International Journal of Contemporary Mathematical Sciences
Vol. 11, 2016, no. 8, 359 - 366
HIKARI Ltd, www.m-hikari.com
http://dx.doi.org/10.12988/ijcms.2016.6634

Introduction to Complex Fuzzy Soft Hypergroup, Complex Fuzzy Soft Hyperring and Complex Fuzzy Soft Hyperideal

P. Thirunavukarasu

Department of Mathematics, Periyar E.V.R College (Autonomous)
Tiruchirappalli – 620 023 Tamilnadu, India

R. Suresh

Department of Mathematics, Kings College of Engineering
Punalkulam,Pudukkottai(Dt) – 613 303 Tamilnadu, India

W. Lilly Merline

Department of Mathematics, Periyar E.V.R College (Autonomous)
Tiruchirappalli – 620 023, Tamilnadu, India

Abstract

In this paper, we develop the initial theory of complex fuzzy soft set by introducing the concept of complex fuzzy soft hypergroup, complex fuzzy soft hyperring and complex fuzzy soft hyperideal. Consequently, a major part of this work is dedicated to extend the theory of complex fuzzy soft set, complex fuzzy hypergroup. Complex fuzzy soft semi hypergroup and complex fuzzy soft sub hypergroup also developed in this paper.

Keywords: complex fuzzy set, complex fuzzy soft set, complex fuzzy soft hypergroup, complex fuzzy soft hyperring and complex fuzzy soft hyperideal

1. Introduction

Soft set theory is a generalization of fuzzy set theory, which was proposed by Molodtsov [7] in 1999 to deal with uncertainty in a non-parametric manner. Soft sets, soft intuitionistic fuzzy set have also been applied to the problem of medical diagnosis for use in medical expert systems. Fuzzy soft sets have also been introduced in [6]. Mappings on fuzzy soft sets [2, 3] were defined and studied in the ground breaking work of Kharal and Ahmad. Complex fuzzy set (CFS) [8]-[9] is a new development in the theory of fuzzy systems [11]. The concept of CFS is an extension of fuzzy set, by which the membership for each element of a complex fuzzy set is extended to complex-valued state. Complex fuzzy hyperstructure [1], complex fuzzy hypergroup [1] is also a latest development in the theory of complex fuzzy set.

This paper is organized as follows: Section 2 reviews the concept of complex fuzzy hypergroup based on complex fuzzy set on complex fuzzy spaces. In section 3, we reviews the complex fuzzy soft set, complex fuzzy soft hypergroup, complex fuzzy soft hyperring and complex fuzzy soft hyperideal based on complex fuzzy soft set. Finally we conclude the paper in section 4.

2. Preliminaries

Definition 2.1: Ramot et al. [8] proposed an important extension of these ideas, the *Complex Fuzzy Sets,* where the membership function μ instead of being a real valued function with the range [0,1] is replaced by a complex-valued function of the form $\mu_s(x) = r_s(x)e^{j\omega_s(x)}$; $j=\sqrt{-1}$, where $r_S(x)$ and $\omega_S(x)$ are both real valued giving the range as the unit circle. However, this concept is different from fuzzy complex number introduced and discussed by Buckley and Zhang. Essentially as explained in [8] this still retains the characterization of the uncertainty through the amplitude of the grade of membership having a value in the range of [0, 1] whilst adding the membership phase captured by fuzzy sets. As explained in Ramot et al [8], the key feature of complex fuzzy sets is the presence of phase and its membership.

Definition 2.2. [1]Let E^2 be the unit disc. Then the Cartesian product $E^2 x E^2$ with partial order defined by:

(i) $\left(r_1e^{i\omega_{r1}}, r_2e^{i\omega_{r2}}\right) \le \left(s_1e^{i\omega_{s1}}, s_2e^{i\omega_{s2}}\right)$ iff $r_1 \le s_1, r_2 \le s_2, \omega_{r1} \le \omega_{s1}$ and $\omega_{r2} \le \omega_{s2}$

whenever $s_1 \neq 0$ and $s_2 \neq 0$ for all $r_1, s_1, \omega_{r1}, \omega_{s1} \in E^2$ and $r_2, s_2, \omega_{r2}, \omega_{s2} \in E^2$

(ii) $\left(0e^{i\omega_{r1}}, 0e^{i\omega_{r2}}\right) = \left(s_1e^{i\omega_{s1}}, s_2e^{i\omega_{s2}}\right)$ whenever, $s_1 = 0$ or $s_2 = 0$ and

$\omega_1 = 0$ or $\omega_2 = 0$ for every $s_1, \omega_1 \in E^2$ and $s_2, \omega_2 \in E^2$.

Definition 2.3. [1] A complex fuzzy space, denoted by (X,E^2), where E^2 is the unit disc, is set of all ordered pairs (x,E^2), $x\in X$, i.e. $(X,E^2)= \left\{\left(x,E^2\right): x\in X\right\}$. We can write $\left(x,E^2\right)=\left\{\left(x,re^{i\omega}\right): re^{i\omega}\in E^2\right\}$, where $i=\sqrt{-1}$, $r\in[0,1]$, and $\omega\in[0,2\pi]$. The ordered pair (x,E^2) is called a complex fuzzy element in the complex fuzzy space (X, E^2).Therefore, the complex fuzzy space is an (ordinary) set of ordered pairs. In each pair the first component indicates the (ordinary) element and the second component indicates a set of possible complex membership values $re^{i\omega}$, where r represents an amplitude term and ω represents a phase term.

Definition 2.4. [1] Let (H,E^2) be a non-empty complex fuzzy space. A complex fuzzy hyperstructure (hypergroupoid), denoted by $((H,E^2);\Diamond)$ is a complex fuzzy space together with a complex fuzzy function having onto co-membership functions(referred as a complex fuzzy hyperoperation) $\Diamond:(H,E^2)X(H,E^2) \rightarrow P^*((H,E^2))$, where $P^*((H,E^2))$ denotes the set of all non-empty complex fuzzy subspaces of (H,E^2) and $\Diamond=\left(\Delta,\nabla_{xy}\right)$ with $\Delta: H\times H\rightarrow P^*(H)$ and $\nabla_{xy}: E^2\times E^2\rightarrow E^2$ for all $x,y\in H$.

Definition: 2.5. [1] A complex fuzzy hypergroup is a complex fuzzy hyperstructure $<(H,E^2);\Diamond>$ satisfying the following axioms:

(i) $((x,E^2)\Diamond(y,E^2))\Diamond(z,E^2)=(x,E^2)\Diamond((y,E^2))\Diamond(z,E^2))$
for all $\left(x,E^2\right),\left(y,E^2\right),\left(z,E^2\right)\in\left(H,E^2\right)$,

(ii) $(x,E^2)\Diamond(H,E^2)=(H,E^2)\Diamond(x,E^2)=(H,E^2))$ for all $\left(x,E^2\right)\in\left(H,E^2\right)$.

3. Complex Fuzzy Soft Hypergroup

In this section, the concept of complex fuzzy soft set, complex fuzzy soft hypergroup, complex fuzzy soft hyperring and complex fuzzy soft hyperideal are studied.

Definition 3.1. [3,10] Let U be an initial set , E be set of parameters and C(U) denotes complex fuzzy power set of U, and let $A\subseteq E$. A pair (C, A) is called a **complex fuzzy soft set** over U, where C is a mapping given by C: A→C (U).

Definition 3.2. For a complex fuzzy soft set (C,A), the set Supp(C,A)= $\left\{\left(x\in A\right): r_x e^{i\omega_x}\neq\phi\right\}$ is called the support of the complex fuzzy soft set (C,A). Thus a non-null complex fuzzy soft set is a complex fuzzy soft set with an empty support and a complex fuzzy soft set (C,A) is said to be non-null if $Supp(C,A)\neq\phi$

Remark 3.3. [1] From now on , Let (C,A) be a non-null complex fuzzy soft set over a hypergroup $<(H,E^2);\Diamond>$, E be a set of parameters and A$\subseteq$E ,Consider $a\in \operatorname{supp}(C,A)$, $(w,E^2),(x,E^2),(y,E^2),(z,E^2)\in(H,E^2)$, then we can assume the complex membership function C_a denoted as, $C_a(w)=r_a(w)e^{i\omega_a(w)}$, $C_a(x)=r_a(x)e^{i\omega_a(x)}$, $C_a(y)=r_a(y)e^{i\omega_a(y)}$, $C_a(z)=r_a(z)e^{i\omega_a(z)}$.

Definition 3.4. Let (C_1,A) and (C_2,B) be complex fuzzy soft sets over U. Then (C_1,A) is called a complex fuzzy soft subset of (C_2,B) if
(i) $A\subseteq B$, (ii) $r_{C_{1a}}(x)\le r_{C_{2a}}(x)$ and $\omega_{C_{1a}}(x)\le\omega_{C_{2a}}(x)$ for every $a\in A$ and $x\in U$, $r_{C_{1a}}(x), r_{C_{2a}}(x), \omega_{C_{1a}}(x), \omega_{C_{2a}}(x)$ are elements of (C_1,A) and (C_2,B) respectively. This relationship is denoted by $(C_1,A)\subseteq(C_2,B)$.

Definition 3.5. Let (C,A) be a non-null complex fuzzy soft set over $<(H,E^2);\Diamond>$. Then (C,A) is called a complex fuzzy soft semi hypergroup over $<(H,E^2);\Diamond>$ if for all $a\in \operatorname{supp}(C,A)$ and $(x,E^2),(y,E^2)\in(H,E^2)$ then $\min\left(r_a(x),r_a(y)\right)\le\inf\left\{r_a(z):(z,E^2)\in(x,E^2)\Diamond(y,E^2)\right\}$, $\min\left(\omega_a(x),\omega_a(y)\right)\le$ $\inf\left\{\omega_a(z):(z,E^2)\in(x,E^2)\Diamond(y,E^2)\right\}$.

Definition 3.6. Let (C,A) be a non-null complex fuzzy soft set over a hypergroup $<(H,E^2);\Diamond>$. Then (C,A) is called a complex fuzzy soft hypergroup over $<(H,E^2);\Diamond>$ if for all $a\in \operatorname{supp}(C,A)$, the following conditions are satisfied:

(i) For all $(x,E^2),(y,E^2)\in(H,E^2)$,
$\min\left(r_a(x),r_a(y)\right)\le\inf\left\{r_a(z):(z,E^2)\in(x,E^2)\Diamond(y,E^2)\right\}$,
$\min\left(\omega_a(x),\omega_a(y)\right)\le\inf\left\{\omega_a(z):(z,E^2)\in(x,E^2)\Diamond(y,E^2)\right\}$.

(ii) For all $(w,E^2),(x,E^2)\in(H,E^2)$, there exists a $(y,E^2)\in(H,E^2)$ such that $(x,E^2)\in(w,E^2)\Diamond(y,E^2)$, $\min\left(r_a(w),r_a(x)\right)\le r_a(y)$, $\min\left(\omega_a(w),\omega_a(x)\right)\le\omega_a(y)$.

(iii) For all $(w,E^2),(x,E^2)\in(H,E^2)$, there exists a $(z,E^2)\in(H,E^2)$ such that $(x,E^2)\in(z,E^2)\Diamond(w,E^2)$, $\min\left(r_a(w),r_a(x)\right)\le r_a(z)$, $\min\left(\omega_a(w),\omega_a(x)\right)$ $\le\omega_a(z)$ .That is, C_a(from Remark 3.3) is a non-null complex fuzzy sub hypergroup[1] of $<(H,E^2);\Diamond>$ for all $a\in \operatorname{supp}(C,A)$.

Definition 3.7. Let (C_1, A) and (C_2, B) be complex fuzzy soft hypergroups over $<(H, E^2);\Diamond>$. Then (C_1, A) is called a complex fuzzy soft sub hypergroup of (C_2, B), which is denoted as $(C_1, A) \leq_s (C_2, B)$ if the following conditions are satisfied:(i) $A \subseteq B,$ (ii) C_{1a} is a non-null complex fuzzy sub hypergroup[1] of C_{2a} for all $a \in \mathrm{supp}(C_1, A)$

Theorem 3.8. Let (C_1, A) and (C_2, B) be complex fuzzy soft hypergroups over $<(H, E^2);\Diamond>$ and (C_1, A) be a complex fuzzy soft subset of $\left(C_2, B\right)$. Then $(C_1, A) \leq_s (C_2, B)$

Proof. Let (C_1, A) and (C_2, B) be complex fuzzy soft hypergroups over $<(H, E^2);\Diamond>$, $(C_1, A) \subseteq (C_2, B)$. Since $(C_1, A) \subseteq (C_2, B)$, by Definition 3.7, it can be concluded that $A \subseteq B,$ and also, since (C_1, A) and (C_2, B) are complex fuzzy soft hypergroups over $<(H, E^2);\Diamond>$ and $(C_1, A) \subseteq (C_2, B)$, C_{1a} is a non-null complex fuzzy subhypergroup of C_{2a} for all $a \in \mathrm{supp}(C_1, A)$.As such, (C_1, A) is a complex fuzzy soft hypergroup over $\left(C_2, B\right)$.Hence, $(C_1, A) \leq_s (C_2, B)$

Definition 3.9. Let (C,A) be a non-null complex fuzzy soft set over $<(R, E^2), +, \Diamond>$. Then (C,A) is called a complex fuzzy soft hyperring (from the definition of fuzzy soft hyperring [4,5]) over R if for all $a \in Supp(C, A)$, the following conditions are satisfied:

(a) $(x, E^2), (y, E^2) \in (R, E^2)$

$$\min\left(r_a(x), r_a(y)\right) \leq \inf\left\{r_a(z) : (z, E^2) \in (x, E^2) + (y, E^2)\right\},$$

$$\min\left(\omega_a(x), \omega_a(y)\right) \leq \inf\left\{\omega_a(z) : (z, E^2) \in (x, E^2) + (y, E^2)\right\}.$$

(b) For all $(w, E^2), (x, E^2) \in (R, E^2),$ there exists a $(y, E^2) \in (R, E^2)$ such that $(x, E^2) \in (w, E^2) + (y, E^2)$, $\min\left(r_a(w), r_a(x)\right) \leq r_a(y),$ $\min\left(\omega_a(w), \omega_a(x)\right) \leq \omega_a(y),$

(c) For all $(w, E^2), (x, E^2) \in (R, E^2),$ there exists a $(z, E^2) \in (R, E^2)$ such that $(x, E^2) \in (z, E^2) + (w, E^2)$ and $\min\left(r_a(w), r_a(x)\right) \leq r_a(z),$ $\min\left(\omega_a(w), \omega_a(x)\right) \leq \omega_a(z),$

(d) For all $(x,E^2),(y,E^2)\in(R,E^2)$, $\min\left(r_a(x),r_a(y)\right)\le \inf r_a(z)$;
$\min\left(\omega_a(x),\omega_a(y)\right)\le \inf \omega_a(z):(z,E^2)\in(x,E^2)\Diamond(y,E^2)$, Then C_a(from Remark 3.3.) is a non-null complex fuzzy sub hyperring of R for each $a\in \operatorname{supp}(C,A)$.

Definition 3.10. Let (C,A) be a non-null complex fuzzy soft set over $<(R,E^2),+,\Diamond>$. Then (C,A) is called a complex fuzzy soft hyperideal (from the definition of fuzzy soft hyperideal [4],[5]) over R if for all $a\in Supp(C,A)$, the following conditions are satisfied:

(a) $(x,E^2),(y,E^2)\in(R,E^2)$, $\min\left(r_a(x),r_a(y)\right)\le \inf\{r_a(z):(z,E^2)$
$\in(x,E^2)+(y,E^2)\}$, $\min\left(\omega_a(x),\omega_a(y)\right)\le \inf\left\{\omega_a(z):(z,E^2)\in(x,E^2)+(y,E^2)\right\}$.

(b) For all $(w,E^2),(x,E^2)\in(R,E^2)$, there exists a $(y,E^2)\in(R,E^2)$ such that
$(x,E^2)\in(w,E^2)+(y,E^2)$, $\min\left(r_a(w),r_a(x)\right)\le r_a(y)$,
$\min\left(\omega_a(w),\omega_a(x)\right)\le\omega_a(y)$,

(c) For all $(w,E^2),(x,E^2)\in(R,E^2)$, there exists a $(z,E^2)\in(R,E^2)$ such that
$(x,E^2)\in(z,E^2)+(w,E^2)$ and $\min\left(r_a(w),r_a(x)\right)\le r_a(z)$,
$\min\left(\omega_a(w),\omega_a(x)\right)\le\omega_a(z)$,

(d) For all $(x,E^2),(y,E^2)\in(R,E^2)$, $\max\left(r_a(x),r_a(y)\right)\le \inf\{r_a(z):(z,E^2)\in$
$(x,E^2)\Diamond(y,E^2)\}$, $\max\left(\omega_a(x),\omega_a(y)\right)\le \inf\{\omega_a(z):(z,E^2)\in(x,E^2)\Diamond(y,E^2)\}$.
That is C_a(from Remark 3.3.) is a complex fuzzy hyperideal of R for each
$a\in\operatorname{supp}(C,A)$.

4. Conclusion

In this paper, the theory of complex fuzzy soft hypergroups is introduced. This is an extension of complex fuzzy hypergroup. We also introduced here are complex fuzzy soft hyperring and complex fuzzy soft hyperideal. It is to be noted that this is among the initial research in the area of complex fuzzy soft hypergroup and hence it serves as a first step in the theory of complex fuzzy soft set.

References

[1] Abdallah Al-Husban, Abdul Razak Salleh, Complex Fuzzy Hypergroups Based on Complex Fuzzy Spaces, *International Journal of Pure and Applied Mathmatics*, **107** (2016), no. 4, 949-958.

[2] N. Cagman, Filiz Citak, Serdar Enginoglu, FP-Soft set Theory and its Applications, *Annals of Fuzzy Mathematics and Informatics*, **2** (2011), no. 2, 219-226.

[3] N. Cagman, S. Enginoglu and F. Citak, Fuzzy Soft Set Theory and its Applications, *Iranian Journal of Fuzzy Systems*, **8** (2011), no. 3, 137-147.

[4] Ganeshsree Selvachandran, Fuzzy Soft Hyperrings and Fuzzy Soft Hyperideals, *Global Journal of Pure and Applied Mathematics*, **11** (2015), no. 2, 807-823.

[5] Ganeshsree Selvachandran, Abdul Razak Salleh, Hypergroup Theory Applied to Fuzzy Soft Sets, *Global Journal of Pure and Applied Mathematics*, **11** (2015), no. 2, 825-835.

[6] A.M. Ibrahim, A.O. Yusuf, Development of Soft Set Theory, *American International Journal of Contemporary Research*, **2** (2012), no. 9, 205-210.

[7] D. A. Molodtsov, Soft set theory – First result, *Computers Math. with Appl.*, **37** (1999), no. 4-5, 19-31.
http://dx.doi.org/10.1016/s0898-1221(99)00056-5

[8] D. Ramot, R. Milo, M. Friedman, and A. Kandel, Complex fuzzy sets, *IEEE Trans. Fuzzy Syst.*, **10** (2002), no. 2, 171–186.
http://dx.doi.org/10.1109/91.995119

[9] D. Ramot, M. Friedman, G. Langholz and A. Kandel, Complex fuzzy logic, *IEEE Trans. Fuzzy Syst.*, **11** (2003), no. 4, 450-461.
http://dx.doi.org/10.1109/tfuzz.2003.814832

[10] P. Thirunavukarasu, R. Suresh, Soft Complex Fuzzy Sets, *IMRF Journal*, **3** (2014), no. 1, 72-76.

[11] L. A. Zadeh, Fuzzy Sets, *Information and Control*, **8** (1965), 338-353. http://dx.doi.org/10.1016/s0019-9958(65)90241-x

Received: July 11, 2016; Published: August 10, 2016