.95 million\ EG, S_2^0 = 1.7\ million\ EG}$ and he wants to keep the minimum of gap poverty value at $\boldsymbol{P_1^* = 0.022}$.

Then the inverse optimization for the above linear programming problem will be:

$$\mathbf{Min\ P_{1_{inv}} = \|c - d\|}$$

S.T

$$\mathbf{22.334\ Y_1 + 20.078\ Y_2 = d_1}$$

$$\mathbf{15\ Y_1 + 35\ Y_2 = d_2}$$

$$\mathbf{Y_j \geq 0 \quad , j = 1, 2}$$

$d_i^*, i = 1,2$ *is* the optimal value of the new coefficient of decision variables in the objective function and $Y_j^*, j = 1,2$ are the optimal dual decision variables can be calculated using equations (16, 17):

$$\mathbf{c_1^* = 0.004431 - \big(22.3(0.000157) + 20.08(0.0000613)\big) = -0.00031}$$

$$\mathbf{c_2^* = 0.005303 - \big(15(0.000157) + 35(0.0000613)\big) = 0.000803}$$

$$\mathbf{d_1^* = c_1 + |c_1^*| = 0.004431 + 0.00031 = 0.004741}$$

$$\mathbf{d_2^* = c_2 - |c_2^*| = 0.005303 - 0.000803 = 0.0045}$$

The new coefficient of the decision variables of the objective function will be (0.004741, 0.0045) instead of (0.004431, 0.005303) that make the feasible possible values of the decision variables (2.95, 1.7) are the optimal solution for the suggested model and satisfy the minimum value of the gap poverty measure at 0.022.

7. Conclusion

Inverse optimization programming problem provides decision makers with an opportunity to take alternative policies. This paper use the inverse optimization problem to make a feasible possible distribution of income aids is the optimal distribution to make the poverty level in a population is minimum value. In our applied example, the coefficients of the decision variables in the objective function are adjusted to be new coefficients. The new coefficient make the possible distribution of income aids is optimal distribution that minimize the poverty level in a population.

Acknowledgements. I would like to thank Dr. Afaf Aly Hassan El-Dash, professor of operations research at the department of mathematics and applied statistics and insurance, faculty of commerce and business administration, Helwan University for her helpful comments on the initial draft and for her advices to complete this research.

References

[1] R. K. Ahuja and J. B. Orlin, *Inverse Optimization, Part I: Linear Programming And General Problems*, Working Paper, Indian Institute of Technology, 1998, 1-33**.**

[2] N. M. Albehery, *Probabilistic Programming to Estimate Poverty Line and Study its Measure,* PhD Thesis from Cairo University, Giza, Egypt, 2003.

[3] N. M. Albehery and T. Wong, Statistical Inference of Poverty Measures Using U-Statistics Approach, *International Journal of Intelligent Technologies and Applied Statistics*, **2** (2011), 245-263.

[4] T. Besley and R. Kanbur, Food subsidies and poverty alleviation, *The Economic Journal*, **98** (1998), 701-719. https://doi.org/10.2307/2233909

[5] D. Burton and P. L. Toint, On an instance of the inverse shortest paths problem, *Mathematical Programming*, **53** (1992), 45-61. https://doi.org/10.1007/bf01585693

[6] J. Duclos and A. Arrar, *Poverty and Equity: Measurement, Policy and estimation with DAD*, Springer Science & Business Media, New York, USA, 2006.

[7] Egyptian Central Agency of Statistics, Income, Consumption and Expenditure Research (2014/2015), Egyptian Central Agency of Statistics, Cairo, Egypt, (2015).

[8] J. E. Foster and E. A. Thorbecke, A class of decomposable poverty measures, *Econometrica*, **52** (1984), 761-766. https://doi.org/10.2307/1913475

[9] C. Heuberger, Inverse combinatorial optimization: A survey on problem, Methods and Results, *Journal of Combinatorial Optimization*, **8** (2004), 329-361. https://doi.org/10.1023/b:joco.0000038914.26975.9b

[10] S. Jain and N. Arya, An Inverse Capacitated Transportation Problem, *IOSR Journal of Mathematics*, **5** (2013), 24-27. https://doi.org/10.9790/5728-0542427

[11] Y. Jiang, X. Xiao, L. Zhang and J. Zhang, A perturbation approach for a type of inverse linear programming problems, *International J. of Computer Mathematics*, **88** (2011), 508–516. https://doi.org/10.1080/00207160903513003

[12] N. Kakwani, *Poverty and Economic Growth*, World Bank, Washington. D. C, USA, 1990.

[13] H. A. Mohammed and N. M. Albehery, *Applications of Inverse Optimization for Convex Nonlinear Problems*, Institute of Statistical Studies and Research, Cairo University, 2016, 72-82.

[14] N. Mohamed, *A Comparative Study of the Inverse Optimization Problems*, PhD Thesis, Ain Shams University, 2006.

[15] J. Zhang and C. Xu, Inverse optimization for linearly constrained convex separable programming problems, *European Journal of Operational Research*, **200** (2010), 671–679. https://doi.org/10.1016/j.ejor.2009.01.043

[16] J. Zhang and Z. Liu, Calculating some inverse linear programming problems, *Journal of Computational and Applied Mathematics*, **72** (1996), 261-273. https://doi.org/10.1016/0377-0427(95)00277-4

Received: February 15, 2019; Published: March 15, 2019