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Delta Generalized Pre-Closed Sets in Topological Spaces

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Abstract

In this paper, a new class of sets called delta generalized pre-closed sets in topological spaces is introduced and some of their basic properties are investigated. This new class of sets lies between the class of gp-closed sets and the class of gpr-closed sets. Further the notions of δgp-neighbourhood, δgp-closure and δgp-interior are introduced and their properties are discussed. Several examples are provided to illustrate the behavior of new sets.

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1 Introduction

The concept of generalized closed sets introduced by Levine [8] plays a significant role in general topology. After the introduction of generazed closed sets many research papers were published which deal with different types of generalized closed sets. Y. Gnanambal [7] introduced the concept of gpr-closed set and investigated its basic properties. H. Maki et al. [11] defined the concept of gp-closed set in topological spaces and established results related to it. These concepts motivated us to define a new class of sets called the delta generalized pre-closed sets.

Throughout this paper, (X,τ)(or simply X) represents topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space (X, τ), we denote the closure of A ,the interior of A and complement of A as cl(A), int(A) and A^c respectively.

2 Preliminaries

Let us recall the following definitions which are useful in the sequel

Definition 2.1 *A subset A of a topological space X is called,*
(i) pre-open [12] *if A⊆int(cl(A)) and Pre-closed if cl(int(A))⊆A.*
(ii)α-open [13] *if A⊆int(cl(int(A))) and α-closed if cl(int(cl(A)))⊆A.*
(iii)b-open [2] *if A ⊆cl(int(A))∪ int(cl(A)) and b-closed if cl(int(A))∩ int(cl(A))⊆A.*
(iv) regular-open [14] *if A=int(cl(A)) and regular-closed if A=cl(int(A)).*
(v)δ-closed[15] *if A=cl_δ(A) where $cl_\delta(A)=\{x\in X{:}int(cl(U))\cap A\neq\phi, U\in\tau$ and $x\in U\}$.*

The pre-closure of a subset A of X,denoted by pcl(A) is the intersection of all pre-closed sets containing A.The pre-interior of a subset A of X,denoted by pint(A) is the union of all pre-open sets contained in A.

Definition 2.2 *A subset A of a topological space X is called,*
(*i*) *generalized closed (briefly,g-closed)*[8] *if cl(A)⊆G whenever A⊆G and G is open in X.*
(*ii*) *generalized α-closed (briefly,gα-closed)*[9] *if αcl(A)⊆G whenever A⊆G and G is α-open in X.*
(*iii*) *α-generalized closed (briefly,αg-closed)*[10] *if αcl(A)⊆G whenever A⊆G and G is open in X.*
(*iv*) *generalized pre-closed (briefly,gp-closed)*[11] *if pcl(A)⊆G whenever A⊆G and G is open in X.*
(*v*) *generalized preregular closed (briefly,gpr-closed)*[7] *if pcl(A)⊆G whenever A⊆G and G is regular open in X.*
(*vi*) *δ-generalized closed (briefly,δg-closed)*[6] *if δcl(A))⊆G whenever A⊆G and G is open in X.*

(vii) generalized b-closed (briefly,gb-closed)[1] *if bcl(A)$\subseteq$G whenever A$\subseteq$G and G is open in X.*

The complements of the above mentioned closed sets are their respective open sets.

Definition 2.3 *A topological space X is said to be*
(1)$T_{\frac{1}{2}}$ space[8] *if every g-closed subset of X is closed.*
(2)T_{gs}-space[5] *if every gp-closed subset of X is pre-closed.*
(3)pre regular $T_{\frac{1}{2}}$-space[7] *if every gpr-closed subset of X is pre-closed.*

3 Delta Generalized Pre-Closed Sets.

Definition 3.1 *A subset A of a topological space X is called a delta generalized pre-closed (briefly, δgp-closed) set if pcl(A)$\subseteq$U whenever A$\subseteq$U and U is δ-open in X.*

The family of all δgp-closed sets in a topological space X is denoted by δGPC(X).

Example 3.2 *Let X=$\{a,b,c\}$ and topology $\tau=\{X,\phi,\{a\},\{b\},\{a,b\}\}$,then δGPC(X)=$\{X, \phi,\{c\},\{a,c\},\{b,c\}\}$.*

Theorem 3.3 *Every pre-closed set is δgp-closed but not conversely.*
Proof: *Let A be a pre-closed set in X such that A$\subseteq$G where G is δ-open in X. Since A is pre-closed,then pcl(A)=A. Therefore pcl(A)$\subseteq$G.*

Example 3.4 *Consider X=$\{a,b,c,d\}$ with the topology $\tau=\{X,\phi,\{a\},\{b\},\{a,b\},\{a,b,c\}\}$. Let A=$\{a,c\}$,then A is δgp-closed but not pre-closed.*

Theorem 3.5 *Every regular-closed(resp,δ-closed,closed,α-closed and gα-closed)set is δgp-closed but not conversely.*
Proof:*Follow from the following implications and Theorem 3.3*
regular-closed$\rightarrow\delta$-closed[15]*$\rightarrow$closed$\rightarrow\alpha$-closed*[13]*$\rightarrow$gα-closed*[9]*$\rightarrow$pre-closed.*

Theorem 3.6 *Every gp-closed set is δgp-closed but not conversely.*
Proof: *Follows from the fact that every δ-open set is open*[15].

Example 3.7 *In Example 3.4, the subset $\{a,c\}$ is δgp-closed but not gp-closed.*

Theorem 3.8 *Every δg-closed(resp,g-closed,αg-closed)set is δgp-closed but not conversely.*
Proof:*Since every δg-closed set is g-closed*[6],*every g-closed set is αg-closed*[10] *and every αg-closed set is gp-closed*[11].*Hence from Theorem 3.6,the proof follows.*

Theorem 3.9 *Every δgp-closed set is gpr-closed but not conversely.*
Proof:*Follows from the fact that every regular open set is δ-open*[15].

Example 3.10 *In Example 3.4,the subset $\{a,b\}$ is gpr-closed but not δgp-closed.*

Remark 3.11 *The following examples show that δgp-closed set is independent of b-closed and gb-closed sets.*

Example 3.12 *In Example 3.4, the subset $\{a\}$ is b-closed and hence gb-closed but not δgp-closed.*

Example 3.13 *In Example 3.4, the subset $\{a,b,c\}$ is δgp-closed but not gb-closed and hence not b-closed.*

Theorem 3.14 *Let $B \subseteq X$ be a δgp-closed set ,then pcl(B)-B contains no non empty δ-closed set.*
Proof:*Let B be a δgp-closed set and M be a δ-closed set in X such that $M \subseteq pcl(B)$-B,then $M \subseteq pcl(B)$ and $M \subseteq X$-B implies $B \subseteq X$-M. Now, B is δgp-closed and X-M is a δ-open set containing B, it follows that $pcl(B) \subseteq X$-M and thus $M \subseteq X$-pcl(B). This implies $M \subseteq pcl(B) \cap (X-pcl(B))=\phi$ and hence $M=\phi$.*

Theorem 3.15 *Let $A \subseteq X$ be a δgp-closed set.Then A is pre-closed if and only if pcl(A)-A is δ-closed.*
Proof:*Let A be a pre-closed set, then we have pcl(A)-A=ϕ which is δ-closed. Conversely, Suppose that pcl(A)-A is δ-closed.Now A is δgp-closed and since pcl(A)-A is a δ-closed subset of itself, then by Theorem 3.14, pcl(A)-A=ϕ.This implies that pcl(A)=A and so A is pre-closed.*

Theorem 3.16 *If $A \subseteq X$ is both δ-open and δgp-closed,then A is pre-closed in X.*
Proof:*Let A be δ-open δgp-closed set in X,then $pcl(A) \subseteq A$.But $A \subseteq pcl(A)$ is always true.Therefore pcl(A)=A and hence A is pre-closed.*

Theorem 3.17 *If $A \subseteq X$ is a δgp-closed set and $A \subseteq B \subseteq pcl(A)$,then B is δgp-closed in X.*
Proof:*Let G be a δ-open set in X such that $B \subseteq G$,then $A \subseteq G$. Since A is δgp-closed,then $pcl(A) \subseteq G$. Now, $pcl(B) \subseteq pcl(pcl(A))=pcl(A) \subseteq G$. So $pcl(B) \subseteq G$.*

The converse of the above theorem need not be true as seen from the following example.

Example 3.18 *Consider* $X=\{a,b,c,d\}$ *with the topology* $\tau=\{X,\phi,\{a\},\{b\},\{a,b\},\{a,c\},\{a,b,c\}\}$*.Let* $A=\{a,b\}$ *and* $B=\{a,b,d\}$ *,then* $A\subseteq B\subseteq pcl(A)$ *and B is* δgp*-closed in X but A is not* δgp*-closed in X.*

Theorem 3.19 *Let Y be an open subspace of a space X and* $A\subseteq Y$*.If A is* δgp*-closed in X,then A is* δgp*-closed in Y.*
Proof:*Let G be a* δ*-open set of Y such that* $A\subseteq G$*.Then* $G=Y\cap H$ *for some* δ*-open set H of X.Since A is* δgp*-closed in X,we have* $pcl(A)\subseteq H$ *and* $pcl_Y(A)=Y\cap pcl(A)\subseteq Y\cap H=G$*.Hence A is* δgp*-closed in Y.*

The converse of the above theorem need not be true as seen from the following example.

Example 3.20 *In Example 3.18,let* $Y=\{a,b,c\}$ *and* $A=\{a,b\}$*,then* $A\subseteq Y\subseteq X$ *and A is* δgp*-closed relative to Y but it is not* δgp*-closed relative to X.*

Remark 3.21 *Union of two* δgp*-closed sets need not be* δgp*-closed.*

Example 3.22 *Consider* $X=\{a,b,c,d,e\}$ *with the topology* $\tau=\{X,\phi,\{a,b\},\{c,d\},\{a,b,c,d\}\}$*.Let* $A=\{a\}$ *and* $B=\{a\}$ *,then A and B are* δgp*-closed sets in X but* $A\cup B=\{a,b\}$ *is not* δgp*-closed in X.*

Remark 3.23 *Intersection of two* δgp*-closed sets need not be* δgp*-closed.*

Example 3.24 *Consider* $X=\{a,b,c,d,e\}$ *with the topology* $\tau=\{X,\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},\{a,b,c\}\}$*. Let* $A=\{a,d\}$ *and* $B=\{a,e\}$ *,then A and B are* δgp*-closed sets in X but* $A\cap B=\{a\}$ *is not* δgp*-closed in X.*

Lemma 3.25 [7]*: For any set A of X, if* $D(A)\subseteq D_p(A)$ *where* $D_p(A)$ *is pre-derived set of A, then* $cl(A)=pcl(A)$*.*

Theorem 3.26 *If A and B are* δgp*-closed sets such that* $D(A)\subseteq D_p(A)$ *and* $D(B)\subseteq D_p(B)$*, then* $A\cup B$ *is* δgp*-closed.*
Proof: *Let A and B be* δgp*-closed subsets of X such that* $D(A)\subseteq D_b(A)$ *and* $D(B)\subseteq D_b(B)$*, then by Lemma 3.25,* $cl(A)=pcl(A)$ *and* $cl(B)=pcl(B)$*. Let* $A\cup B\subseteq G$ *where G is a* δ*-open in X,then* $A\subseteq G$ *and* $B\subseteq G$*. Since A and B are* δgp*-closed, then* $pcl(A)\subseteq G$ *and* $pcl(B)\subseteq G$*. Now,* $pcl(A\cup B)\subseteq cl(A\cup B)=cl(A)\cup cl(B)=pcl(A)\cup pcl(B)\subseteq G\cup G=G$ *Thus* $pcl(A\cup B)\subseteq G$ *whenever* $A\cup B\subseteq G$ *and G is* δ*-open in X and hence* $A\cup B$ *is* δgp*-closed .*

Theorem 3.27 *For a space X, the following statements are equivalent:*
(i) $\delta O(X)\subseteq\{F\subseteq X$*:F is pre-closed*$\}$
(ii) Every subset of X is δgp*-closed*
Proof: *(i)*$\rightarrow$*(ii): Let A be any subset of X such that* $A\subseteq G$ *where G is* δ*-open in X,then by(i),* $G\in\{B\subseteq X$*: B is pre-closed*$\}$ *and hence* $pcl(A)\subseteq G$ *which*

implies A is δgp-closed.
(ii)→(i):Let G∈$\delta O(X)$.Then by(ii), G is δgp-closed and since G⊆G, then pcl(G)⊆G but G⊆pcl(G) is always true. Therefore pcl(G)=G and hence G∈{F⊆X:F is pre-closed}.

Theorem 3.28 *For a space X, the following statements are equivalent:*
(i) Every δgp-closed set is gδ-closed
(ii)Every pre-closed set is gδ-closed.
Proof: *(i)→(ii) :Follows from the Theorem 3.3.*
(ii)→(i):Let A be a δgp-closed set in X such that A⊆G where G is δ-open in X, then pcl(A)⊆G. Since pcl(A) is pre-closed, then by (ii),pcl(A) is gδ-closed. Therefore cl(A)⊆cl(pcl(A)⊆G. That is cl(A)⊆G.

Theorem 3.29 *For any x∈X ,the set X-{x} is δgp-closed or δ-open.*
Proof: *Suppose X-{x} is not δ-open,then X is the only δ-open set containing X-{x}.This implies pcl(X-{x})⊆X. Hence X-{x} is δgp-closed.*

Theorem 3.30 *If A is δgp-closed,then $cl_\delta\{x\}\cap A\neq\phi$, for every x∈pcl(A).*
Proof: *Let x∈pcl(A).Suppose that $cl_\delta\{x\}\cap A=\phi$,then A⊆X-$cl_\delta\{x\}$ and X-$cl_\delta\{x\}$ is δ-open. Since A is δgp-closed,then pcl(A)⊆X-$cl_\delta\{x\}$ so, x∉pcl(A) which is a contradiction. Therefore $cl_\delta\{x\}\cap A\neq\phi$.*

Definition 3.31 [3].*The intersection of all δ-open subsets of X containing A is called the δ kernel of A and it is denoted by $ker_\delta(A)$.*

Theorem 3.32 *A subset A of a space A is δgp-closed if and only if pcl(A)⊆$ker_\delta(A)$.*
Proof:*Suppose that A is δgp-closed set in X such that x∈pcl(A). If possible,let x∉$ker_\delta(A)$,then there exists a δ-open set G in X such that A⊆G and x∉G. Since A is a δgp-closed set in X,pcl(A)⊆G implies x∉pcl(A) which is a contradiction.*
Conversely,let pcl(A)⊆$ker_\delta(A)$ be true and G is a δ-open set containing A, then $ker_\delta(A)$⊆G,which implies pcl(A)⊆G. Hence A is δgb-closed .

4 Delta Generalized Pre-Open Sets.

Definition 4.1 *A subset A of a topological space X is called a delta generalized pre-open (briefly,δgp-open)set if A^c is δgp-closed.*

Theorem 4.2 *A set A⊆X is δgp-open if and only if G⊆pint(A) whenever G is δ-closed and G⊆A.*
Proof: *Let A be a δgp-open set and suppose G⊆A where G is δ-closed. Then X-A is a δgp-closed set contained in the δ-open set X-G,pcl(X-A)⊆X-G. Since pcl(X-A)=X-pint(A)*[9]*,then X-pint(A)⊆X-G. That is G⊆pint(A).*

Conversely,let $G \subseteq pint(A)$ be true whenever $G \subseteq A$ and G is δ-closed,then X-pint(A)$\subseteq$X-G.That is pcl(X-A)$\subseteq$X-G.This implies X-A is δgp-closed and A is δgp-open in X.

Theorem 4.3 *If A is δgp-open and B is any set in X such that pint(A)$\subseteq$B$\subseteq$A, then B is δgp-open in X.*
Proof: *Follows from the definition and Theorem 3.17.*

Theorem 4.4 *If A is δgp-open and B is any set in X such that pint(A)$\subseteq$B, then A$\cap$B is δgp-open in X.*
Proof: *Let A be a δgp-open set of X and pint(A)$\subseteq$B, then A$\cap$ pint(A)$\subseteq$A$\cap$B$\subseteq$A. Since pint(A)$\subseteq$A,then pint(A)$\subseteq$A$\cap$B$\subseteq$A and from Theorem 4.3, A$\cap$B is δgp-open in X.*

Lemma 4.5 *For any set A, pint(pcl(A)-A)=ϕ*

Theorem 4.6 *If a set A$\subseteq$X is δgp-closed,then pcl(A)-A is δgp-open in X.*
Proof:*Suppose that A is δgp-closed and M is δ-closed such that M$\subseteq$bcl(A)-A,then by Theorem 3.14, M=ϕ and hence M$\subseteq$pint(pcl(A)-A).Therefore by Theorem 4.2, pcl(A)-A is δgp-open.*

Definition 4.7 [3]*Let A and B be two non void subsets of a toplogical space X.Then A and B are said to be δ separated if A$\cap\delta$cl(B)=δcl(A)$\cap$B=ϕ.*

Theorem 4.8 *If A and B are δ separated δgp-open sets,then A$\cup$B is δgp-open.*
Proof:*Let F be a δ-closed suset of A$\cup$B.Then F$\cap\delta$cl(A)$\subseteq$(A$\cup$B)$\cap\delta$cl(A) =(A$\cap\delta$cl(A))$\cup$(B$\cap\delta$cl(A))=A$\cup\phi$=A.That is,F$\cap\delta$cl(A)$\subseteq$A. Therefore F$\cap\delta$cl(A) is a δ-closed set contained in A and A is δgp-open,then by Theorem 4.2, F$\cap\delta$cl(A)$\subseteq$pint(A).Similarly F$\cap\delta$cl(B)$\subseteq$pint(B).Thus we have F=F$\cap$(A$\cup$B) =(F$\cap$A)$\cup$(F$\cap$B)$\subseteq$(F$\cap\delta$cl(A))$\cup$(F$\cap\delta$cl(B))$\subseteq$pint(A)$\cup$pint(B)$\subseteq$pint(A$\cup$B). That is F$\subseteq$pint(A$\cup$B).Hence by Theorem 4.2, A$\cup$B is δgp-open.*

5 Some Related Separation Axioms.

Definition 5.1 *A topological space X is said to be $\delta gpT_{\frac{1}{2}}$-space if every δgp-closed subset of X is pre-closed.*

Theorem 5.2 *: A space X is $\delta gpT_{\frac{1}{2}}$-space if and only if every singleton set is either δ-closed or pre-open.*
Proof:*Necessity:Let x$\in$X. If $\{x\}$ is not δ-closed,then X-$\{x\}$ is not δ-open.*

By Theorem 3.29,X-$\{x\}$ is δgp-closed.Since X is $\delta gpT_{\frac{1}{2}}$,X-$\{x\}$ is pre-closed. Therefore $\{x\}$ is pre-open.
Sufficiency: Let A⊆X be δgp-closed and x∈pcl(A).
Suppose that x∉A,thus A⊆X-$\{x\}$.
Case i)If $\{x\}$ is δ-closed then X-$\{x\}$ is δ-open. Since A is δgb-closed, pcl(A)⊆X-$\{x\}$, then x∉pcl(A) which is a contradiction.
Case ii) If $\{x\}$ is pre-open then X-$\{x\}$ pre-closed.Therefore pcl(A)⊆X-$\{x\}$ This implies that x∉pcl(A) ,also a contradiction.So pcl(A)⊆A.Therefore pcl(A)=A.

Theorem 5.3 *: A space X is $\delta gpT_{\frac{1}{2}}$-space if and only if PO(X)=δGPO(X).*

Theorem 5.4 *: Every $\delta gpT_{\frac{1}{2}}$-space is T_{gs} space but not conversely.*
Proof: *Let X be $\delta gpT_{\frac{1}{2}}$-space and A be gp-closed.Since every gp-closed set is δgp-closed[Theorem 3.6] ,then A is pre-closed.*

Example 5.5 *Consider X=$\{a,b,c\}$ with the topology $\tau=\{X,\phi,\{a\},\{b\},\{a,c\},\{a,b\}\}$. Then X is T_{gs} space but not $\delta gpT_{\frac{1}{2}}$-space,since $\{a\}$ is δgp-closed but not pre-closed.*

Theorem 5.6 *: Every pre regular $T_{\frac{1}{2}}$-space is $\delta gpT_{\frac{1}{2}}$-space but not conversely.*
Proof:*Follows from Theorem 3.9.*

Example 5.7 *In Example 3.22, X is $\delta gpT_{\frac{1}{2}}$-space but not pre regular $T_{\frac{1}{2}}$-space,since $\{a,b,c,d\}$ is gpr-closed but not pre-closed.*

Remark 5.8 *The following examples show that $\delta gpT_{\frac{1}{2}}$-space is independent of $T_{\frac{1}{2}}$-space .*

Example 5.9 *(i)In Example 3.22, X is $\delta gpT_{\frac{1}{2}}$-space but not $T_{\frac{1}{2}}$-space, since $\{a,b,c,e\}$ is g-closed but not closed .*
(ii) In example 5.5, X is $T_{\frac{1}{2}}$-space but not $\delta gpT_{\frac{1}{2}}$-space, since $\{a\}$ is δgp-closed but not pre-closed.

Definition 5.10 *A topological space X is said to be $T_{\delta gp}$-space if every δgp-closed subset of X is closed.*

Theorem 5.11 *: Every $T_{\delta gp}$-space is $T_{\frac{1}{2}}$-space but not conversely.*
Proof: *Let X be $T_{\delta gp}$-space space. Let A be g-closed, since every g-closed set is δgp-closed then A is closed. Therefore X is $T_{\frac{1}{2}}$-space.*

Example 5.12 *In Example 5.5, X is $T_{\frac{1}{2}}$-space but not $T_{\delta gp}$-space, since $\{a\}$ is δgp-closed but not closed.*

Theorem 5.13 *:Every $T_{\delta gp}$-space is $\delta gpT_{\frac{1}{2}}$-space but not conversely.*
Proof: *Let X be $T_{\delta gp}$-space and A be δgp-closed, then A is closed.Therefore A is pre-closed and hence X is $\delta gbT_{\frac{1}{2}}$-space.*

Example 5.14 *In Example 3.22, X is $\delta gpT_{\frac{1}{2}}$-space but not $T_{\delta gp}$-space,since $\{a\}$ is δgp-closed but not closed.*

6 Related Nbhds,Closure and Interior.

Definition 6.1 *A subset M of a topological space X is called δgp- neighbourhood (briefly, δgp-nbhd) of a point $x\in X$, if there exists a δgp-open set U such that $x\in U\subseteq M$.*

The collection of all δgp-nbhds of $x\in X$ is called δgb-nbhd system of x and is denoted by $\delta gbN(x)$.

Theorem 6.2 *Every nbhd A of a point $x\in X$ is δgp-nbhd of that point.*
Proof: *Follows from the definition and the fact that every open set is δgp-open.*

The converse of the above theorem need not be true as seen from the following example.

Example 6.3 *Consider $X=\{a,b,c,d\}$ with the topology $\tau=\{X,\phi,\{a\},\{b\},\{a, b\},\{a,b,c\}\}$. The subset $\{c,d\}$ is a δgb-nbhd of $c\in X$ but it is not a nbhd of c, as there is no open set G such that $c\in G\subseteq\{c,d\}$.*

Theorem 6.4 *If A is δgp-open set,then it is δgb-nbhd of each of its points.*
Proof: *Let A be any δgp-open set of X ,then for each $x\in A$, $x\in A\subseteq A$. Therefore A is δgp-nbhd of each of its points.*

The converse of the above theorem need not be true as seen from the following example.

Example 6.5 *In Example 6.3,$\{c,d\}$ is δgp-nbhd of each of its points but it is not δgb-open.*

Theorem 6.6 *If $A\subseteq X$ is a δgp-closed set and $x\in A^C$,then there exists a δgp-nbhd F of x such that $F\cap A=\phi$.*
Proof: *Let $A\subseteq X$ be a δgp-closed set,then A^C is δgp-open.Therefore ,By Theorem 6.4, A^C is δgp-nbhd of each of its points. Let $x\in A^C$ then there exists a δgp-open set F such that $x\in F\subseteq A^C$.That is, $F\cap A=\phi$.*

Theorem 6.7 *Let x be a point in a space X ,then*
(i) $\delta gpN(x)\neq\phi$.
(ii) If $A\in\delta gpN(x)$,then $x\in A$.

(iii) If $A\in\delta gpN(x)$ and $B\supseteq A$,then $B\in\delta gpN(x)$.
(iv) If $A_\lambda\in\delta gpN(x)$ for each $\lambda\in\wedge$,then $\bigcup A_\lambda\in\delta gpN(x)$.
Proof: *(i) Since $X\in\delta gpN(x)$, $\delta gpN(x)\neq\phi$.*
(ii) Let $A\in\delta gpN(x)$,then there exists a δgp-open set G such that $x\in G\subseteq A$.This implies $x\in A$.
(iii) Let $A\in\delta gpN(x)$,then there exists a δgp-open set G such that $x\in G\subseteq A$. Since $A\subseteq B$,then $x\in G\subseteq B$.This shows $B\in\delta gpN(x)$.
iv) Since for each $\lambda\in\wedge$, A_λ is δgp-nbhd of x, then there exists a δgp-open set G_λ such that $x\in G_\lambda\subseteq A_\lambda$. Which implies that $x\in G_\lambda\subseteq\bigcup A_\lambda$ and hence $\bigcup A_\lambda\in\delta gpN(x)$.

Remark 6.8 *Intesection of δgp-nbhd of $x\in X$ need not be a δgp-nbhd of that point,as seen from the following example.*

Example 6.9 *In Example 3.22,the subsets $\{a,c,e\}$ and $\{b,d,e\}$ are δgp-nbhd of $e\in X$,but their intersection$\{a,c,e\}\cap\{b,d,e\}=\{e\}$ is not δgp-nbhd of e, as there is no δgp-open set G such that $e\in G\subseteq\{e\}$.*

Definition 6.10 *Let A be a subset of a topological space X.Then the δgp closure of A,denoted by$\delta gpCl(A)$ is defined to be the intersection of all δgp-closed sets containing A.*

Theorem 6.11 *Let A and B be subsets of a topological space X.Then*
(i) $\delta gpCl(X)=X$ and $\delta gpCl(\Phi)=\Phi$.
(ii) If $A\subseteq B$,then $\delta gpCl(A)\subseteq\delta gpCl(B)$.
(iii)$\delta gpCl(A)\cup\delta gpCl(B)\subseteq\delta gpCl(A\cup B)$.
(iv) $\delta gpCl(A\cap B)\subseteq\delta gpCl(A)\cap\delta gpCl(B)$.
(v) If A is δgp closed,then $\delta gpCl(A)=A$.
(vi) $\delta gpCl(\delta gpCl(A))=\delta gpCl(A)$.
(vii) $A\subseteq\delta gpCl(A)\subseteq pcl(A)$.
Proof: *The easy verification is omitted.*

Remark 6.12 *The equalities do not hold in results (iii) and(iv).Also converse of (v) need not be true in general as seen from the following examples.*

Example 6.13 *(iii) In Example 3.22,let $A=\{a\}$and $B=\{b\}$.Then $\delta gpCl(A)=\{a\}$, $\delta gpCl(B)=\{b\}$ and $\delta gpCl(A\cup B)=\{a,b,e\}$.*
Thus we have $\delta gpCl(A\cup B)=\{a,b,e\}\neq\{a,b\}=\delta gpCl(A)\cup\delta gpCl(B)$.
(iv) Consider $X=\{a,b,c,d\}$ with the topology $\tau=\{X,\phi,\{b\},\{c\},\{a,b\},\{b,c\},\{a,b,c\},\{b,c,d\}\}$, let $A=\{b\}$ and $B=\{c\}$.Then $\delta gpCl(A)=\{b,d\}$, $\delta gpCl(B)=\{c,d\}$ and $\delta gpCl(A\cap B)=\phi$.Thus we have $\delta gpCl(A\cap B)=\phi\neq\{d\}=\delta gpCl(A)\cap\delta gpCl(B)$.
(v)In Example 3.4,let $A=\{a\}$.Then $\delta gpCl(A)=A$.But it is not a δgp-closed set in X.

Theorem 6.14 *Let A be a subset of a space X.Then* $x \in \delta gpCl(A)$ *if and only if* $U \cap A \neq \phi$*, for every* δgp*-open set U containing x.*
Proof:*Let* $x \in \delta gpCl(A)$*.Suppose that there exists a* δgp*-open set U containing x such that* $U \cap A = \phi$*,then* $A \subseteq X\text{-}U$ *and X-U is* δgp*-closed.Therefore* $\delta gpCl(A) \subseteq X\text{-}U$*,which implies* $x \notin \delta gpCl(A)$*,a contradiction.*
Conversely,suppose that $x \notin \delta gpCl(A)$*.Then there exists a* δgp*-closesd set F containing A such that* $x \notin F$*. Hence* F^C *is a* δgp*-open set containing x.*
Therefore $F^C \cap A = \phi$*,which contradicts the hypothesis.*

Definition 6.15 *Let A be a subset of a topological space X.Then the* δgp *interior of A,denoted by* $\delta gpInt(A)$ *is defined to be the union of all* δgp*-open sets contained in A.*

Theorem 6.16 *Let A and B be subsets of a space X.Then*
(i) $\delta gpInt(X)=X$ *and* $\delta gpInt(\Phi)= \Phi$.
(ii) If $A \subseteq B$*,then* $\delta gpInt(A) \subseteq \delta gpInt(B)$.
(iii) $\delta gpInt(A) \cup \delta gpInt(B) \subseteq \delta gpInt(A \cup B)$.
(iv)$\delta gpInt(A \cap B) \subseteq \delta gpInt(A) \cap \delta gpInt(B)$.
(v)If A is δgp *open ,then* $\delta gpInt(A)=A$.
(vi) $\delta gpInt(\delta gpInt(A))= \delta gpInt(A)$.
Proof: *The easy verification is omitted.*

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