

# The Determination of the Reorder Point Using the Marshall–Olkin Extended Weibull Distribution in Inventory Control

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## Abstract

This paper aims to determine the reorder level  $R$  when the lead time demand has the Marshall–Olkin extended Weibull distribution which is introduced by Marshall and Olkin in 1997. Using some methods of estimations such as maximum likelihood, method of moments, percentile based estimation and least squares the three unknown parameters of the distribution are estimated.

Using the value of  $R$  and the estimated parameters according to each method the most important functions in inventory models such as the protection lost sales  $P_R$  (i.e., the probability of not going out of stock) and the potential lost sales  $S_R$  (i.e., the unsatisfied demand) were obtained. Finally some conclusions are presented.

**Keywords:** the Marshall–Olkin extended Weibull distribution; Inventory control; Reorder Point; Protection lost sales; Mean and the variance of potential lost sales; Lead time demand; Methods of estimation

## 1. Introduction

In classical inventory models our main focus is on identifying the protection lost sales (i.e., the probability of not going out of stock) and the potential lost sales (i.e., the unsatisfied demand) when both demand and lead time have a certain proba-

bility distribution. If the probability density function (*p.d.f*) of demand during lead time (i.e., lead time demand) is  $f(x)$ , then for a reorder level system of control with reorder  $R$  the protection  $P_R$ , the mean of potential lost sales  $S_R$  and the variance of potential lost sales  $V_R$  are given by Burgin and Wild (1967) as:

$$P_R = \int_0^R f(x) dx, \quad 0 \leq x \leq \infty \quad (1)$$

$$S_R = \int_R^\infty (x - R) f(x) dx, \quad 0 \leq x \leq \infty \quad (2)$$

$$V_R = \int_R^\infty (x - R - S_R)^2 f(x) dx \quad (3)$$

Where,  $x$  is random variable that represents the demand for a particular time period, and  $R$  is the reorder level.

In order to determine the previous functions the distribution of lead time demand distribution must has the following general characteristics [Burgin and Wild (1975)]:

- (1) It should be able to represent only non-negative values of demand.
- (2) It should be able to represent frequency distribution changing from;
  - (a) mono- tonically decreasing to (b) unimodal distributions heavily skewed to the right and finally to (c) normal type distributions (truncated to zero).

Here, the main obstacle is to find a probability distribution to which these characteristics apply to. Since these previous characteristics are satisfied in the Marshall–Olkin extended Weibull (MOEW) distribution which is introduced by Marshall and Olkin in 1997, with the following probability density function;

$$f(x; \alpha, \gamma, \lambda) = \frac{\alpha \gamma \lambda x^{\gamma-1} e^{-\lambda x^\gamma}}{(1 - \bar{\alpha} e^{-\lambda x^\gamma})^2}, \quad x > 0, \quad \gamma, \alpha, \lambda > 0 \quad (4)$$

where,  $\bar{\alpha} = 1 - \alpha$ ,  $\gamma$  and  $\alpha$  are the shape parameters and  $\lambda$  is the scale parameter. Then the cdf  $F(x)$  of MOEW distribution for  $x > 0$  is given by;

Where,  $\gamma$  is the shape parameter and  $\lambda$  is the scale parameter. Then the cdf  $F(x)$  of MOEW distribution for  $x > 0$  is given by;

$$F(x) = \frac{1 - e^{-\lambda x^\gamma}}{1 - \bar{\alpha} e^{-\lambda x^\gamma}} \quad (5)$$

The MOEW hazard rate function takes the form;

$$h(x) = \frac{\gamma \lambda x^{\gamma-1}}{1 - \bar{\alpha} e^{-\lambda x^\gamma}}, \quad x > 0 \quad (6)$$

For all values of  $\alpha \geq 1$  and  $\gamma \geq 1$  we find that the function  $h(x)$  is increasing, in contrast the function turns into decreasing function if  $\alpha \leq 1$  and  $\gamma \leq 1$ .

Figures (1) below illustrate the graphical representation for some of selected parameter values for functions (4) and (6) respectively. The Figure indicate that the MOEW distribution is very versatile and the value of  $\alpha$  has essential effect on its skewness and kurtosis. From the figure we note that this distribution can be used in several problems since its hazard rate function hesitant between increasing and decreasing.

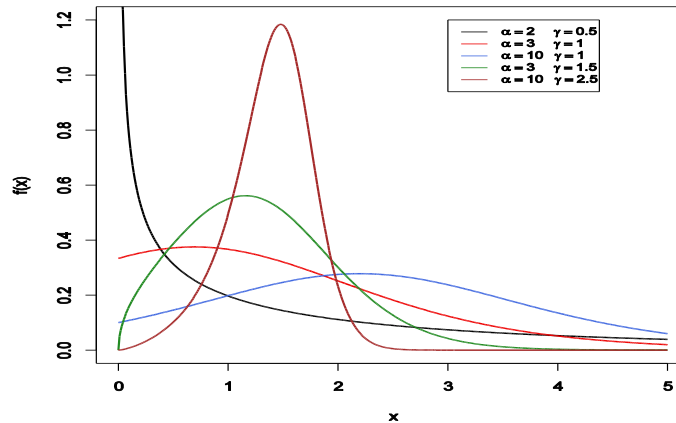


Figure (1): Graphical representation of the p.d.f for MOEW distribution at different values of parameters

## 2. Methods of Estimation

In this section, we will introduce some methods for estimating the parameters,  $\lambda, \gamma$  and  $\alpha$  of the MOEW distribution. Let  $x = (x_1, x_2, \dots, x_n)$  is a random sample of size  $n$  from the MOEW distribution with unknown parameters  $\lambda, \gamma$  and  $\alpha$ .

### 2.1. Maximum Likelihood Estimators

The method of maximum likelihood is the most frequently used method of parameter estimation. Its success stems from their well-estimated characteristics, including consistency, asymptotic efficiency and invariance property. Using the maximum likelihood method, the estimators of three unknown parameters  $\lambda, \gamma$  and  $\alpha$ , can be obtained by taking the natural log of the likelihood function of a random sample consisting of observation  $x_i$ ,  $i = 1, 2, \dots, n$  from a distribution with p.d.f. (4) is;

$$L(x_i; \lambda, \gamma, \alpha) = \prod_{i=1}^n f(x_i; \alpha, \gamma, \lambda) = \prod_{i=1}^n \frac{\alpha \gamma \lambda x_i^{\gamma-1} e^{-\lambda x_i^\gamma}}{(1 - \bar{\alpha} e^{-\lambda x_i^\gamma})^2} \quad (7)$$

The log-likelihood function without constant term is given by;

$$\ell n(L) = n \ell n(\alpha, \gamma, \lambda) + (\gamma - 1) \sum_{i=1}^n \ell n(x_i) - \lambda \sum_{i=1}^n x_i^\gamma - 2 \sum_{i=1}^n \ell n(1 - \bar{\alpha} e^{-\lambda x_i^\gamma}) \quad (8)$$

By taking the partial derivatives of the log-likelihood function with respect to the three parameters in  $L$ , to get;

$$\left\{ \begin{aligned} \frac{\partial \ell n L}{\partial \alpha} &= \frac{n}{\alpha} - 2 \sum_{i=1}^n \left( \frac{e^{-\lambda x_i^\gamma}}{1 - \bar{\alpha} e^{-\lambda x_i^\gamma}} \right), \\ \frac{\partial \ell n L}{\partial \lambda} &= \frac{n}{\lambda} - \sum_{i=1}^n x_i^\gamma - 2 \bar{\alpha} \sum_{i=1}^n x_i^\gamma \left( \frac{e^{-\lambda x_i^\gamma}}{1 - \bar{\alpha} e^{-\lambda x_i^\gamma}} \right), \text{ and} \\ \frac{\partial \ell n L}{\partial \gamma} &= \frac{n}{\gamma} - \sum_{i=1}^n \ell n(x_i) - \lambda \sum_{i=1}^n x_i^\gamma \ell n(x_i) - 2 \bar{\alpha} \lambda \sum_{i=1}^n x_i^\gamma \left( \frac{e^{-\lambda x_i^\gamma}}{1 - \bar{\alpha} e^{-\lambda x_i^\gamma}} \right) \ell n(x_i), \end{aligned} \right\} \quad (9)$$

Setting the above derivatives in equations (9) equal to zero and then, solving these three nonlinear likelihood estimating equations numerically to yield the maximum likelihood estimates.

## 2.2. Ordinary and Weighted Least-Squares Estimators

The least square and weighted least square estimators were proposed by Swain, Venkatraman, and Wilson (1988) to estimate the parameters of Beta distributions. In this paper, we apply the same technique for the MOEW distribution. The least square estimators of the unknown parameters  $\lambda$ ,  $\gamma$ , and  $\alpha$  of MOEW distribution can be obtained by minimizing the function;

$$\sum_{j=1}^n \left[ F(X_{(j)}) - \frac{j}{n+1} \right]^2 \quad (10)$$

With respect to three unknown parameters  $\lambda$ ,  $\gamma$ , and  $\alpha$ .

Suppose that  $F(X_{(j)})$  denotes the distribution function of the ordered random variables  $X_{(1)} < X_{(2)} < \dots < X_{(n)}$ , where  $\{X_{(1)}, X_{(2)}, \dots, X_{(n)}\}$  is a random sample of size  $n$  from a distribution function  $F(\cdot)$ . Therefore, in this case, the least square estimators  $\hat{\lambda}_{OLSE}$ ,  $\hat{\gamma}_{OLSE}$ , and  $\hat{\alpha}_{OLSE}$ , of the parameters  $\lambda$ ,  $\gamma$ , and  $\alpha$  can be obtained by minimizing the function;

$$\sum_{j=1}^n \left[ \frac{1 - e^{-\lambda x_{(j)}^\gamma}}{1 - \bar{\alpha} e^{-\lambda x_{(j)}^\gamma}} - \frac{j}{n+1} \right]^2 \quad (11)$$

With respect to  $\lambda$ ,  $\gamma$ , and  $\alpha$ .

The weighted least square estimators of the three unknown parameters  $\lambda$ ,  $\gamma$ , and  $\alpha$  can be obtained by minimizing the function;

$$\sum_{j=1}^n w_j \left[ F(X_{(j)}) - \frac{j}{n+1} \right]^2 \quad (12)$$

With respect to unknown parameters to  $\lambda$ ,  $\gamma$ , and  $\alpha$ . The weights ( $w_j$ ) are equal to  $\frac{1}{V(X_{(j)})} = \frac{(n+1)^2(n+2)}{(n-j+1)}$ . Therefore, in this case, the weighted least square estimators  $\hat{\lambda}_{WLSE}$ ,  $\hat{\gamma}_{WLSE}$ , and  $\hat{\alpha}_{WLSE}$ , of the parameters  $\lambda$ ,  $\gamma$ , and  $\alpha$  can be obtained by minimizing the function;

$$\sum_{j=1}^n \frac{(n+1)^2(n+2)}{(n-j+1)} \left[ \frac{1 - e^{-\lambda x_{(j)}^\gamma}}{1 - \bar{\alpha} e^{-\lambda x_{(j)}^\gamma}} - \frac{j}{n+1} \right]^2 \quad (13)$$

With respect to  $\lambda$ ,  $\gamma$ , and  $\alpha$ .

### 3. Simulation Study

In this section, we will conduct a Monte Carlo simulation study to evaluate the performance of the different methods of estimation discussed in the previous section the simulation study is conducted by using Mathcad program version 14. We evaluate the performance of the different estimators in terms of their mean squared errors (MSEs). We generate 1000 samples of MOEW distribution, when  $n = (20, 50, 100, 150)$  and by choosing  $(\lambda, \gamma, \alpha) = (2, 2.5, 10)$  respectively. The average value of estimates and RMSEs of MLEs, LSEs and WLSEs are obtained and presented in tables (1) to (4).

Table (1): Average values estimates and the corresponding RMSEs for  $n = 20$

	$\hat{\lambda}$		$\hat{\gamma}$		$\hat{\alpha}$	
MLE	1.967	(0.367)	2.702	(0.656)	8.903	(14.952)
OLSE	2.010	(0.351)	2.492	(0.668)	8.928	(2.836)
WLSE	2.021	(0.353)	2.476	(0.634)	9.505	(1.927)

Table (2): Average values estimates and the corresponding RMSEs for  $n = 50$ 

	$\hat{\lambda}$		$\hat{\gamma}$		$\hat{\alpha}$	
MLE	1.978	(0.216)	2.592	(0.342)	8.863	(14.708)
OLSE	1.997	(0.212)	2.510	(0.381)	9.197	(2.453)
WLSE	2.00	(0.216)	2.512	(0.354)	9.730	(1.423)

Table (3): Average values estimates and the corresponding RMSEs for  $n = 100$ 

	$\hat{\lambda}$		$\hat{\gamma}$		$\hat{\alpha}$	
MLE	1.996	(0.154)	2.542	(0.242)	8.515	(16.585)
OLSE	2.003	(0.157)	2.509	(0.275)	9.572	(1.791)
WLSE	2.005	(0.158)	2.509	(0.259)	9.835	(1.112)

Table (4): Average values estimates and the corresponding RMSEs for  $n = 150$ 

	$\hat{\lambda}$		$\hat{\gamma}$		$\hat{\alpha}$	
MLE	1.996	(0.127)	2.525	(0.186)	8.050	(14.687)
OLSE	2.003	(0.127)	2.496	(0.216)	9.723	(1.443)
WLSE	2.003	(0.128)	2.503	(0.203)	9.910	(0.822)

From previous tables it can be noted that, all the estimates shows the property of consistency i.e., the larger the sample size, the lower RMSEs. By comparing between the different methods of estimation, the results presented in the previous tables show that the WLSEs method is the best method for estimating the parameters  $\lambda$ ,  $\gamma$ , and  $\alpha$  in terms of RMSEs in most cases. If we arrange the three estimation methods based on the RMSEs criterion from the best to the worst, according to  $\lambda$  are OLSE, WLSE and MLE, according to  $\gamma$  are MLE, WLSE and OLS, finally according to  $\alpha$  are WLSE, OLSE and MLE.

#### 4. Determination of Reorder Level $R$ , Mean and Variance of Potential Lost Sales for the Marshall–Olkin extended Weibull distribution

If we operate a classic reorder level system of inventory control then the protection  $P_R$  for the the Marshall–Olkin extended Weibull distribution defined by p.d.f. (4) and a reorder level  $R$  is;

$$P_R = \int_0^R \frac{\alpha \gamma \lambda x^{\gamma-1} e^{-\lambda x^\gamma}}{(1 - \bar{\alpha} e^{-\lambda x^\gamma})^2} \cdot dx \quad (14)$$

In the case of out of stock, there should be a measure of the unmet demand which is called (potential lost sales), so for a reorder level  $R$  the mean of the potential lost sales  $S_R$  of the MOEW distribution is given by;

$$S_R = \int_R^{\infty} (x - R) \cdot \frac{\alpha \gamma \lambda x^{\gamma-1} e^{-\lambda x^\gamma}}{(1 - \bar{\alpha} e^{-\lambda x^\gamma})^2} dx \quad (15)$$

The variance of potential lost sales for the MOEW distribution can be obtained by the following function;

$$V_R = \int_R^{\infty} (x - R - S_R)^2 \cdot \frac{\alpha \gamma \lambda x^{\gamma-1} e^{-\lambda x^\gamma}}{(1 - \bar{\alpha} e^{-\lambda x^\gamma})^2} dx \quad (16)$$

Now after estimating the three unknown parameters  $\lambda$ ,  $\gamma$ , and  $\alpha$  for the MOEW distribution we are interested in determining the reorder level  $R$  for a given protection levels  $P_R$  (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9). Our main focus is on finding  $R$  for a given  $P_R$  in equation (1). Therefore;

$$R = F^{-1}(P_R) \quad (17)$$

The value of  $R$  in (17) can easily be obtained in case of MOEW distribution using Mathcad program version 14. Table (5) shows the different values of the reorder level  $R$  which was estimated based on estimates of the parameter values for MOEW distribution according to each of the three previous estimation methods. Using MLE, OLSE and WLSE estimates, then for the protection lost sales  $P_R$  with reorder level system of  $R$  the mean of potential lost sales  $S_R$ , and the variance of potential lost sales  $V_R$  can be obtained also in table (5) as follow,

Table (5): The Reorder level  $R$  for  $(n = 20; n = 50; n = 100; n = 150)$  using MLE, LSE and WLSE methods of estimation.

	$P_R$	$R$				$S_R$				$V_R$			
		$n = 20$	$n = 50$	$n = 100$	$n = 150$	$n = 20$	$n = 50$	$n = 100$	$n = 150$	$n = 20$	$n = 50$	$n = 100$	$n = 150$
MLE	0.1	0.678	0.665	0.649	0.637	0.37188	0.38563	0.39233	0.39605	0.03985	0.04350	0.04550	0.04671
	0.2	0.825	0.816	0.802	0.791	0.24572	0.25599	0.26142	0.26471	0.02874	0.03151	0.03306	0.03401
	0.3	0.920	0.914	0.902	0.891	0.17425	0.18204	0.18628	0.18889	0.02199	0.02419	0.02542	0.02618
	0.4	0.994	0.991	0.981	0.971	0.12610	0.13202	0.13527	0.13727	0.01709	0.01885	0.01984	0.02045
	0.5	1.058	1.058	1.049	1.040	0.09079	0.09523	0.09766	0.09914	0.01316	0.01455	0.01533	0.01580
	0.6	1.119	1.121	1.114	1.105	0.06359	0.06681	0.06856	0.06962	0.00978	0.01084	0.01144	0.01179
	0.7	1.181	1.186	1.180	1.173	0.04199	0.04419	0.04538	0.04608	0.00679	0.00755	0.00797	0.00822
	0.8	1.251	1.259	1.255	1.250	0.02458	0.02591	0.02663	0.02704	0.00412	0.00459	0.00485	0.00500
	0.9	1.347	1.360	1.359	1.355	0.01056	0.01116	0.01148	0.01165	0.00178	0.00199	0.00211	0.00217
OLSE	0.1	0.651	0.660	0.667	0.668	0.37266	0.39047	0.40879	0.42276	0.04452	0.04594	0.04832	0.05092
	0.2	0.806	0.815	0.821	0.823	0.24882	0.26111	0.27612	0.28841	0.03274	0.03378	0.03611	0.03867
	0.3	0.906	0.915	0.921	0.923	0.17897	0.18722	0.19915	0.20950	0.02568	0.02628	0.02829	0.03056
	0.4	0.986	0.993	0.999	1.002	0.13189	0.13707	0.14631	0.15480	0.02049	0.02075	0.02237	0.02430
	0.5	1.055	1.062	1.067	1.070	0.09717	0.09998	0.10687	0.11364	0.01622	0.01622	0.01746	0.01902
	0.6	1.120	1.127	1.132	1.135	0.07008	0.07110	0.07596	0.08112	0.01245	0.01226	0.01315	0.01436
	0.7	1.187	1.193	1.198	1.201	0.04813	0.04786	0.05096	0.05464	0.00899	0.00868	0.00925	0.01012
	0.8	1.264	1.269	1.273	1.277	0.02981	0.02877	0.03038	0.03269	0.00577	0.00540	0.00570	0.00624
	0.9	1.369	1.373	1.377	1.381	0.01415	0.01292	0.01340	0.01445	0.00275	0.00244	0.00253	0.00276
WLSE	0.1	0.659	0.670	0.672	0.673	0.38014	0.39776	0.41225	0.42523	0.04574	0.04676	0.04875	0.05107
	0.2	0.815	0.825	0.826	0.827	0.25537	0.26738	0.27914	0.29051	0.03408	0.03476	0.03662	0.03889
	0.3	0.916	0.925	0.926	0.927	0.18464	0.19252	0.20173	0.21123	0.02693	0.02721	0.02877	0.03077
	0.4	0.994	1.003	1.003	1.005	0.13672	0.14145	0.14847	0.15620	0.02159	0.02156	0.02279	0.02448
	0.5	1.063	1.070	1.071	1.073	0.10118	0.10350	0.10862	0.11472	0.01715	0.01688	0.01780	0.01916
	0.6	1.128	1.135	1.135	1.137	0.07331	0.07381	0.07731	0.08190	0.01321	0.01277	0.01342	0.01445
	0.7	1.195	1.201	1.201	1.203	0.05060	0.04981	0.05194	0.05515	0.00957	0.00905	0.00945	0.01018
	0.8	1.272	1.276	1.276	1.279	0.03152	0.02999	0.03100	0.03296	0.00616	0.00564	0.00582	0.00627
	0.9	1.377	1.379	1.379	1.382	0.01508	0.01348	0.01369	0.01453	0.00296	0.00255	0.00258	0.00277



The probability of going out of stock  $H_R$  is the complement of probability of not going out of stock  $P_R$  which is computed as;

$$H_R = 1 - \int_0^R \frac{\alpha \gamma \lambda x^{\gamma-1} e^{-\lambda x^\gamma}}{(1 - \bar{\alpha} e^{-\lambda x^\gamma})^2} \cdot dx \quad (18)$$

We note that there is an inverse relationship between the reorder point  $R$  and both of the complement of protection lost sales  $H_R$ , and the mean of potential lost sales  $S_R$ .

The results can be displayed graphically for this case, in figure (2) and (3) as follows;

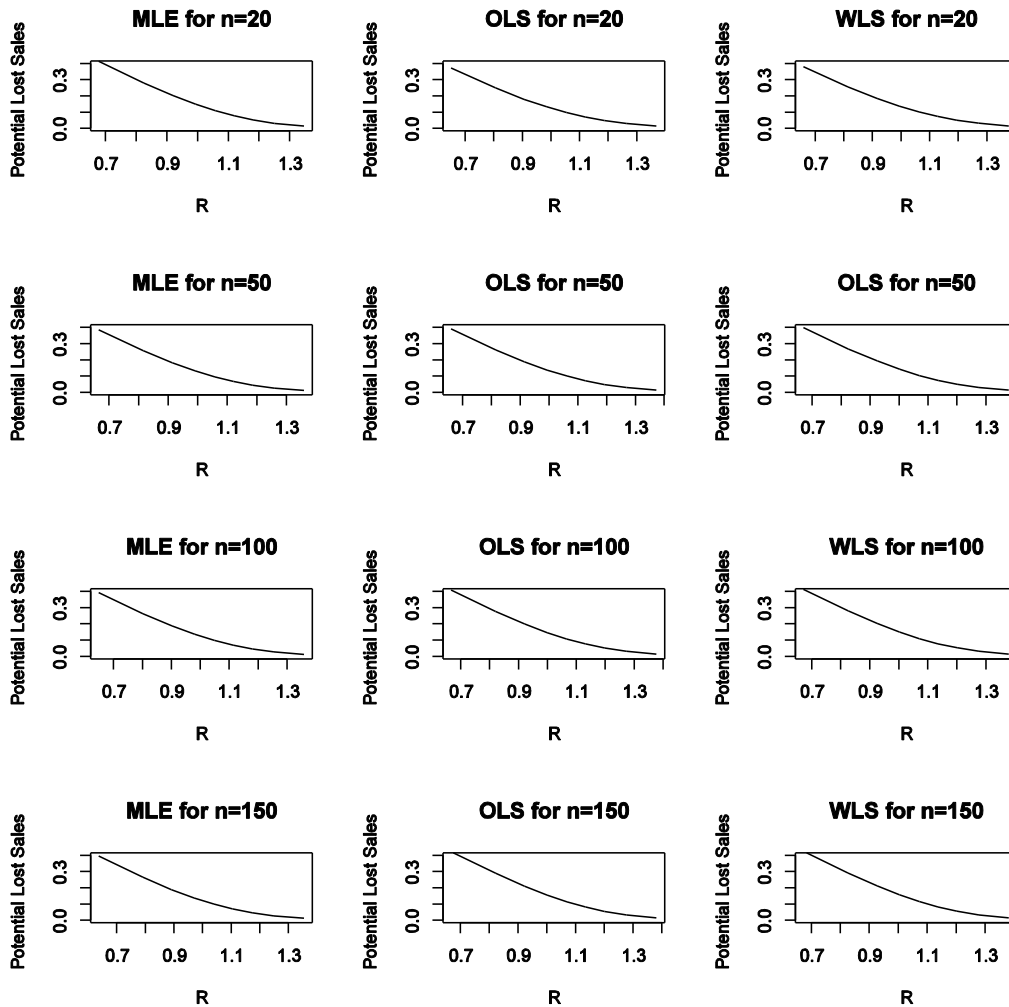


Figure (2): the relation between reorder level ( $R$ ) and potential lost sales ( $S_R$ ) for different methods of estimation and different samples sizes.

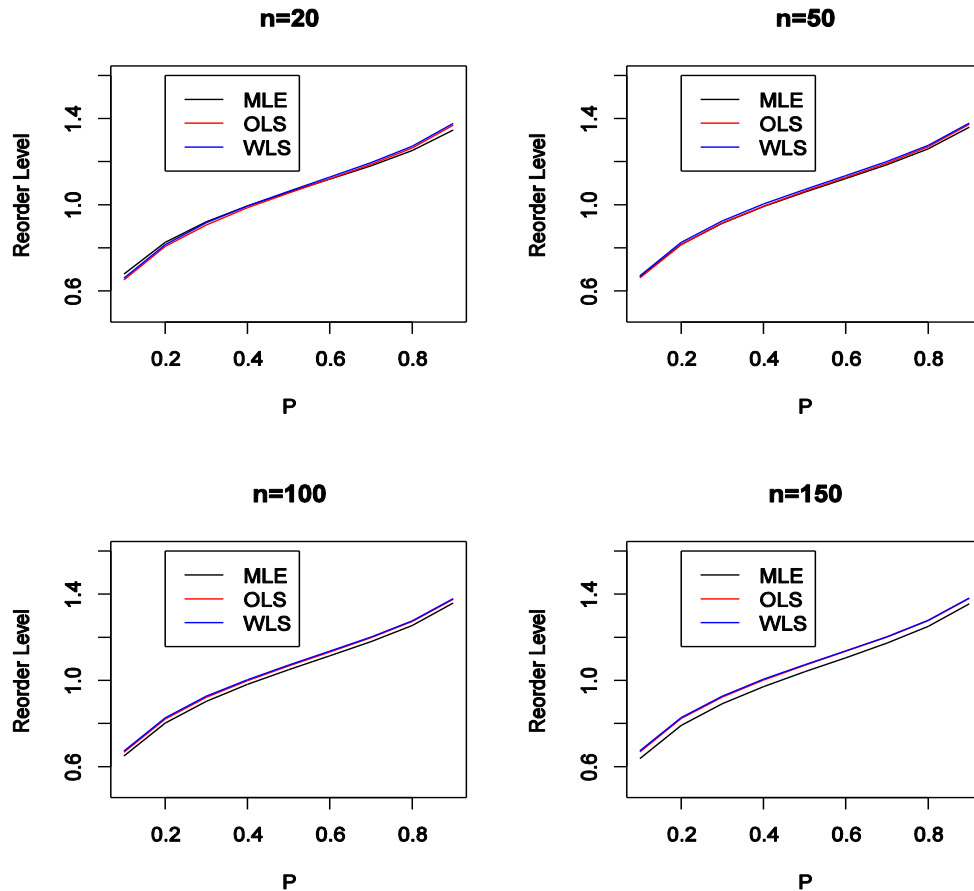


Figure (3): the relation between reorder level ( $R$ ) and protection lost sales ( $P_R$ ) for different methods of estimation and different samples sizes.

## 5. Summary and Conclusions

Marshall–Olkin extended Weibull distribution, introduced by Marshall and Olkin in 1997, is an important distribution that can be used in inventory model applications as an alternative to gamma distribution, which was presented by Burgin in 1973 because it represent only for non–negative values of demand, its frequency distribution changing from; mono–tonically decreasing to uni–modal distributions heavily skewed to the right and finally to normal type distributions (truncated to zero). These are the characteristics that must be met in the distribution of lead time demand in inventory control.

A Monte Carlo simulation study was conducted to evaluate the performance of the different three methods of estimation (maximum likelihood, least square and weighted least square) the simulation study is conducted by using Mathcad program version 14. The performance of the different estimators in terms of their mean squared errors (MSEs) was evaluated. Thousand samples of MOEW distribution, when  $n = (20, 50, 100, 150)$  were generated and by choosing  $(\lambda, \gamma, \alpha) = (2, 2.5, 10)$  respectively.

The average value of estimates and RMSEs of the methods of estimation are obtained and presented.

Using the estimated values of the three distribution parameters  $\hat{\lambda}$ ,  $\hat{\gamma}$  and  $\hat{\alpha}$  it was possible to obtain the important functions of the inventory models such as reorder level  $R$ , mean  $S_R$  and variance  $V_R$  of the unsatisfied demand were obtained at different values for the probability of not going out of stock  $P_R$ .

The results have shown that;

- The WLSEs method is the best method for estimating the parameters  $\lambda, \gamma$ , and  $\alpha$  in terms of RMSEs in most cases.
- There is an inverse relationship between the reorder level  $R$  and both of the complement of protection lost sales  $H_R$ , and the mean of potential lost sales  $S_R$ .
- MOWE distribution provides a very good distribution for lead time distribution in inventory control.

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**Received: September 1, 2018; Published: September 12, 2018**