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Bayesian Inference from the Kumaraswamy-Weibull Distribution Based on Censored Samples with Applications Real Data

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Abstract

This paper is concerned with the Bayesian analysis for the Kumaraswamy-Weibull (Kum-W) distribution under type II censored samples. Approximate Bayes estimates are computed using the Gibbs sampling procedure. This procedure generates samples from the posterior distributions. The approximate Bayes estimators are obtained under the assumptions of non-informative priors. Also, using Bayesian framework, the posterior density function, the predictive density for a single future response, i^{th} ordered future response, and several future responses are derived under type II doubly censored samples. The predictive means, standard deviations, prediction intervals, and the shape characteristics for a single future response are determined. Applications to real data sets are utilized to illustrate the potentiality of the Bayesian analysis and the predictive results.

Keywords: Kumaraswamy-Weibull (Kum-W) distribution; Bayesian approach; predictive inference; censored samples

1. Introduction

The Bayesian approach to statistical inference considers uncertainties associated with all unknown quantities whether they are unobserved parameters or observable future values. What is known about the parameters includes best information about the phenomena of interest and best data concerning the unknowns, availability considered. Using what is known about unobservable parameters, inference is drawn about them by constructing their joint probability distribution. The distribution of unknowns given the knowns is called the posterior distribution. The unknown quantities may include future observations (that are currently unknown). Inference about future observations is referred to as prediction and their marginal distribution is referred to as the predictive distribution. Several authors discussed the predictive inference. For example, Khan *et al*. (2003), Khan (2012), Khan *et al*. (2013), Khan *et al*. (2014a), Khan *et al*. (2014b), Mandouh (2016), among others in the case of complete samples and Khan (2006), Khan *et al*. (2010), Khan *et al*. (2011), Khan (2013), Khan (2014) , among others in the case of censored samples.

This paper is concerned with Kumaraswamy-Weibull (Kum-W) introduced by Cordeiro *et al.* (2010) as a special case of the Kumaraswamy-Generated family (see Cordeiro and de Castro (2011)) with cumulative distribution function (cdf):

$$F(x) = 1 - \left\{1 - \left[1 - e^{-(\lambda x)^c}\right]^a\right\}^b, \tag{1.1}$$

and the corresponding density and hazard function are respectively:

$$f(x) = abc\lambda^c x^{c-1} e^{-(\lambda x)^c}\left[1 - e^{-(\lambda x)^c}\right]^{a-1}\left\{1 - \left[1 - e^{-(\lambda x)^c}\right]^a\right\}^{b-1} \tag{1.2}$$

and

$$h(x) = \frac{abc\lambda^c x^{c-1} e^{-(\lambda x)^c}\left[1-e^{-(\lambda x)^c}\right]^{a-1}}{1-\left[1-e^{-(\lambda x)^c}\right]^a}. \tag{1.3}$$

The Weibull, exponentiated Weibull (EW) and exponentiated exponential (EE) distributions are the most important sub-models of (1.2) for a=b=1, b=1, and c=b=1, respectively. The Kum-W distribution has three shape parameters, *a*, *b* and *c*. These three shape parameters allow for a high degree of flexibility of the Kum-W distribution. This distribution allows for all five major hazard shapes: constant, increasing, decreasing, bathtub and unimodal failure rates. (see Cordeiro *et al.* (2010)).

This paper is organized as follows. In section 2, we compute approximate Bayes estimates under type II censored samples using the Gibbs sampling procedure. In section 3, the predictive density for a single future response, ith ordered future response, and several future responses are derived under type II doubly censored samples. In section 4, applications to real data sets are considered to illustrate the potentiality of the Bayesian analysis and the predictive results.

2. Bayesian Analysis for the Kum-W Distribution Based on Type II Censored Samples

Suppose that n items, whose life times follow Kum-Weibull distribution (1.2), are put on test. The test is terminated when the rth item fails for some fixed value of r. The lifetimes of these first r failed items, say $x_{(1)}, x_{(2)}, \ldots \ldots, x_{(r)}$, are observed. The likelihood function is given by:

$$L(a,b,c,\lambda) \propto \prod_{i=1}^{r} f(x_i)\,[1-F(x_r)]^{n-r}$$

$$\propto (abc\lambda^c)^r e^{-\sum_{i=1}^{n}(\lambda x_{(i)})^c} \prod_{i=1}^{r} x_{(i)}{}^{c-1} \left[1-e^{-(\lambda x_{(i)})^c}\right]^{a-1} \left[1-\left[1-e^{-(\lambda x_{(i)})^c}\right]^a\right]^{b-1}$$

$$\times \left[\left\{1-\left[1-e^{-(\lambda x_{(r)})^c}\right]^a\right\}^b\right]^{n-r}.$$

The log likelihood function can be written as follows:

$$l(a,b,c,\lambda) = rln(abc\lambda^c) + (c-1)\sum_{i=1}^{r}\ln(x_i) - \sum_{i=1}^{r}(\lambda x_i)^c + (a-1)\sum_{i=1}^{r}\ln(w_i)] +$$
$$(b-1)\sum_{i=1}^{r}\ln[1-\mathrm{w}_i{}^a] + (n-r)\ln[[1-\{1-\exp(-(\lambda x_r)^c\}^a]^b], \qquad (2.1)$$

where $w_i = [1-\exp(-(\lambda x_i)^c]$.

Now, approximate Bayes estimates are computed using the Gibbs sampling procedure which generates samples from the posterior distributions. The approximate Bayes estimators are obtained under the assumptions of non-informative priors.

We consider the Kum-W model with density function (1.2) and a non-informative joint prior distribution for a, b, c and λ given by:

$$\pi_0(a,b,c,\lambda) \propto \frac{1}{abc\lambda}, \qquad (2.2)$$

where $a, b, c \ and\ \lambda > 0$. The joint posterior distribution for these parameters can be written as

$$\pi(\,a,b,c,\lambda \mid \boldsymbol{x}\,) \propto \pi_0(a,b,c,\lambda) exp\{l(x;a,b,c,\lambda)\} \qquad (2.3)$$

where $l(x;a,b,c,\lambda)$ as given by (2.1).

Consider the reparametrization $\rho_1 = \log(a)$ and $\rho_2 = \log(b)$, $\rho_3 = \log(c)$ and $\rho_4 = \log(\lambda)$. We obtain from (3.1) a non-informative prior for ρ_1, ρ_2, ρ_3 and ρ_4, namely

$$\pi(\rho_1,\rho_2,\rho_3,\rho_4) = costant, \quad where \ -\infty < \rho_1,\rho_2,\rho_3 and\ \rho_4 < \infty.$$

The choice of the values of hyper-parameters of the uniform priors is vital for the convergence of the Gibbs sampling algorithm. In practical terms, one can consider a uniform prior distribution $U(-a_i, a_i)$ for i=1, 2, 3, 4 with larger values for a_i to produce approximate non-informative priors for ρ_1, ρ_2, ρ_3 and ρ_4 and proper joint posterior distribution.

Using the above reparamertization, the joint posterior distributions for ρ_1, ρ_2, ρ_3 and ρ_4 reduces to

$$\pi(\rho_1,\rho_2,\rho_3,\rho_4 \mid x) \propto \pi(\rho_1,\rho_2,\rho_3,\rho_4)\exp\{ r\rho_3 + r\rho_1 + r\,\rho_2 + r.exp(\rho_3)\rho_4 + \left(\exp(\rho_3)-1\right)\sum_{i=1}^{r}\ln(x_i) - \sum_{i=1}^{r}\left(\exp(\rho_4)\,x_i\right)^{\exp(\rho_3)} + \left(\exp(\rho_1)-1\right)\sum_{i=1}^{r}\ln[1-e^{-\left(\exp(\rho_4)x_i\right)^{\exp(\rho_3)}} + (\exp(\rho_2)-1)\sum_{i=1}^{r}\ln\left[1-\left(1-e^{-(\exp(\rho_4)x_i)^{\exp(\rho_3)}}\right)^{\exp(\rho_1)}\right] + (n-r)\ln[\left[1-\left(1-e^{-(\exp(\rho_4)x_i)^{\exp(\rho_3)}}\right)^{\exp(\rho_1)}\right]^{\exp(\rho_2)}]\} \tag{2.4}$$

If we assume the prior $\pi(\rho_1,\rho_2,\rho_3,\rho_4) = costant$, the conditional posterior distributions used in the Gibbs sampling algorithm are:

$$\pi(\rho_1 \mid \rho_2,\rho_3,\rho_4,x) \propto \exp\left\{\mathrm{n}\rho_1 + (\exp(\rho_1)-1)\sum_{i=1}^{r}\ln[1-e^{-(\exp(\rho_4))^{\exp(\rho_3)}}\right] + (\exp(\rho_2)-1)\sum_{i=1}^{r}\ln\left[1-\left(1-e^{-(\exp(\rho_4)x_i)^{\exp(\rho_3)}}\right)^{\exp(\rho_1)}\right] + (n-r)\ln[\left[1-\left(1-e^{-(\exp(\rho_4)x_i)^{\exp(\rho_3)}}\right)^{\exp(\rho_1)}\right]^{\exp(\rho_2)}]\},$$

$$\pi(\rho_2 \mid \rho_1,\rho_3,\rho_4,x) \propto \exp\{r\rho_2 + (\exp(\rho_2)-1)\sum_{i=1}^{r}\ln\left[1-\left(1-e^{-(\exp(\rho_4)x_i)^{\exp(\rho_3)}}\right)^{\exp(\rho_1)}\right] + (n-r)\ln[1-\left[1-\left(1-e^{-(\exp(\rho_4)x_i)^{\exp(\rho_3)}}\right)^{\exp(\rho_1)}\right]^{\exp(\rho_2)}\},$$

$$\pi(\rho_3 \mid \rho_1,\rho_2,\rho_4,x) \propto \exp\{r\rho_3 + nexp(\rho_3)\rho_4 + (\exp(\rho_3)-1)\sum_{i=1}^{r}\ln(x_i) - \sum_{i=1}^{r}(\exp(\rho_4)x_i)^{\exp(\rho_3)} + (\exp(\rho_1)-1)\sum_{i=1}^{r}\ln\left[1-e^{-(\exp(\rho_4)x_i)^{\exp(\rho_3)}}\right] + (\exp(\rho_2)-1)\sum_{i=1}^{r}\ln[1-\left(1-e^{-(\exp(\rho_4)x_i)^{\exp(\rho_3)}}\right)^{\exp(\rho_1)} + (n-r)\ln\left[\left[1-\left(1-e^{-(\exp(\rho_4)x_i)^{\exp(\rho_3)}}\right)^{\exp(\rho_1)}\right]^{\exp(\rho_2)}\right\},$$

and

$$\pi(\rho_4 \mid \rho_1, \rho_2, \rho_3, x) \propto \exp\{r.\exp(\rho_3)\rho_4 - \sum_{i=1}^{r}(\exp(\rho_4)\, x_i)^{\exp(\rho_3)} + (\exp(\rho_1) - 1)\sum_{i=1}^{r}\ln[1 - e^{-(\exp(\rho_4)x_i)^{\exp(\rho_3)}}] + (\exp(\rho_2) - 1)\sum_{i=1}^{r}\ln\left[1 - \left(1 - e^{-(\exp(\rho_4)x_i)^{\exp(\rho_3)}}\right)^{\exp(\rho_1)}\right] + (n-r)\ln[\left[1 - \left(1 - e^{-(\exp(\rho_4)x_i)^{\exp(\rho_3)}}\right)^{\exp(\rho_1)}\right]^{\exp(\rho_2)}]\}.$$

Posterior summaries of interest can be derived from the generated samples for the joint posterior distribution for the new parameters using the Gibbs sampling procedure. However, this may involves very lengthy and complicated computations. A considerable simplification in the computation can be achieved using the WinBUGS software which requires only the specification of the joint distribution for the data and the prior distributions for the model parameters.

3. The Prediction Densities of the Kum-Weibull distribution based on Type II Doubly Censored Samples

Let $x_{(1)}, x_{(2)}, \dots, x_{(n)}$ be an ordered random sample of size n from model (1.2). When the k smallest ordered observations $(x_{(1)}, \dots, x_{(k)})$ and the n - r largest ordered observations $(x_{(r+1)}, \dots, x_{(n)})$ are lost or removed from the sample, only the remaining ordered observations $(x_{(k+1)}, \dots, x_{(r)})$ are observed and are used in the statistical analysis. The likelihood function takes the form:

$$L(x; a, b, c, \lambda) \propto [F(x_{k+1})]^k \prod_{i=k+1}^{r} f(x_i)\,[1 - F(x_r)]^{n-r}$$

$$\propto (abc\lambda^c)^{r-k} e^{-\sum_{i=1}^{n}(\lambda x_{(i)})^c} \prod_{i=k+1}^{r} x_{(i)}{}^{c-1} \left[1 - e^{-(\lambda x_{(i)})^c}\right]^{a-1} \left[1 - \left[1 - e^{-(\lambda x_{(i)})^c}\right]^a\right]^{b-1}$$

$$\times \left[1 - \left\{1 - \left[1 - e^{-(\lambda x_{(r)})^c}\right]^a\right\}^b\right]^k \times \left[\left\{1 - \left[1 - e^{-(\lambda x_{(r)})^c}\right]^a\right\}^b\right]^{n-r}. \qquad (3.1)$$

Using the prior distribution in (2.1), the posterior distribution is given by

$$\pi(a, b, c, \lambda \mid \boldsymbol{x}) \propto \pi_0(a, b, c, \lambda) L(x; a, b, c, \lambda) \qquad (3.2)$$

where $L(x; a, b, c, \lambda)$ is as given by (3.1).

3.1 Predictive Density for Single future response

Let z be a single future response from the model given by (1.2), where z is independent of the observed data. Then, the predictive density for a single future response (z) given $\boldsymbol{x} = (x_{k+1}, x_{k+2}, \dots, x_r)$ is

$$p(z|\boldsymbol{x}) = \int_{b=0}^{\infty}\int_{a=0}^{\infty}\int_{\lambda=0}^{\infty}\int_{c=0}^{\infty} p(z|a,b,c,\lambda)\, \pi(\,a,b,c,\lambda \mid \boldsymbol{x}\,)\, dcd\lambda dadb,$$

where $p(z|a,b,c,\lambda)$ may be defined from model (1.2), see Khan (2014). Thus, the predictive density for a single future response is defined as

$$p(z|\boldsymbol{x}) = \begin{cases} \eta_1(x)\int_{b=0}^{\infty}\int_{a=0}^{\infty}\int_{\lambda=0}^{\infty}\int_{c=0}^{\infty}(abc)^{r-k}\,\lambda^{c(r-k+1)-1}z^{c-1}e^{-(\lambda z)^c}\left[1-e^{-(\lambda z)^c}\right]^{a-1} \\ \times\left[1-\left[1-e^{-(\lambda z)^c}\right]^a\right]^{b-1}e^{-\sum_{i=1}^{r}(\lambda x_{(i)})^c}\prod_{i=k+1}^{r}x_{(i)}{}^{c-1}\left[1-e^{-(\lambda x_{(i)})^c}\right]^{a-1}\left[1-\left[1-e^{-(\lambda x_{(i)})^c}\right]^a\right]^{b-1} \\ \times\left[1-\left\{1-\left[1-e^{-(\lambda x_{(r)})^c}\right]^a\right\}^b\right]^k\times\left[\left\{1-\left[1-e^{-(\lambda x_{(r)})^c}\right]^a\right\}^b\right]^{n-r} dadbd\lambda dc, for\ z\geq 0; c,\lambda,a, and\ b>0, \\ 0 \qquad eleswhere, \end{cases}$$

(3.3)

where $\eta_1(x)$ is a normalizing constant and when n = r, (3.3) reduces to complete samples.

The predictive estimates for a future response will be discussed separately in section (4) based on two real data sets. A numerical integration procedure “NIntegrate” in Mathematica software version 8.0, Wolfram Research (2012), is applied to plot the predictive density graph. Also the Mathematica Package is utilized to carry out all related calculations such as the predictive means, standard deviation, predictive intervals, and the measures of skewness and kurtosis.

3.2 Predictive Density for ith Ordered Future Response

One may be interested in a clinical experiment to obtain ith patient’s future survival time. Consider the model (1.2) and let z_i be the ith ordered future response in a set of m future responses. Then the pdf of z_i given a, b, c, and λ is

$$f(z_i|a,b,c,\lambda) = \frac{l!}{(i-1)!\,(l-i)!}[F(z_i)]^{i-1}f(z_i)[1-F(z_i)]^{l-i}$$

$$= \frac{l!}{(i-1)!\,(l-i)!}\left[1-\left\{1-\left[1-e^{-(\lambda z_{(i)})^c}\right]^a\right\}^b\right]^{i-1} abc\lambda^c z_{(i)}{}^{c-1}e^{-(\lambda z_{(i)})^c}$$
$$\times\left[1-e^{-(\lambda z_{(i)})^c}\right]^{a-1}\left[1-\left[1-e^{-(\lambda z_{(i)})^c}\right]^a\right]^{b-1}\left[\left\{1-\left[1-e^{-(\lambda z_{(i)})^c}\right]^a\right\}^b\right]^{l-i}$$

Thus, the predictive density for z_i is given by

$p(z_i|\boldsymbol{x})$

$$
= \begin{cases} \eta_2(x)\dfrac{l!}{(i-1)!\,(l-i)!}\displaystyle\int_{b=0}^{\infty}\int_{a=0}^{\infty}\int_{\lambda=0}^{\infty}\int_{c=0}^{\infty}(abc)^{r-k}\,\lambda^{c(r-k+1)-1} {z_i}^{c-1} e^{-(\lambda z_i)^c}\left[1-e^{-(\lambda z_i)^c}\right]^{a-1} \\ \quad \times [1-\left[1-e^{-(\lambda z_i)^c}\right]^a]^{b-1} e^{-\sum_{i=1}^{r}(\lambda x_{(i)})^c} \displaystyle\prod_{i=k+1}^{r} {x_{(i)}}^{c-1}\left[1-e^{-(\lambda x_{(i)})^c}\right]^{a-1} \\ \quad \times \left[1-\left[1-e^{-(\lambda x_{(i)})^c}\right]^a\right]^{b-1}\left[1-\left\{1-\left[1-e^{-(\lambda x_{(k+1)})^c}\right]^a\right\}^b\right]^{k+i-1} \\ \quad \times \left[\left\{1-\left[1-e^{-(\lambda x_{(r)})^c}\right]^a\right\}^b\right]^{n-r+l-i} \quad da db d\lambda dc, for\ z_i \geq 0; c, \lambda, a, and\ b > 0, \\ 0 \qquad eleswhere, \end{cases}
$$

(3.4)

where $\eta_2(x)$ is a normalizing constant.

3.3 Predictive Density for m Future Responses

Let $z_{(1)}, z_{(2)}, \ldots\ldots, z_{(m)}$ be the m ordered future responses from model (1.2). Then the pdf of $\boldsymbol{z} = (z_{(1)}, z_{(2)}, \ldots\ldots, z_{(m)})$ given a, b, c, and λ is

$$
p(\boldsymbol{z}|a,b,c,\lambda) = m!\prod_{i=1}^{m} p(z_i|a,b,c,\lambda)
$$

$$
= m!\,(abc\lambda^c)^m \prod_{i=1}^{m} {z_i}^{c-1}\, e^{-\sum_{i=1}^{m}(\lambda z_i)^c}\left[1-e^{-(\lambda z_i)^c}\right]^{m(a-1)}[1-\left[1-e^{-(\lambda z_i)^c}\right]^a]^{m(b-1)}
$$

and the predictive density of z is given by

$p(\boldsymbol{z}|\boldsymbol{x})$

$$
= \begin{cases} \eta_3(x)\dfrac{l!\,m!}{(i-1)!\,(l-i)!}\displaystyle\int_{b=0}^{\infty}\int_{a=0}^{\infty}\int_{\lambda=0}^{\infty}\int_{c=0}^{\infty}(abc)^{r-k+m-1}\,\lambda^{c(r-k+m)-1}\prod_{i=1}^{m} {z_i}^{c-1}\, e^{-\sum_{i=1}^{m}(\lambda z_i)^c}\left[1-e^{-(\lambda z_i)^c}\right]^{m(a-1)} \\ \quad \times [1-\left[1-e^{-(\lambda z_i)^c}\right]^a]^{m(b-1)}\, e^{-\sum_{i=1}^{r}(\lambda x_{(i)})^c}\displaystyle\prod_{i=k+1}^{r} {x_{(i)}}^{c-1}\left[1-e^{-(\lambda x_{(i)})^c}\right]^{a-1} \\ \quad \times \left[1-\left[1-e^{-(\lambda x_{(i)})^c}\right]^a\right]^{b-1}\left[1-\left\{1-\left[1-e^{-(\lambda x_{(k+1)})^c}\right]^a\right\}^b\right]^{k+i-1} \\ \quad \times \left[\left\{1-\left[1-e^{-(\lambda x_{(r)})^c}\right]^a\right\}^b\right]^{n-r+l-i} \quad da db d\lambda dc, for\ z_i \geq 0; c, \lambda, a, and\ b > 0, \\ 0 \qquad eleswhere, \end{cases}
$$

(3.5)

where $\eta_3(x)$ is a normalizing constant. For m = 1, (3.5) reduces to the predictive density for a single future response given by (3.3).

4. Applications to Real Data

Consider the data sets mentioned by Cordeiro *et al.* (2010). These data sets were fitted to the Kum-W distribution. The first studied by Meeker and Escobar (1998, p. 383) and the second studied by Murthy *et al*. (2004, p. 154).

Data Set 1 (voltage data): This data gives the times of failure and running times for a sample of devices from a field-tracking study of a larger system. At a certain point in time, 30 units were installed in normal service conditions. The times (Thousands of cycles) are: 275, 13, 147, 23, 181, 30, 65, 10, 300, 173, 106, 300, 300, 212, 300, 300, 300, 2, 261, 293, 88, 147, 28, 143, 300, 23, 300, 80, 245, 266. Note that: data were censored at 300.

Data Set 2 (test stopped data): This data represents failure times and are taken from Murthy *et al* (2004, p. 154). The data set is: 0.0014, 0.0623, 1.3826, 2.0130, 2.5274, 2.8221, 3.1544, 4.9835, 5.5462, 5.8196, 5.8714, 7.4710, 7.5080, 7.6667, 8.6122, 9.0442, 9.1153, 9.6477, 10.1547and 10.7582.

We consider the kum-W distribution with density (1.2) under the reparametrization $\rho_1 = \log(a)$, $\rho_2 = \log(b)$, $\rho_3 = \log(c)$ and $\rho_4 = \log(\lambda)$. We assume approximate non-informative prior uniform U(0,2), U(0,0.01), U(0,0.01) and U(-4,-3) distributions for ρ_1, ρ_2, ρ_3 and ρ_4 respectively.

A set of 10000 Gibbs samples was generated after a “burn-in-sample” of size 1000 to eliminate the initial values considered for the Gibbs sampling algorithm. All the calculations are performed using the WinBUGS software.

Once the convergence is achieved, one needs to run the simulation for a further number of iterations to obtain samples that can be used for posterior inference. One way to assess the accuracy of the posterior estimates is by calculating the Monte Carlo error (MC error) for each parameter. This is an estimate of the difference between the mean of sampled values and the true posterior mean. The simulation should be run until the MC error for each parameter of interest is less than about 5% of the sample standard deviation. One can note in our examples that MC error less than 5% of the sample standard deviation.

The following tables 1-2 list the posterior descriptive summaries of interest for the Kum-W model under type II censored. The posterior kernel densities for the parameters are given in figures 1-2.

Table1: Summary results for the posterior parameters in the case of Kum-W model based on 30 data points (voltage data) with r=22

Parameter	Estimate	Standard Deviation	MC error	95% Credible Interval
A	1.031	0.0300100	5.302E-4	(1.001, 1.110)
B	1.004	0.0027920	4.036E-5	(1.000, 1.010)
C	1.002	0.0021910	3.196E-5	(1.000, 1.008)
λ	0.01848	0.0001684	2.859E-6	(0.01832, 0.0859)

Table2: Summary results for the posterior parameters in the case of Kum-W model based on 20 data points (test stopped data) with r=16

Parameter	Estimate	Standard Deviation	MC error	95% Credible Interval
a	1.007	0.006717	1.254E-4	(1.000, 1.025)
b	1.005	0.002900	2.597E-5	(1.000, 1.010)
c	1.004	0.002685	3.543E-5	(1.000, 1.009)
λ	0.04826	0.001508	2.434E-5	(0.04425, 0.04974)

We estimated the predictive inference for a future response and their results are given in table (3). We determined certain levels of predictive interval for a single future response given a type II doubly censored, and their results are reported in table (4). The predictive densities for the future response are given in figures 3-4.

Table (3): Summary Results of the Predictive Inference for a Single Future Response

Data set 1 (n=20, r=16, k=2)		**Data set 2 (n=30, r=22, k=3)**	
Raw Moments	**Central Moments**	**Raw Moments**	**Central Moments**
$\acute{\mu}_1$ =19.8329	$\mu_2 = 1771.15$	$\acute{\mu}_1$ =31.4867	μ_2 =2101.53
$\acute{\mu}_2$ =2164.50	$\mu_3 = 468085$	$\acute{\mu}_2$ =3092.94	$\mu_3 = 413527$
$\acute{\mu}_3 = 581267$	$\mu_4 = 2.4856E8$	$\acute{\mu}_3$ =643253	$\mu_4 = 1.69811$
$\acute{\mu}_4$ =2.90028		$\acute{\mu}_4$ =2.35377E8	
Skewness & Kurtosis β_1 =39.4350 β_2 =79.2354 $\gamma_1 = \sqrt{\beta_1}$ = 6.27973 $\gamma_2 = \lvert\beta_2 - 3\rvert$ =76.2354		**Skewness & Kurtosis** $\beta_1 = 18.4248$ $\beta_2 = 38.4501$ $\gamma_1 = \sqrt{\beta_1}$ =4.29241 $\gamma_2 = \lvert\beta_2 - 3\rvert = 35.4501$	
Mean = 19.8329 **Standard deviation** = 42.0850 **Coefficient of Skewness** = 6.27973 **Coefficient of Kurtosis** = 76.2354		**Mean** = 31.4867 **Standard deviation** = 45.8424 **Coefficient of Skewness** = 4.29241 **Coefficient of Kurtosis** = 35.4501	

Table (4): Summary Results of the Prediction Intervals at Different Credibility Levels for a Single Future Response

Data Set 1: Prediction Intervals		Data Set 2: Prediction Intervals	
90%	(0, 52.4727)	90%	(0, 85.2389)
95%	(0, 87.0270)	95%	(0, 125.046)
98%	(0, 145.788)	98%	(0, 187.424)
99%	(0, 200.127)	99%	(0, 242.032)

Posterior Densities:

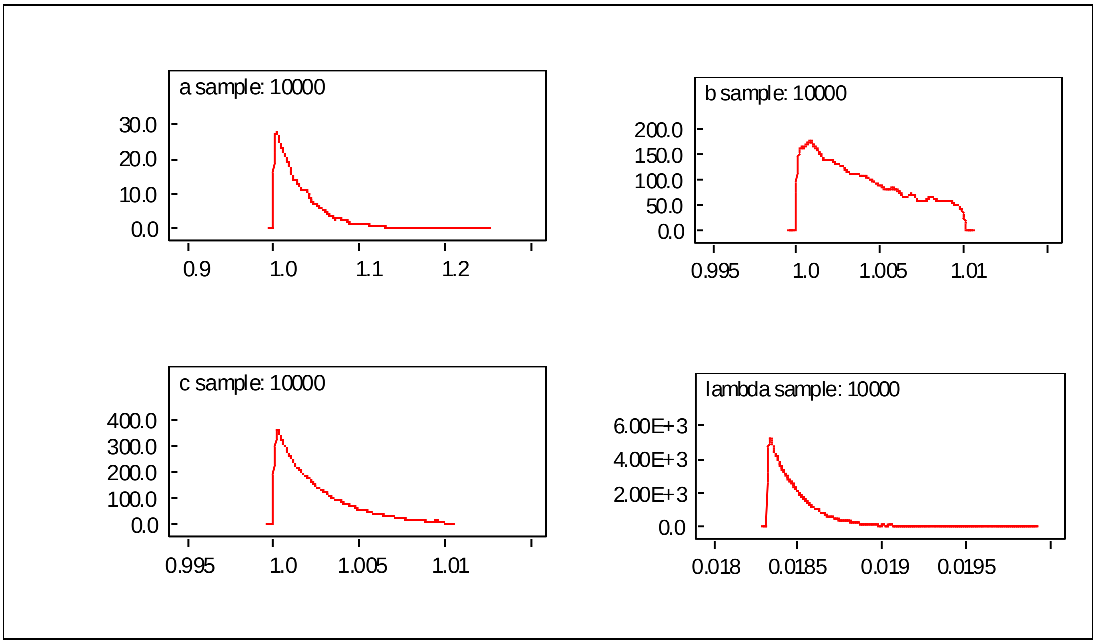

Figure 1: Posterior kernel density for the parameters in the case of Kum-W model based on 30 voltage data

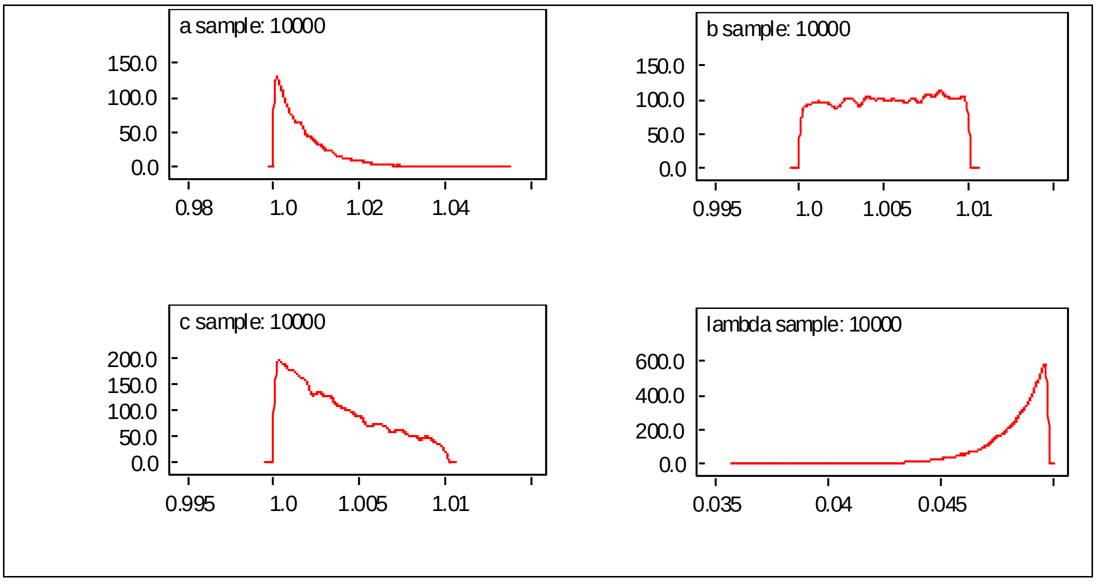

Figure 2: Posterior kernel density for the parameters in the case of Kum-W model based on 20 test stopped data

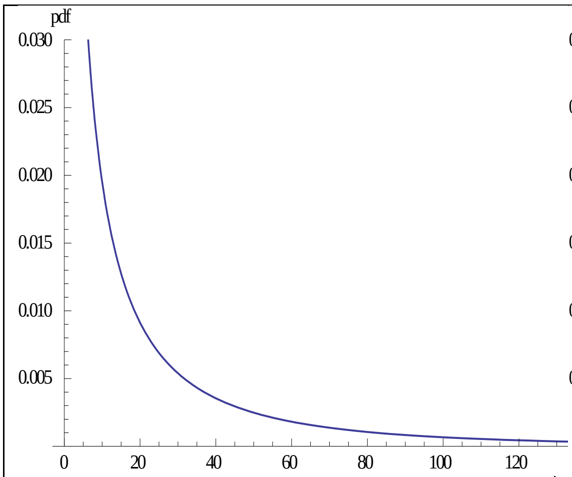

Figure 3: Predictive density function based on 20 data points (data set 1)

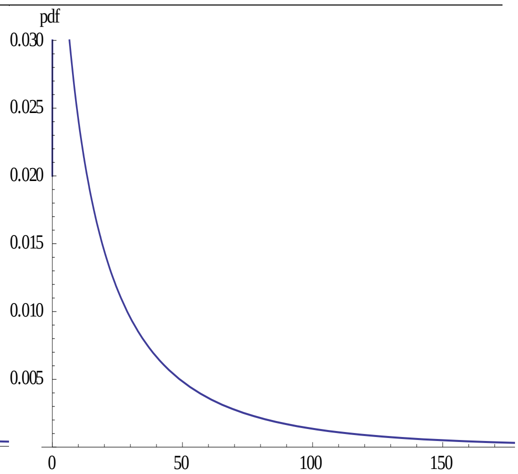

Figure 4: Predictive density function based on 30 data points (data set 2)

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