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Density Estimation of the Egyptian Annual Maximum Earthquake Magnitudes (1964 – 2011) Using a Suggested Nonparametric Gumbel Estimator

Samah M. Abo-El-Hadid

Department of Mathematics, Insurance and Applied Statistics
Faculty of Commerce and Business Administration
Helwan University, Egypt

Abstract

Earthquake has many terrible effects on both environment and human beings. As a matter of fact, a lot of people get killed and injured; also buildings, hospitals, schools, etc get destroyed as a result of its consequences. So in this paper, we going to estimate the probability distribution of the annual maximum earthquake magnitudes in Egypt from the year of 1964 to 2011 using a suggested standard Gumbel kernel density estimator. The statistical properties of the suggested estimator are obtained.

Keywords: Maximum earthquake magnitudes, Gumbel distribution, Nonparametric Density Estimation

1. Introduction

The earthquake can be defined as a shaking in the ground, caused by the movement of the earth's crust. The most widely-used method to measure earthquake magnitudes is the Richter scale. A severe earthquake may have catastrophic effects like: death of hundreds of people, injuries, destruction

and enormous damage to the economies of the affected area, surface faulting, and tsunamis. The objective of this study is to estimate the distribution of annual maximum earthquake magnitude in Egypt with the suggested standard Gumbel kernel density estimator and studying its statistical properties.
Koravos and others (2003) analyze the tectonic moment release rates using historical earthquakes data for Greece over the time period 1899–1986 using the parametric generalized Gamma distribution. Wang and others (2011) estimate the distribution of the annual maximum earthquake magnitudes around Taiwan from 1900 to 2009 using five parametric distributions which were: normal, lognormal, Gamma, Beta, and Pareto distributions. According to the Kolmogorov–Smirnov and Chi-Square tests, the normal, lognormal, and Gamma distributions are acceptable among the five probability distributions.
Haktanir and others (2012) estimate the distribution of Turkish annual maximum earthquake magnitudes from 1907 to 2010 using five parametric distributions which are: Generalized Extreme Value (GEV), Gumbel, log-normal, Pearson, and log-Pearson distributions. They found that the five distributions are suitable but the most suitable one according to Chi-square, Kolmogorov–Smirnov, and probability plot correlation coefficient, is the GEV distribution.
Chiou and Miao (2013) estimate the distribution of the annual maximum earthquake magnitudes in Southern California (from the year of 1932 to 2011) using the Gumbel, Frechet, and generalized extreme value distributions. The Kolmogorov-Smirnov and Anderson-Darling goodness-of-fit tests are used to compare the observed and theoretical cumulative frequencies, the three distributions seem to fit the data equally, but the GEV distribution seems to be the best probability distribution.
Serkan and others (2014) estimate the distribution of Turkish annual maximum earthquake magnitudes, occurred between 1900–2014 years, using various parametric distributions (Generalized Extreme Value, Gumbel, Gamma and Pearson III). The Chi-Square test is applied to these four distributions and the Gumbel and Gamma distributions are accepted but the best fitted distribution is the Gamma distribution.
El-Quliti and others (2016) applied a frequency analyses to records of earthquakes at the Kingdom of Saudi Arabia for years 1913-2016 to predict the annual frequency; the percentage probability; and the probability of a certain-magnitude using Weibull equation. Al Abbasi and others (2018) applied a new distribution, namely, Kumaraswamy-Reflected Weibull distribution to maximum earthquakes magnitude in Iraq for the period 1944-2015. It is found that the new distribution fits the data better than either Gumbel or Reflected Weibull distributions particularly in the case of larger range of maximum earthquake magnitudes.
In this paper, a new nonparametric distribution, namely, nonparametric standard Gumbel distribution is introduced and used to estimate the distribution of maximum earthquake magnitudes in Egypt for the period from 1964 to 2011.

The rest of this paper is organized as follows: In Section 2 the new nonparametric standard Gumbel distribution is introduced and its statistical properties are studied. In Section 3 this new estimator is applied to earthquake data. Finally, in section 4, a brief conclusion is provided.

2. The standard Gumbel kernel estimator

The nonparametric kernel density estimator method was first introduced by Rosenblatt (1956). Rosenblatt's kernel estimator of the unknown density f(x) is given by (Härdle et. al. 2004):

$$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x-x_i}{h}\right) \qquad , -\infty \le x \le \infty \qquad (1)$$

Here n is the sample size; K(·) is the kernel function; and h is the bandwidth.
In this section, we propose the use of standard Gumbel kernel for density estimation. The standard Gumbel kernel function is defined as (Gumbel 1958):

$$K(u) = e^{-(u+e^{-u})} \qquad , \ -\infty \le u \le \infty \qquad (2)$$

where

$$E(u) = \int_{-\infty}^{\infty} uK(u)\, du = \gamma \qquad (3)$$

$$E(u^2) = \int_{-\infty}^{\infty} u^2 K(u)\, du = \gamma^2 + \frac{\pi^2}{6} \qquad (4)$$

$$\mathrm{var}(u) = \frac{\pi^2}{6} \qquad (5)$$

where $\gamma \approx 0.577$ is the Euler-mascheroni constant.
Then the standard Gumbel kernel density estimator is given by:

$$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n} e^{-\left(\frac{x-x_i}{h} + e^{-\left(\frac{x-x_i}{h}\right)}\right)} \qquad , \ -\infty \le x \le \infty \qquad (6)$$

The expected value of the kernel density estimator is (Silverman 1986):

$$E[\hat{f}(x)] = E\left[\frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x-x_i}{h}\right)\right] = \frac{1}{h}\int K\left(\frac{x-x_i}{h}\right) f(x)dx$$

Let $u = \frac{x-x_i}{h}$ then

$$E[\hat{f}(x)] = \frac{1}{h}\int K(u)\, f(x-uh)h du \qquad (7)$$

Using Taylor expansion for $f(x-uh)$ yields that:

$$\mathrm{Bias}[\hat{f}(x)] \simeq -hf'(x)\int uK(u)\, du + \frac{h^2}{2} f''(x)\int u^2 K(u)\, du \qquad (8)$$

Substitute from equations (3) and (4) into equation (8) yields:

$$\mathrm{Bias}\left[\hat{f}(x)\right] \simeq -\gamma\, h\, f'(x) + \frac{h^2}{2}\left(\gamma^2 + \frac{\pi^2}{6}\right) f''(x) \qquad (9)$$

Also, it can be shown that the variance of the kernel density estimator is (Wand and Johnes 1995):

$$\mathrm{Var}\left[\hat{f}(x)\right] = \frac{1}{nh^2}\left\{E\left[K\left(\frac{x-x_i}{h}\right)\right]^2 - \left[EK\left(\frac{x-x_i}{h}\right)\right]^2\right\} \qquad (10)$$
$$= \frac{1}{nh^2}\left\{\int\left[K\left(\frac{x-x_i}{h}\right)\right]^2 f(x_i)dx_i - \left[\int K\left(\frac{x-x_i}{h}\right) f(x_i)dx_i\right]^2\right\}$$

Let $u = \frac{x-x_i}{h}$, then

$$\mathrm{Var}\left[\hat{f}(x)\right] = \frac{1}{nh^2}\{h\int K^2(u)f(x-uh)du - [h\int K(u)\, f(x- uh)du]^2\}$$

Again by Taylor expansion: $f(x-uh) = f(x) - uhf'(x) + \frac{u^2h^2}{2!} f''(x) + \cdots$
Then

$$\mathrm{Var}\left[\hat{f}(x)\right] \simeq \frac{f(x)}{nh}\int K^2(u)du \qquad (11)$$

Using the standard Gumbel kernel function:

$$\int_0^\infty K^2(u)\,du = \int_{-\infty}^\infty \left(e^{-(u+e^{-u})}\right)^2 du = \frac{1}{4} \qquad (12)$$

Substitute equation (12) into equation (11), then the asymptotic variance of the Gumbel kernel density estimator is:

$$\mathrm{Var}\left[\hat{f}(x)\right] = \frac{f(x)}{4nh} \qquad (13)$$

Combining (9) and (13), the mean squared errors for $\hat{f}(x)$ is (Simonoff 1996):

$$\mathrm{MSE}\left[\hat{f}(x)\right] = \mathrm{Var}\left[\hat{f}(x)\right] + \mathrm{Bias}^2\left[\hat{f}(x)\right]$$
$$= \frac{f(x)}{4nh} + h^2\gamma^2\left(f'(x)\right)^2 \qquad (14)$$

Also, the asymptotic integrated mean squared error IMSE for $\hat{f}(x)$ is:

$$\mathrm{IMSE}\left[\hat{f}(x)\right] = \frac{1}{4nh} + h^2\gamma^2\int_{-\infty}^\infty \left(f'(x)\right)^2 dx \qquad (15)$$

The optimal bandwidths which minimize the IMSE for $\hat{f}(x)$ is obtained as follows:

$$\frac{\partial\ \mathrm{IMSE}\left[\hat{f}(x)\right]}{\partial h} = -\frac{1}{4nh^2} + 2\,h\gamma^2\int_{-\infty}^\infty \left(f'(x)\right)^2 dx = 0 \qquad (16)$$

Then:

$$h_{opt} = \left[8n\,\gamma^2 \int_{-\infty}^{\infty} \left(f'(x)\right)^2\, dx\right]^{-\frac{1}{3}} \qquad (17)$$

Now let us replace the unknown term $\int \left(f'(x)\right)^2 dx$ in (17) by the standard Gumbel density as a reference distribution. Let:

$$f(x) = e^{-(x+e^{-x})} \qquad , \; -\infty \le x \le \infty \qquad (18)$$

then

$$f'(x) = (e^{-x} - 1)e^{-(x+e^{-x})} \qquad (19)$$

$$\therefore \left(f'(x)\right)^2 = (e^{-x} - 1)^2 \; e^{-2(x+e^{-x})} \qquad (20)$$

and hence

$$\int_{-\infty}^{\infty} \left(f'(x)\right)^2 dx = \frac{1}{8} \qquad (21)$$

Substituting (21) into (17), the optimal smoothing parameter is:

$$h_{opt} = [n\gamma^2]^{-\frac{1}{3}} \qquad (22)$$

3. Nonparametric maximum earthquake distribution

In this section, the suggested nonparametric standard Gumbel distribution which was introduced in section 2 is used to estimate the distribution of the annual maximum earthquake magnitudes in Egypt. The dataset for the time period from the year 1964 to 2003 is obtained from Dorra (2011); and for the time period from 2004 to 2011 from Abdelazim and others (2016). Figure (1) shows the histogram and the estimated distribution of the maximum earthquake magnitudes using both the parametric Gumbel distribution and the nonparametric Gumbel distribution.

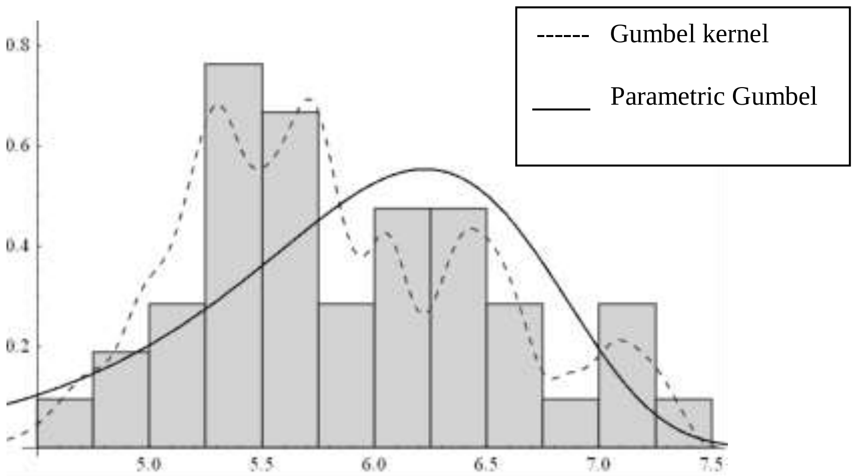

Fig. (1): The histogram; parametric and nonparametric Gumbel estimator of annual maximum earthquake magnitude

The above figure shows that the nonparametric Gumbel estimator matches with the annual maximum earthquake magnitudes data compared with the parametric one.

4. Conclusions

In this paper, the distribution of the maximum earthquake magnitudes in Egypt is estimated using a suggested new nonparametric estimator of the probability density function using the standard Gumbel kernel function. Also, the theoretical properties and the optimal smoothing parameter of this proposed estimator are obtained.

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