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New Generalized Sub Class of Cyclic-Goppa Code

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Abstract

Cyclic codes can be viewed as ethics in an excess class of polynomials. Moreover, it is a challenging issue to construct Sub class cyclic Goppa or related Codes.Tzeng and Zimmerman proposed an idea that multiple error correcting Goppa codes can be converted in to cyclic and reversible Goppa codes by adding extra parity check. In this correspondence, we extended the idea given by Tzeng and Zimmermann and construct new generalized sub classes of cyclic Goppa codes .It is observed code (L, G) contains subclass of all cyclic codes which are reversible and cyclic

Keywords: Goppa Codes, Cyclic Goppa Codes, Subclass, Reversible Codes

1 Introduction

For many years, the Goppa codes have been interesting in the field of coding theory, as these codes are generally non cyclic; however, class of Goppa codes is cyclic and is known as BCH codes. . By extending an overall parity check the double error-correcting binary Goppa codes, with defined Goppa polynomial and location sets, can become cyclic as proved by Berlekamp and Moreno. After that Tzeng and Zimmermann shown that a number of multiple error correcting Goppa codes converted in to cyclic when these codes are extended by an overall parity check. They also proved that these codes are reversible. In this paper we extended the idea proposed by Tzeng and Zimmermann and constructed a new subclass of Goppa code with extended parity check matrix and proved that these codes are reversible and cyclic. We give the following brief detail about the Goppa codes. The following polynomial with degree t over the extended field $GF(q^m)$

$$g(x) = g_0 + g_1 x + g_2 x^2 + ... + g_t x^t \tag{1}$$

represents Goppa polynomial of the Goppa code$\Gamma(L, g(z)$

$$L(X) = W_1, W_2.....W_n \subset GF(q^m \tag{2}$$

Such that $g(\alpha_n) \neq 0$ for all$\alpha_n \in L$ with a vector $C = \odot c_1, c_2....c_n$ over $GF(q)$we associate the function

$$R_c(x) = \sum_{n=1}^{n} \frac{c_i}{x - w_n} \tag{3}$$

In which $\dfrac{1}{x - w_n}$ is the unique polynomial with $(x - w_i)\dfrac{1}{x - w_n} \equiv 1(modg(x))$ The Goppa code$\Gamma(L, g(z))$ consists of all vector c such that $R_c(x) \equiv 0(modg(x))$ Argument 1.1 The code $\Gamma(L, g(z))$ represents a Goppa code of length n over GF(q)with added further conditions of dimension $k \geqslant i - jt$ and the minimum distance of the code satisfies $d \geqslant t + 1$

Proof we would like to prove first the lower bound on the dimension as pointed earlier $\dfrac{1}{x - w_n}$can be represented as a polynomial $P_i(x) modulog(x)$

$$\frac{1}{x - w_n} = P_{n_1} + P_{n_2} x + ...P_{n_t} x^{t-1} \tag{4}$$

Thus we can rewrite equation as $\sum_{n=1}^{i} c_n P_n(x) \equiv 0 mod(g(x))$

$$\sum_{n=1}^{i} c_n P_{mn} \equiv 0, for 1 \leqslant m \leqslant t \tag{5}$$

clearly $(L, g(z))$termed as t linear equation over the GF(x) which convert to $\leqslant mt$ linear equation over the field $GF(q)$

Parity check matrix of the Goppa code
In the proof of above Theorem 1.1 we proved that c proposes a codeword if and only if

$$\sum_{n=1}^{i} c_n P_{mn} \equiv 0, for 1 \leqslant m \leqslant t \tag{6}$$

with P_{mn} such that $\frac{1}{x-w_n} = P_{n_1} + P_{n_2}x + ...P_{n_t}x^{t-1}$ The parity check matrix H satisfies $cH^t = 0$

$$\begin{pmatrix} p_{11}......p_{n1} \\ .... \\ p_{1t}......p_{nt} \end{pmatrix}$$

To determine the factor we rewrite

$$P_n(X) \equiv (x-\omega_n)^{-1} \equiv \frac{g(x)-g(x_n}{x-\omega_n}.g(x)^{-1} \tag{7}$$

This can be checked by multiplication with $(x-\omega_n)$
$(x-\omega_n)^{-1}.\frac{g(x)-g(x_n}{x-\omega_n}.g(x)^{-1} = -g(x)g(\alpha_n)^{-1} + 1 \equiv 1 mod(g(x))$ We now define $h_i = g(\omega_n)^{-1}$ and recall that $g(x) = g_0 + g_1x....... + g_nx^t$
$P_n(x) = -\frac{g_t(x^t-\omega_n^t)... + g_1(x-\omega_n)}{x-\omega_n}h_n$
if we now substitute $P_n(x) = P_{n_1} + P_{n_2}x + ... + P_{n_t}x^{t-1}$, we find the following expression for P_{mn}
$P_{n_1} = -(g_t\omega_n^{t-1} + g_{t-1}\omega^{t-2} + ... + g_2\omega_n + g_n)h_n$
$P_{n_2} = -(g_t\omega_n^{t-2} + g_{t-1}\omega^{t-3} + ... + g_2)h_n$
$P_{n(t-1)} = -(g_t\omega_n + g_{t-1})h_n$
$P_{nt} = -(g_t)h_n$ after calculating we find that $H = CXY$ for

$$\mathbf{C} = \begin{pmatrix} -g_t & -g_{t-1} & -g_{t-2}... - g_1 \\ 0 & -g_t & -g_{t-1}...... - g_2 \\ 0 & 0 & -g_t...............g_3 \\ \vdots & \vdots & \vdots .... \vdots \\ 0 & 0 & 0............. - g_t \end{pmatrix}$$

$$\mathbf{X} = \begin{pmatrix} \alpha_1^{t-1} & \alpha_2^{t-1}......\alpha_n^{t-1} & \\ \alpha_1^{t-2} & \alpha_2^{t-2}......\alpha_n^{t-2} & \\ \vdots & \vdots & .... \vdots \\ \alpha_1 & \alpha_2............\alpha_n & \\ 1 & 1..........1 & \end{pmatrix}$$

$$\mathbf{Y} = \begin{pmatrix} h_1 & 0 & 0.....0 \\ 0 & h_2 & 0.........0 \\ 0 & 0 & h_3.....0 \\ : & : & : .... : \\ 0 & 0 & 0............h_n \end{pmatrix}$$

obviously C is invertible and in addition code can be represented by another parity check matrix that is

$$\mathbf{H} = \begin{pmatrix} \alpha_1^{t-1}h_1 & \alpha_2^{t-1}h_2....\alpha_n^{t-1}h_n \\ \alpha_1^{t-2}h_1 & \alpha_2^{t-2}h_2....\alpha_n^{t-2}h_n \\ : & : .... : \\ \alpha_1 h_1 & \alpha_2 h_2...\alpha_n h_n \\ h_1 & h_2............h_n \end{pmatrix}$$

A.The following matrix H_E is known as the parity check matrix of the classical Goppa code with added parity check.

$$\mathbf{H_E} = \begin{pmatrix} H_{(L,G)} & 0 \\ 1..1 & 1 \end{pmatrix}$$

Where H_E represents as the parity check matrix of a separable Goppa code of $G(x)$ with degree two and location set $LGF(q^m)$
B. The H_{PC} also represents parity check matrix of the classical cyclic Goppa code

$$\mathbf{H_{PC}} = \begin{pmatrix} H_{(L,G)} \\ 1..1 \end{pmatrix}$$

2 Reversible Goppa Codes

Let $L = \omega_1, \omega_2...\omega_n, \omega_{n+1}...\omega_{2n}$ be a subset of $GF(q^m)$, and Let g(z) be a polynomial in z over the above said field $GF(q^m)$ with a condition that no root of g(z) is in L and also degree of g(z) is less than 2n. A Goppa code is hence defined as the set of n-tuples$x_1, x_2...x_n, x_{n+1}, x_{n+2}..x_{2n}$ with $x_i GF(q)$ such that $\sum_{i=1}^{n} \frac{x_{2i}}{z-\omega_{2i}} \equiv 0 mod g(z)$where g(z)

$$g(z) = (z-\theta_1)^{q_1} + (z-\theta_2)^{q_2} + ...(z-\theta_n)^{q_n} + (z-\theta_{n+1})^{q_{n+1}} + ...(z-\theta_{2n})^{q_{2n}} \quad (8)$$

$$\mathbf{H} = \begin{pmatrix} (\theta_1-\omega_1)^{-1} & (\theta_1-\omega_2)^{-1}...(\theta_1-\omega_n)^{-1} & (\theta_1-\omega_{n+1})^{-1} & (\theta_1-\omega_{n+2})^{-1}....(\theta_1-\omega_{2n})^{-1} \\ (\theta_1-\omega_1)^{-2} & (\theta_1-\omega_2)^{-2}...(\theta_1-\omega_n)^{-2} & (\theta_1-\omega_{n+1})^{-2} & (\theta_1-\omega_{n+2})^{-2}....(\theta_1-\omega_{2n})^{-2} \\ : & :....: & : & :........: \\ (\theta_1-\omega_1)^{-q_1} & (\theta_1-\omega_2)^{-q_1}...(\theta_1-\omega_n)^{-q_1} & (\theta_1-\omega_{n+1})^{-q_1} & (\theta_1-\omega_{n+2})^{-q_1}....(\theta_1-\omega_{2n})^{-q_1} \\ (\theta_2-\omega_1)^{-1} & (\theta_2-\omega_2)^{-1}...(\theta_2-\omega_n)^{-1} & (\theta_2-\omega_{n+1})^{-1} & (\theta_2-\omega_{n+2})^{-1}....(\theta_2-\omega_{2n})^{-1} \\ (\theta_2-\omega_1)^{-2} & (\theta_2-\omega_2)^{-2}...(\theta_2-\omega_n)^{-2} & (\theta_2-\omega_{n+1})^{-2} & (\theta_2-\omega_{n+2})^{-2}....(\theta_2-\omega_{2n})^{-2} \\ : & :....: & : & :........: \\ (\theta_2-\omega_1)^{-q_1} & (\theta_2-\omega_2)^{-q_1}...(\theta_2-\omega_n)^{-q_1} & (\theta_2-\omega_{n+1})^{-q_1} & (\theta_2-\omega_{n+2})^{-q_1}....(\theta_2-\omega_{2n})^{-q_1} \\ : & :......: & : & :........: \\ (\theta_n-\omega_1)^{-1} & (\theta_n-\omega_2)^{-1}...(\theta_n-\omega_n)^{-1} & (\theta_n-\omega_{n+1})^{-1} & (\theta_n-\omega_{n+2})^{-1}....(\theta_n-\omega_{2n})^{-1} \\ (\theta_n-\omega_1)^{-2} & (\theta_n-\omega_2)^{-2}...(\theta_n-\omega_n)^{-2} & (\theta_n-\omega_{n+1})^{-2} & (\theta_n-\omega_{n+2})^{-2}....(\theta_n-\omega_{2n})^{-2} \\ : & :....: & : & :........: \\ (\theta_n-\omega_1)^{-q_1} & (\theta_n-\omega_2)^{-q_1}...(\theta_n-\omega_n)^{-q_1} & (\theta_n-\omega_{n+1})^{-q_1} & (\theta_n-\omega_{n+2})^{-q_1}....(\theta_n-\omega_{2n})^{-q_1} \end{pmatrix}$$

We consider a class of Goppa code with g(z) $(\theta_1, \theta_2) \in GF(q^m)$ and $L = GF(q^m) - (\theta_1, \theta_2)$ the polynomial $(z-\theta_1)(z-\theta_2)$ is irreducible over the field $GF(q^m)$ with added condition that $\theta_2 = \theta_1^{q^m}$ and $\theta_1 = \theta_2^{q^m}$ and $n + n = q^m$ therefore the parity check matix of above considered Goppa code is

$$\mathbf{H} = \begin{pmatrix} (\theta_1-\omega_1)^{-1} & (\theta_1-\omega_2)^{-1}...(\theta_1-\omega_n)^{-1} & (\theta_1-\omega_{n+1})^{-1} & (\theta_1-\omega_{n+2})^{-1}....(\theta_1-\omega_{2n})^{-1} \\ (\theta_1-\omega_1)^{-2} & (\theta_1-\omega_2)^{-2}...(\theta_1-\omega_n)^{-2} & (\theta_1-\omega_{n+1})^{-2} & (\theta_1-\omega_{n+2})^{-2}....(\theta_1-\omega_{2n})^{-2} \\ : & :....: & : & :........: \\ (\theta_1-\omega_1)^{-q_1} & (\theta_1-\omega_2)^{-q_1}...(\theta_1-\omega_n)^{-q_1} & (\theta_1-\omega_{n+1})^{-q_1} & (\theta_1-\omega_{n+2})^{-q_1}....(\theta_1-\omega_{2n})^{-q_1} \\ (\theta_2-\omega_1)^{-1} & (\theta_2-\omega_2)^{-1}...(\theta_2-\omega_n)^{-1} & (\theta_2-\omega_{n+1})^{-1} & (\theta_2-\omega_{n+2})^{-1}....(\theta_2-\omega_{2n})^{-1} \\ (\theta_2-\omega_1)^{-2} & (\theta_2-\omega_2)^{-2}...(\theta_2-\omega_n)^{-2} & (\theta_2-\omega_{n+1})^{-2} & (\theta_2-\omega_{n+2})^{-2}....(\theta_2-\omega_{2n})^{-2} \\ : & :....: & : & :........: \\ (\theta_2-\omega_1)^{-q_1} & (\theta_2-\omega_2)^{-q_1}...(\theta_2-\omega_n)^{-q_1} & (\theta_2-\omega_{n+1})^{-q_1} & (\theta_2-\omega_{n+2})^{-q_1}....(\theta_2-\omega_{2n})^{-q_1} \end{pmatrix}$$

we should be noted that for each i,j$1 \leqslant (i,j) \leqslant 2n$ and if $(\theta_1-\omega_1) = -(\theta_2-\omega_j)$
$\theta_1 - \omega_j = -(\theta_2 - \omega_i)$
$(\theta_1 - \omega_j)^d = [(\theta_2 - \omega_i)]^d forthevaluesd = 2, 3...q_1$
Therefore the considered two cases we have $\theta_1 + \theta_2 \in GF(q^m)$ for each $\omega_j \in L$ there must exist $\omega_i = \theta_1 + \theta_2 - \omega_j \in L$These codes becomes reversible if we ordered them as $\omega_{2n+1-j} = \theta_1 + \theta_2 - \omega_i$

3 Extended Reversible Goppa Codes

we extended the considered Goppa codes by adding overall parity check and the new extended Parity check matix denoted as H_E is

$$\mathbf{H_E} = \begin{pmatrix} 1 & 1 & 1..... \quad 1 & 1 & 1.........1 \\ 0 & (\theta_1-\omega_1)^{-1} & (\theta_1-\omega_2)^{-1}...(\theta_1-\omega_n)^{-1} & (\theta_1-\omega_{n+1})^{-1} & (\theta_1-\omega_{n+2})^{-1}....(\theta_1-\omega_{2n})^{-1} \\ 0 & (\theta_1-\omega_1)^{-2} & (\theta_1-\omega_2)^{-2}...(\theta_1-\omega_n)^{-2} & (\theta_1-\omega_{n+1})^{-2} & (\theta_1-\omega_{n+2})^{-2}....(\theta_1-\omega_{2n})^{-2} \\ : & :....: & : & :........: & \\ 0 & (\theta_1-\omega_1)^{-q_1} & (\theta_1-\omega_2)^{-q_1}...(\theta_1-\omega_n)^{-q_1} & (\theta_1-\omega_{n+1})^{-q_1} & (\theta_1-\omega_{n+2})^{-q_1}....(\theta_1-\omega_{2n})^{-q_1} \\ 0 & (\theta_2-\omega_1)^{-1} & (\theta_2-\omega_2)^{-1}...(\theta_2-\omega_n)^{-1} & (\theta_2-\omega_{n+1})^{-1} & (\theta_2-\omega_{n+2})^{-1}....(\theta_2-\omega_{2n})^{-1} \\ 0 & (\theta_2-\omega_1)^{-2} & (\theta_2-\omega_2)^{-2}...(\theta_2-\omega_n)^{-2} & (\theta_2-\omega_{n+1})^{-2} & (\theta_2-\omega_{n+2})^{-2}....(\theta_2-\omega_{2n})^{-2} \\ : & :....: & : & :........: & \\ 0 & (\theta_2-\omega_1)^{-q_1} & (\theta_2-\omega_2)^{-q_1}...(\theta_2-\omega_n)^{-q_1} & (\theta_2-\omega_{n+1})^{-q_1} & (\theta_2-\omega_{n+2})^{-q_1}....(\theta_2-\omega_{2n})^{-q_1} \end{pmatrix}$$

To prove H_E represents a cyclic code we have to conclude that above said matrix is equivalent to the proposed matrix i.e

$$\begin{pmatrix}
1 & 1 & 1 \cdots\cdots\cdots & 1 \\
1 & 1+\frac{\theta_2-\theta_1}{\theta_1-\omega_1} & 1+\frac{\theta_2-\theta_1}{\theta_1-\omega_2} \ldots & 1+\frac{\theta_2-\theta_1}{\theta_1-\omega_n} \\
1 & (1+\frac{\theta_2-\theta_1}{\theta_1-\omega_1})^2 & (1+\frac{\theta_2-\theta_1}{\theta_1-\omega_2})^2 \ldots\ldots & (1+\frac{\theta_2-\theta_1}{\theta_1-\omega_n})^2 \\
1 & (1+\frac{\theta_2-\theta_1}{\theta_1-\omega_1})^n & (1+\frac{\theta_2-\theta_1}{\theta_1-\omega_2})^n \ldots\ldots & (1+\frac{\theta_2-\theta_1}{\theta_1-\omega_n})^n \\
1 & 1+\frac{\theta_2-\theta_1}{\theta_1-\omega_1} & 1+\frac{\theta_2-\theta_1}{\theta_1-\omega_2} \ldots & 1+\frac{\theta_2-\theta_1}{\theta_1-\omega_n} \\
1 & (1-\frac{\theta_2-\theta_1}{\theta_2-\omega_1})^2 & (1-\frac{\theta_2-\theta_1}{\theta_2-\omega_2})^2 \ldots\ldots & (1-\frac{\theta_2-\theta_1}{\theta_2-\omega_n})^2 \\
1 & (1-\frac{\theta_2-\theta_1}{\theta_2-\omega_1})^n & (1-\frac{\theta_2-\theta_1}{\theta_2-\omega_2})^n \ldots\ldots & (1-\frac{\theta_2-\theta_1}{\theta_2-\omega_n})^n
\end{pmatrix}$$

$$1+\frac{\theta_2-\theta_1}{\theta_1-\omega_d}=\frac{\theta_2-\omega_d}{\theta_1-\omega_d}$$
$$1-\frac{\theta_2-\theta_1}{\theta_1-\omega_d}=\frac{\theta_1-\omega_d}{\theta_2-\omega_d}$$
$$=(1-\frac{\theta_2-\theta_1}{\theta_1-\omega_d})^{-1}$$

further
$$\frac{\theta_2-\omega_i}{\theta_1-\omega_i}\neq\frac{\theta_2-\omega_j}{\theta_1-\omega_j} \text{ for } i\neq j$$
conjugates over $GF(q^m)$ we get
$$(\frac{\theta_2-\omega_d}{\theta_1-\omega_d})^{q^m+1}=(\frac{\theta_2^{q^m}-\omega_d^{q^m}}{\theta_1^{q^m-\omega_d^{q^m}}})(\frac{\theta_2-\omega_d}{\theta_1-\omega_d})$$
$$=(\frac{\theta_1-\omega_d}{\theta_2-\omega_d})(\frac{\theta_2-\omega_d}{\theta_1-\omega_d})$$
$= 1$ Suppose N= n(n+1) and then $N = q^m - 1$ have definitely N-1 different Nth roots of unity which are not equal to one. Consequently, the location set is expressed as,
$$\frac{\theta_2-\omega_d}{\theta_1-\omega_d}=\rho^d$$
therefore $\omega_d+\omega_{n+1-d}=\frac{\theta_2-\theta_1\rho^d}{1-\rho^d}+\frac{\theta_2-\theta_1\rho^{n+1-d}}{1-\rho^{n+1-d}}$
$$=\frac{\theta_2-\theta_1\rho^d}{1-\rho^d}+\frac{\theta_2-\theta_1\rho^{-d}}{1-\rho^{-d}}$$

$= \dfrac{\theta_2 - \theta_1\rho^d}{1-\rho^d} + \dfrac{\theta_1 - \theta_2\rho^d}{1-\rho^d}$
$= \theta_1 + \theta_2$
Thus, we follow the H_E

$$\mathbf{H} = \begin{pmatrix} 1 & 1 & 1.....1 & 1 \quad .....1 \\ 1 & \rho & \rho^2...\rho^{N-1} & \rho^{(N+1)-1}...\rho^{(N+N)-1} \\ 1 & \rho^2 & (\rho^2)^2...(\rho^2)^{N-1} & (\rho^2)^{(N+1)-1}...(\rho^2)^{(N+N)-1} \\ : & : & : .... : & : ...... : \\ 1 & \rho^d & (\rho^d)^d...(\rho^d)^{N-1} & (\rho^d)^{(N+1)-1}...(\rho^d)^{(N+N)-1} \\ 1 & \rho^{-1} & \rho^{-2}...\rho^{-(N-1)} & \rho^{-(N+1)+1}...\rho^{-(N+N)+1} \\ 1 & \rho^{-2} & (\rho^{-2})^2...(\rho^{-2})^{N-1} & (\rho^{-2})^{(N+1)-1}...(\rho^{-2})^{(N+N)-1} \\ : & : & : .... : & : ...... : \\ 1 & \rho^{-d} & (\rho^{-d})^d...(\rho^{-d})^{N-1} & (\rho^{-d})^{(N+1)-1}...(\rho^{-d})^{(N+N)-1} \end{pmatrix}$$

Example The Goppa code with $L = GF(2^4)$,let $g(z) = (z-\theta)(z-\theta^4)$ as θ is a primitive in $GF(2^4)$ and suppose we ordered the location set L such as
$\theta_i = \dfrac{\theta^3 - \theta\theta^i}{1-\theta^i} \; for 1 \leqslant i \leqslant 14$
let$g(z) = [(z-\theta)(z-\theta^3)]^4$ then we get

$$\mathbf{H} = \begin{pmatrix} (\theta-\theta^2)^{-1} & (\theta-0)^{-1}....(\theta-\theta^{11})^{-1} \\ (\theta-\theta^2)^{-2} & (\theta-0)^{-2}....(\theta-\theta^{11})^{-2} \\ (\theta-\theta^2)^{-3} & (\theta-0)^{-3}....(\theta-\theta^{11})^{-3} \\ (\theta-\theta^2)^{-4} & (\theta-0)^{-4}....(\theta-\theta^{11})^{-4} \\ (\theta^3-\theta^2)^{-1} & (\theta^3-0)^{-1}....(\theta^3-\theta^{11})^{-1} \\ (\theta^3-\theta^2)^{-2} & (\theta^3-0)^{-2}....(\theta^3-\theta^{11})^{-2} \\ (\theta^3-\theta^2)^{-3} & (\theta^3-0)^{-3}....(\theta^3-\theta^{11})^{-3} \\ (\theta^3-\theta^2)^{-4} & (\theta^3-0)^{-4}....(\theta^3-\theta^{11})^{-4} \end{pmatrix}$$

$$\mathbf{H} = \begin{pmatrix} \theta^{10} & \theta^{14}....\theta^9 \\ \theta^5 & \theta^{13}......\theta^3 \\ 1 & \theta^{12}.....\theta^{12} \\ \theta^4 & \theta^{11}.....\theta^{11} \\ \theta^9 & \theta^{11}.....\theta^{10} \\ \theta^3 & \theta^9.....\theta^5 \\ \theta^{12} & \theta^6.....1 \\ \theta^{10} & \theta^5....\theta^{12} \end{pmatrix}$$

$$\mathbf{H_E} = \begin{pmatrix} 1 & 1.........1 \\ 1 & (1+\theta^9\theta^{10})^1...(1+\theta_9\theta^9)^1 \\ 1 & (1+\theta^9\theta^{10})^2...(1+\theta_9\theta^9)^2 \\ 1 & (1+\theta^9\theta^{10})^3...(1+\theta_9\theta^9)^3 \\ 1 & (1+\theta^9\theta^{10})^4...(1+\theta_9\theta^9)^4 \\ 1 & (1-\theta^9\theta^{10})^1...(1-\theta_9\theta^9)^1 \\ 1 & (1-\theta^9\theta^{10})^2...(1-\theta_9\theta^9)^2 \\ 1 & (1-\theta^9\theta^{10})^3...(1-\theta_9\theta^9)^3 \\ 1 & (1-\theta^9\theta^{10})^4...(1-\theta_9\theta^9)^4 \end{pmatrix}$$

$$\mathbf{H_E} = \begin{pmatrix} 1 & 1..........1 \\ 1 & \theta^1........\theta^{14} \\ 1 & \theta^2.....(\theta^{14})^2 \\ 1 & \theta^3.....(\theta^{14})^3 \\ 1 & \theta^4.....(\theta^{14})^4 \\ 1 & \theta^{-1}.....(\theta^{14})^{-1} \\ 1 & \theta^{-2}.....(\theta^{14})^{-2} \\ 1 & \theta^{-3}.....(\theta^{14})^{-3} \\ 1 & \theta^{-4}.....(\theta^{14})^{-4} \end{pmatrix}$$

which defines a reversible $(16, 2)$ cyclic code generated by
$g(x) = (x+1)(x^4+x+1)(x^4+x^3+1)(x^4+x^3+x^2+x+1)(x^5+x^4+x^3+x^2+x+1)$

4 Conclusion

Goppa codes and a set of extended reversible Goppa codes have been formulated and a new extended subclass of cyclic and reversible codes is presented in this study, resulting in the representation of extended reversible Goppa codes. An additional example given above is considered,where $g(x) = (x+1)(x^4+x+1)(x^4+x^3+1)(x^4+x^3+x^2+x+1)(x^5+x^4+x^3+x^2+x+1)$is termed as a reversible code with Goppa polynomial of the degree. This work arise a certain number of new research problems. In future, we are going to enhance a moderated extended reversible code for a generalized cyclic Goppa code.

References

[1] Sergey V. Bezzateev and Natalia A. Shekhunova, Subclass of cyclic Goppa codes, *IEEE Transactions on Information Theory*, **59** (2013), no. 11,

7379-7385. http://dx.doi.org/10.1109/tit.2013.2278176

[2] Elwyn R. Berlekamp, Goppa codes, *IEEE Transactions on Information Theory*, **19** (1973), no. 5, 590-592. http://dx.doi.org/10.1109/tit.1973.1055088

[3] Sergei V. Bezzateev and Natalia A. Shekhunova, Subclass of binary Goppa codes with minimal distance equal to the design distance, *IEEE Transactions on Information Theory*, **41** (1995), no. 2, 554-555. http://dx.doi.org/10.1109/18.370170

[4] Sergey Bezzateev and Natalia Shekhunova, Chain of separable binary Goppa codes and their minimal distance, *IEEE Transactions on Information Theory*, **54** (2007), no. 12, 5773-5778. http://dx.doi.org/10.1109/tit.2008.2006442

[5] Sergei V. Bezzateev and Natalia A. Shekhunova, One generalization of Goppa codes, *Proceedings IEEE International Symposium on Information Theory*, (1997). http://dx.doi.org/10.1109/isit.1997.613221

[6] K. K Tzeng and K. Zimmermann, On Extendind Goppa Codes to Cyclic Codes, *IEEE Transactions on Information Theory*, **21** (1975), 712-716. http://dx.doi.org/10.1109/tit.1975.1055449

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