International Journal of Contemporary Mathematical Sciences
Vol. 11, 2016, no. 6, 251 - 265
HIKARI Ltd, www.m-hikari.com
http://dx.doi.org/10.12988/ijcms.2016.638

# Core in Rough Graph and Weighted Rough Graph

**S. Robinson Chellathurai**

Scott Christian College (Autonomous), Nagercoil, India

**L. Jesmalar**

Holy Cross College (Autonomous), Nagercoil, India



**Abstract**

The main objective of this study is to show the usefulness of rough set theory in graph theory. The aim of this paper is to construct the structure of rough graph and introduced weighted rough graph. Also discuss about the distance and core in rough graph and weighted rough graph.

**Keywords**: Undefinable (rough) spanning sub graph, weighted rough graph, weighted eccentricity, weighted center, Rough graph core & weighted rough graph core

## 1. Introduction

Rough set theory, proposed by Zdzislaw Pawlak (1982), models uncertainty by equivalence relations. The primary notion is the partitioning of the domain in to equivalence classes. Objects belonging to the same equivalence cannot be distinguished. Hence Rough set theory is a theory of multiple memberships.

A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. When there is vagueness in the description of the objects or in its relationships or in both, it is natural that we need to design a “Rough graph model”.

An attribute set was first assigned to edges of a graph. Given a subset R of the attribute set, there is associated an equivalence relation, based in which R-rough graph is defined. If the vertices of a graph are connected through edges denoting different possible relationship.

In this paper the core is taken up the central structure of a rough graph & weighted rough graph. The core of a graph G is a path of minimum distance. Minieka & patel [6] have investigated the problem of finding a tree core with a specified length. EliezerA. Albacea [2] has presented a parallel algorithm for finding the core of the tree with **weights only on edges**.

In a particular application, villages may be considered as vertices and roads as edges. In such cases the population of a village will then be the **rough weight of the vertex**, the bus route, which goes adjacent to the villages may be treated as central structure .now the people of several villages have to reach the bus route traversing the lowest possible distance.

In this edge & vertex weighted rough graphs, it is proposed to design & develop to find the central structures like radius, diameter, center and core.

## 2. Definition and main results

### 2.1 Definition –Rough set

A rough set is a formal approximation of a crisp set in terms of a pair of sets which the lower and upper approximation of the original set

Let U denote the set of objects called universe and let R be an equivalence relation on U. The pairs A= (U,R) is called an approximation space. For u, v ∈U & (u,v) ∈ R, u and belong to the same equivalence class and we say that they are indistinguishable in A. The relation R is called an indiscernibility relation.

Let $[x]_R$ denote an equivalence class of R containing element x, then lower & upper approximation for a subset X ⊆ U in A denoted by $\underline{A}$(X) & $\overline{A}$(X) respectively

Where $\underline{A}(X) = \{x \in U / [x]_R \subset X\}$

$\overline{A}(X) = \{ x \in U / [x]_R \cap X \neq \phi\}$Thus if an object x∈ $\underline{A}$(X) then "x surely belongs to X in A" If x∈ $\overline{A}$(X) then "x possibly belong to X in A"

### 2.2 Membership value

The membership value of X is $\mu(X) = \frac{|\underline{A}(X)|}{|\overline{A}(X)|}$

The membership value of each element of X in A $\mu_X(x) = \frac{\#([x]_R \cap X)}{\#[x]_R}$

### 2.3 Results

1. If $\overline{A}$(X) = $\underline{A}$(X) then X is definable on attribute set A.
2. If $\overline{A}$(X) $\neq$ $\underline{A}$(X) then X is undefinable (roughly definable) on attribute set A

i. If $\underline{A}$(X)$\neq \phi$&$\overline{A}$(X) $\neq$U then X is undefinable (R- roughly definable)on attribute set A

ii. If $\underline{A}$(X) $\neq \phi$ &$\overline{A}$(X)= =U then X is internally undefinable ($R_i$) on attribute set A

iii. If $\underline{A}$(X) $= \phi$ &$\overline{A}$(X)$\neq$U then X is externally undefinable ($R_e$) on attribute set A

iv. If $\underline{A}$(X) $= \phi$ &$\overline{A}$(X)=U then X is totally undefinable ($R_t$) on attribute set A

## 3. R-Rough graph

### 3.1 Definition –R-Rough graph

Let U= (V,E) be the universal graph . Where V= $\{v_1, v_2, v_{3,} \ldots v_n\}$ is the set of vertices & E={$\{e_1, e_2, \ldots\ e_n$ } is the set edges on U where the edge $e_k$is endued with vertex attribute($v_i, v_j$).Let **R** =$\{r_1, r_2, \ldots\ r_{|R|}\}$ be the attribute set on U. For any attribute set R⊆**R** on E, the elements of E can be divided into different equivalence classes $[e]_R$.

For any graph T= (W,X) ,where W⊆Vand X⊆ E,If X is the sum of equivalence classes then the graph is called R-definable graph or R-exact graph. If not, the graph is called R-un definable graph or R-rough graph. For R-rough graph, two exact graphs

$\underline{R}$(T)=(W,$\underline{R}(X)$) &$\overline{R}$(T)=(W, $\overline{R}(X)$) can be used to define it approximately.
Where $\underline{R}$(X) = {e∈E/ $[e]_R \subset$ X}
$\overline{R}$(X)= {e∈E/ $[e]_R \cap$ X$\neq \phi$}The graph $\underline{R}$(T) & $\overline{R}$(T)are called R-Lower & R-Upper approximation of graph T. The pair of graph ($\underline{R}$(T) ,$\overline{R}$(T)) is called R-rough graph

### 3.2 Results

1. The graph T =($\underline{R}$(T),$\overline{R}$(T)) is exact iff ($\underline{R}$(T)= $\overline{R}$(T).
2. The graph T =($\underline{R}$(T),$\overline{R}$(T)) is called a rough graph iff ($\underline{R}$(T)$\neq$ $\overline{R}$(T).
3. The graph T =($\underline{R}$(T),$\overline{R}$(T)) is a classical graph iff all the edges of T belong to the same equivalence class with respect to R

**3.3 Example**

Let us consider the graph G=(W,X) where W⊆Vand X⊆ E
W={$v_1, v_2, v_{3,}\ v_4,\ v_5, v_6,\ v_7, v_8,\ v_9$}and E={$e_1, e_2, e_{3,}\ e_4,\ e_5, e_6,\ e_7, e_8,\ e_{9,} e_{10}$}

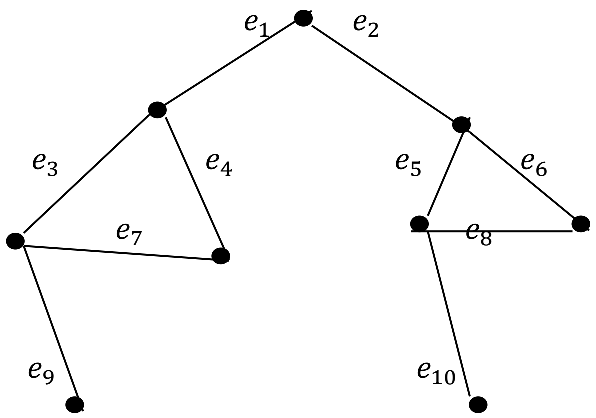


Fig 1

Let X={$e_{1,}\ e_3,\ e_{4,} e_6,\ e_{10}$} Let R be the equivalence relation, here R=edge coloring

The equivalence classes are{{$g, g, g$},{$r, r$},{$y, y, y$},{ $b, b$}}
E={{$e_{1,}\ e_{6,}\ e_{10}$},{$e_{2,}\ e_3$},{$e_{4,}\ e_{5,}\ e_9$},{ $e_7, e_8$}}

R-Lower & R-Upper approximation of graph G are $\underline{R}$(G)& $\overline{R}$(G)
$\underline{R}$(G)=(W,$\underline{R}(X)$) &$\overline{R}$(G)=(W, $\overline{R}(X)$) can be used to define it approximately.

Where $\underline{R}$(X)={e∈E/[e]$_R$ ⊂ X}
={$e_{1,}\ e_{6,}\ e_{10}$}
$\overline{R}$(X)={ e∈E/[e]$_R$ ⋂ X≠ $\phi$}
={$e_1, e_2, e_{3,}\ e_4,\ e_5, e_6,\ \ e_{9,} e_{10}$}

The membership values are $\mu_X(e_1)= \frac{\#([e_1]_R \cap X)}{\#[e_1]_R}$ =1, $\mu_X(e_6)=\mu_X(e_{10})=1$

$\mu_X(e_2)=\mu_X(e_3)=\frac{1}{2}$

$\mu_X(e_4)=\mu_X(e_5)= \mu_X(e_9)=\frac{1}{3}$

**Results 3.4**

(i)$\underline{R}$(X)$\subset X \subseteq \overline{R}$(X) (ii)$\underline{R}(\phi)= \phi = \overline{R}(\phi)$(iii)$\underline{R}(U)= U = \overline{R}(U)$(iv)$\underline{R}$(X$\cap Y$)= $\underline{R}$(X) $\cap$ $\underline{R}(Y)$

(v)$\overline{R}$(X$\cup Y$)= $\overline{R}(X) \cup \overline{R}(Y)$ (vi)X$\subseteq$Y $\Rightarrow$ $\underline{R}$(X) $\subseteq$ $\underline{R}(Y)$ &$\overline{R}(X)\subseteq$ $\overline{R}(Y)$

(vii)$\underline{R}$(X$\cup Y$)$\supseteq$ $\underline{R}$(X) $\cup$ $\underline{R}(Y)$(viii)$\overline{R}$(X$\cap Y$)$\subseteq$ $\overline{R}(X) \cap \overline{R}(Y)$(ix)$\underline{R}(\underline{R}$(X))= $\overline{R}(\underline{R}$(X))= $\underline{R}$(X)

(x)$\underline{R}(\overline{R}$(X))= $\overline{R}(\overline{R}$(X))= $\overline{R}$(X).

## 4. Rough -Spanning sub graph

**Theorem 4.1: Let** G be a complete graph with n≥2 vertices then

(i) Number of definable spanning sub graph under minimum edge coloring is

$$n(d(K_n))=\begin{cases} 2^n & if\ n\ is\ odd \\ 2^{n-1} & if\ n\ is\ even \end{cases}$$

(ii) Number of undefinable (rough) spanning sub graph under minimum edge coloring is

$$n(R(K_n))=\begin{cases} 2^{\frac{n(n-1)}{2}} - 2^n & if\ n\ is\ odd \\ 2^{\frac{n(n-1)}{2}} - 2^{n-1} & if\ n\ is\ even \end{cases}$$

**Theorem 4.2:** Let G be a cycle with n≥3 vertices then

(i) Number of definable spanning sub graph under minimum edge coloring is

$$n(d(C_n))=\begin{cases} 2^3 & if\ n\ is\ odd \\ 2^2 & if\ n\ is\ even \end{cases}$$

(ii) Number of undefinable (rough) spanning sub graph under minimum edge coloring is

$$n(R(C_n))=\begin{cases} 2^n - 2^3 & if\ n\ is\ odd \\ 2^n - 2^2 & if\ n\ is\ even \end{cases}$$

**Theorem 4.3:** Let G(V,E) be any graph .If the number of edges is equal to the number the equivalence classes under attribute R then all the spanning sub graph of the given graph is definable under the attribute R.

**Example 4.4**

**(i) In $K_3$:** All the edge sub graph of $K_3$ are definable under minimum edge coloring n(d($K_4$ ))=$2^3$

**(ii) In $K_4$:** n(d($K_4$ ))=$2^3$ , n(R($K_4$ ))=12, n($R_i$($K_4$ ))=18 n($R_e$($K_4$ ))=18 n($R_t$($K_4$ ))=8

**(iii) In $K_5$ ;**n(d($K_4$ ))= $2^5$ , n(R($K_4$ ))=620, n($R_i$($K_4$ ))=170 n($R_e$($K_4$ ))=172 n($R_t$($K_4$ ))=30

**(iv) In $C_4$:**n(d($C_4$ ))=$2^2$ , n(R($C_4$ ))=0, n($R_i$($C_4$ ))=4 n($R_e$($C_4$ ))=4 n($R_t$($C_4$ ))=4

**(v) In $C_5$ :** n(d($C_5$ ))=$2^3$ , n(R($C_5$ ))=8, n($R_i$($C_5$ ))=8 n($R_e$($C_5$ ))=8 n($R_t$($C_5$ ))=0

## 5. Weighted Rough graph

**Definition 5.1**:A Rough graph is a pair R(G)=($\underline{R}$(G),$\overline{R}$(G)) where $\underline{R}$(G):($\underline{V}$,$\underline{E}$) is a crisp graph and $\overline{R}$(G):($\overline{V}$,$\overline{E}$) is a fuzzy graph and $\underline{R}$(G) ⊆ $\overline{R}$(G)) .

A weighted rough graph is of the form G : ($\underline{R}$,$\overline{R}$ ,$V,\sigma,\mu$) Where $\mu$ is a fuzzy subset of a nonempty set $E$ & $\sigma$ is a fuzzy relation on $\mu$. Define $\mu_1$ : $\overline{R}$ →(0,1) , $\mu_2$ : $\underline{R}$ → 1

& $\sigma(u) \geq$ max{$\mu(u\ ,u_i)$ / $u$ and $u_i$ are adjacent, where 1≤ i ≤ n}

**Definition5.2:** The complement of a weighted rough graph G : ($\underline{R}$,$\overline{R}$,V,$\sigma,\mu$) is G':($\underline{R}$ ',$\overline{R}$ ' V',$\sigma$ ',$\mu$ ' ) Where (i) V' = V (ii) $\mu$ ' =1- $\mu$ (iii) $\sigma$ '($u$) ≥ { max{$\mu$'($u$ ,$u_i$) / $u$ and $u_i$ are adjacent where $u_i$ ∈ V }.

**Definition5.3:** Let G be a weighted rough graph. The **degree** of an edge e=uv is $d_{R(G)}$(uv)= $\sigma$(u)+$\sigma$(v). Minimum degree of G is $\delta(G)$= ∧{$d(uv)/uv$ ∈ $E$}. Maximum degree of G is ∆ $(G)$= ∨{$d(uv)/uv$ ∈ $E$}. Let G be a weighted rough graph. The **degree** of an vertex '$v$' is defined by $d_{(R(G))_{u\neq v}}(v)= \sum \mu(u\ ,v)$

**Definition 5.4:**Let $R$(G) :($\sigma,\mu$) be a rough graph .If $d_{R(G)}$(e)=k for all e ∈ $E$,then $R$G) is said to be an regular weighted edge rough graph of degree k or a k- regular weighted edge rough graph.

**Remark 5.5:** ($\overline{R}$(G)) is a k-edge regular weighted rough graph iff $\delta$=∆=k.

**Definition 5.6:** A weighted rough graph is **strong** if $\sigma(u) = \max\{\mu(u, u_i) \,/\, u$ and $u_i$ are adjacent, where $1 \le i \le n\}$

**Definition 5.7:** A weighted rough graph is **complete** if $\sigma(u) = \max\{\mu(u, u_i) \,/\, u$ and $u_i$ are adjacent, $\forall\ i = 1,2 \dots n\}$ $u, u_1 u_2 \dots\dots u_n$ ∈ σ* and every adjacent edges $uu_1, uu_2, \dots\dots uu_n$ ∈ μ*where G:( σ*, μ*) is a crisp graph

**Theorem 5.8:** Every rough graph is a fuzzy graph. But the converse need not be true.

**Definition 5.9:** A path P of **length** n is a sequence of distinct nodes $u_0 u_1 \dots\dots u_n$ s.t $\mu(u_{i-1}, u_i)$>0,i=1,2…n & the degree of membership of a weakest edge is defined as its **strength**. The strength of connectedness between two nodes u & v is defined as the maximum of the strength of all path between u & v [4]

## 6. Distance in Rough graph

**Definition6.1:Eccentricity** of a vertex v, denoted by $e_R$(v) is the distance from v to a vertex farthest from it $e_R$(v)= max{$d_R$(u,v) :u∈ V}, **$d_R$(u,v) =$\wedge_p$ {$l_p$ x $s_p$ where p is a(u, v) path, $l_p$ - length of a path & $s_p$-strength of a path** }.**Diameter** is the maximum eccentricity & **radius** is the minimum eccentricity . Each vertex in V at which the eccentricity function is minimized is called a **center.** A vertex v is called peripheral vertex if $e_R$(v)=diam(R(G)).Average distance is defined by A(R(G)) $=\frac{1}{n_{C_2}} \sum d_R(u, v)$. Each vertex in V at which the distance function is minimized is called a **median** of R(G). The path of minimum distance is called a **core** (or) path median of R(G).

**Theorem6.2:** In a rough graph R(G), $d_R$:VxV→ [0,1] is a metric on V , ∀ u,v,w ∈ V

(i) $d_R(u, v) \ge 0$ ∀ u,v ∈ V

(ii) $d_R(u, v) = 0$ $iff$ u=v

(iii) $d_R(u, v) = d_R(v, u)$

(iv) $d_R(u, w) \le d_R(u, v) + d_R(v, w)$

**Theorem6.3**:Let R(G)=($\underline{R}$(G),$\overline{R}$(G)) be a complete rough graph then diam($K_n$) ≤ 1& rad($K_n$)< 1

**Theorem6.4**:Let R(G)=($\underline{R}$(G),$\overline{R}$(G)) be a complete bipartite rough graph then diam($K_{m,n}$) ≤ 2 & rad($K_{m,n}$)<2

**Theorem6.5**: Let R(G)=($\underline{R}$(G),$\overline{R}$(G)) be a path on n vertices then diam($P_n$)=$\sum_{i=1}^{n} \mu(u_1, u_i)$
rad($P_n$)$\leq \sum_{i=1}^{n} \mu(u_1, u_i)$

**Theorem6.6**: Let R(G)=($\underline{R}$(G),$\overline{R}$(G)) be a complete rough graph & rad(R(G) =1 then R(G) is a self centered graph.

**Definition6.7:** Let R(G)=($\underline{R}$(G),$\overline{R}$(G)) be a connected rough graph. If there is at most one strongest path between any two nodes of R(G) ,then R(G) is a rough graph tree

## 7. Distance in Weighted Rough graph

**Eccentricity** of a vertex v, denoted by $e_{wR}$(v) is the distance from v to a vertex farthest from it $\boldsymbol{e_{wR}(v)}$**= max{**$\boldsymbol{d_{wR}(u,v)}$ **:**$\boldsymbol{u}$ **∈ V},** $\boldsymbol{d_{wR}(u,v)}$ **=**$\boldsymbol{d_R(u,v)}$ **x** $\boldsymbol{\sigma(v)}$ **∀** $\boldsymbol{u, v}$ **∈ V** where $d_R(u, v)$ is distance in rough graph.**Diameter** is the maximum eccentricity & **radius** is the minimum eccentricity .Each vertex in V at which the eccentricity function is minimized is called a **center** of R(G).A vertex v is called peripheral vertex if e(v)=diam(R(G).Average distance is defined by A(R(G)) =$\frac{1}{n_{C_2}} \sum d_{wR}(u, v)$. Each vertex in V at which the distance function is minimized is called a **median.** The path of minimum distance is called a **w-core** (or) path median.

**Theorem7.1:** For any weighted rough graph G, the radius and diameter satisfy rad(G)≤diam(G)≤ 2rad(G)**.**

**Theorem7.2:** Let G be a weighted rough graph, center of graph need not be same as the center of its underlying graph.

**Theorem7.3**

For any two adjacent vertices u,v in a weighted rough graph ,$|e_{wR}(u) - e_{wR}(v)| \leq 1$.

**Theorem7.4**

For any two vertices u,v in a weighted rough graph ,$|e_{wR}(u) - e_{wR}(v)| \leq d_{wR}(u, v)$

**Remark7.5**

In ordinary trees, the center is $K_1$ (or) $K_2$, but in weighted rough graph trees, the center need not be $K_1$ (or) $K_2$.

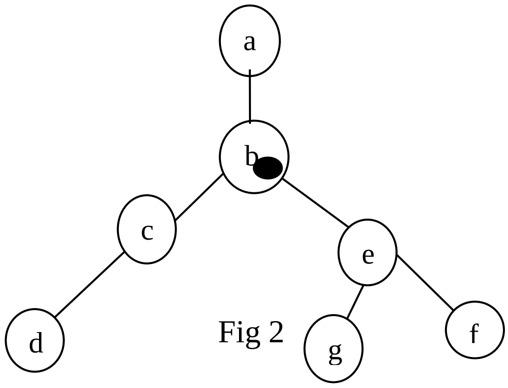


Fig 2

Center of a tree in a crisp graph

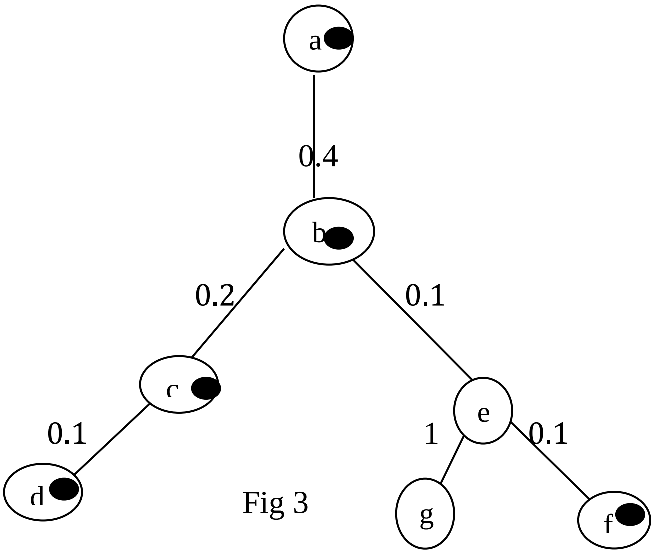


Fig 3

Center of a tree in a rough graph

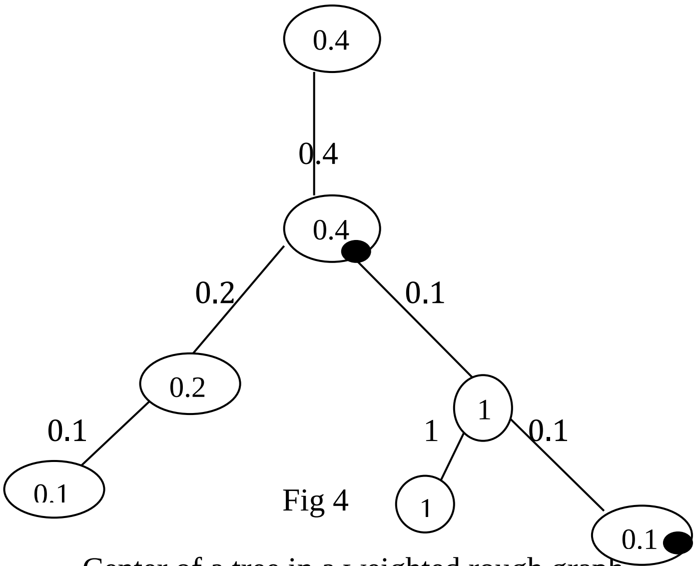


Fig 4

Center of a tree in a weighted rough graph

**Remark7.6:** The center of a rough graph and a weighted rough graph are need not be same

In fig 3, the central vertices of a rough graph are a, b, c, d &f. But in fig 4 the cental vertices of a weighted rough are b &f

**Theorem7.2:** A rough graph which is self centered need not be a self centered weighted rough graph.

## 8. Find the core of the given rough graph tree

Finding **core** of the given tree consists of the following four steps.

1. Find the end point for the given tree; call it $e_1$.
2. Re-root the tree at $e_1$.
3. Find the end point of the new tree; call it $e_2$.
4. The path from $e_1$ to $e_2$ forms the **core** of the tree.

These steps are explained in detail with the help of an example. Finding the endpoint of a core includes the following four steps.

(i)For each vertex $v \in V$, compute the size (number of nodes) of the sub tree rooted at v, size(v)(ii)For each vertex $v \in V$, compute reduction (r,v).(iii)Compute max r =max{reduction (r,v); $v \in V$. (iv)The end point is the vertex v with reduction (r,v)= max r

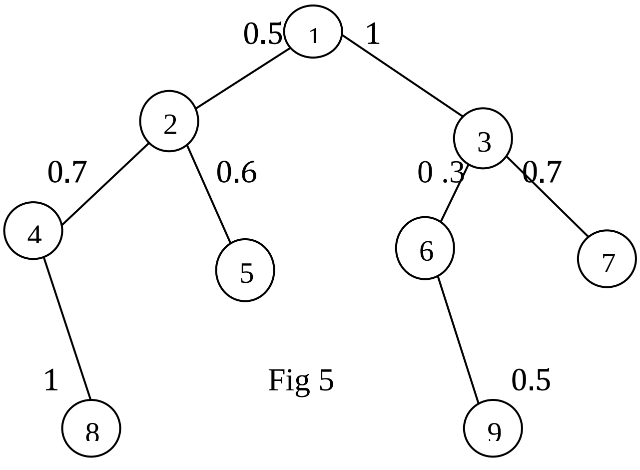


Fig 5

Step 1: Find the end point of the given tree (i)Compute size(v)

| V | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| size | 9 | 4 | 4 | 2 | 1 | 2 | 1 | 1 | 1 |

(ii)Compute reduction (r,v) where r=1. Reduction: $(r,u_2)$=.5x4=2, $(r,u_3)$=1x4=4,$(r,u_4)$=2x.7+4x.5=3.4,$(r,u_5)$=1x.6+4x.5=2.6,$(r,u_6)$=2x.3+4x1=4.6,$(r,u_7)$=1x.2+4x1=4.7, $(r,u_8)$=1x1+2x.7+4x.5=4.4 , $(r,u_9)$=1x.5+2x.3+4x1=5.1

(iii)Since the highest reduction is 5.1 ,the corresponding vertex 9 is the end point ,$e_1$=9

Step 2: Re-root the tree at $e_1$

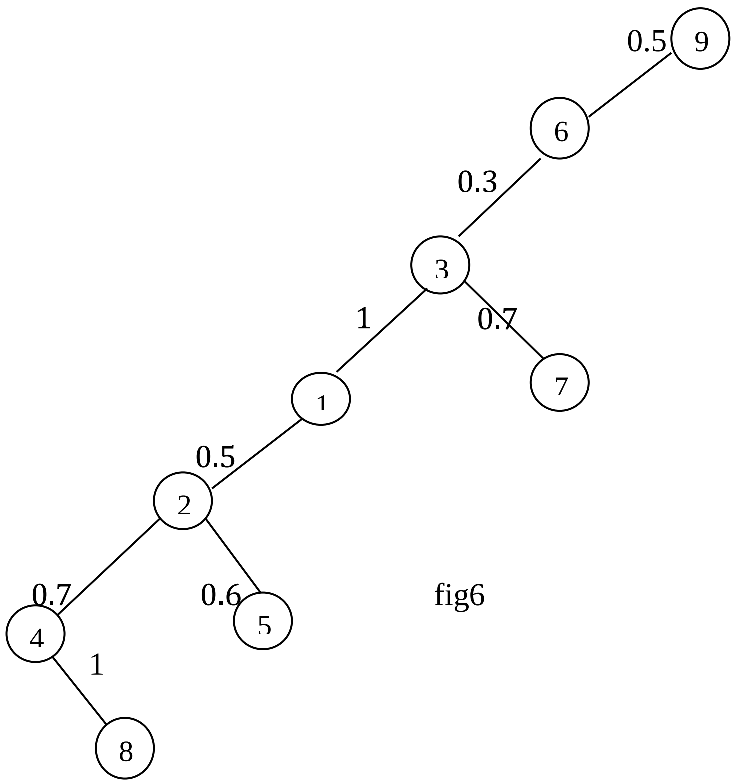


fig6

Step 3: Find the end point of the re-rooted tree.(i)Compute size(v)

| V | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| size | 5 | 4 | 7 | 2 | 1 | 8 | 1 | 1 | 9 |

(ii)Compute reduction (r,v) where r=9.
Reduction:$(r,u_6)$=4,$(r,u_3)$=4+2.1=6.1,$(r,u_7)$=6.1+.7=6.8,$(r,u_1)$=6.1+5=11.1$(r,u_2)$=13 .1, $(r,u_4)$=13.7, $(r,u_5)$=14.1, $(r,u_8)$=15.5.Since the highest reduction is 15.5 the corresponding vertex 8 is the end point,$e_2$ =8

Step4: The path from $e_1$ to $e_2$ forms the core of the tree

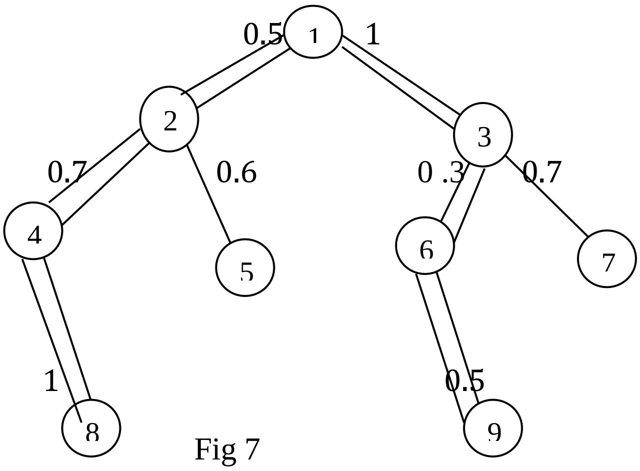


Fig 7

## 9. Find the core of the given weighted rough graph tree

Finding **core** of the given tree consists of the following four steps.

1. Find the end point for the given tree; call it $e_1$.
2. Re-root the tree at $e_1$.
3. Find the end point of the new tree; call it $e_2$.
4. The path from $e_1$ to $e_2$ forms the **w-core** of the tree.

These steps are explained in detail with the help of an example. Finding the endpoint of a core includes the following four steps.

i. For each vertex $v \in V$, compute the sub tree weighted distance at v, stwrd(v)
ii. For each vertex $v \in V$, compute reduction (r,v).
iii. Compute max r =max{reduction (r,v); $v \in V$.
iv. The end point is the vertex v with reduction (r,v)= max r

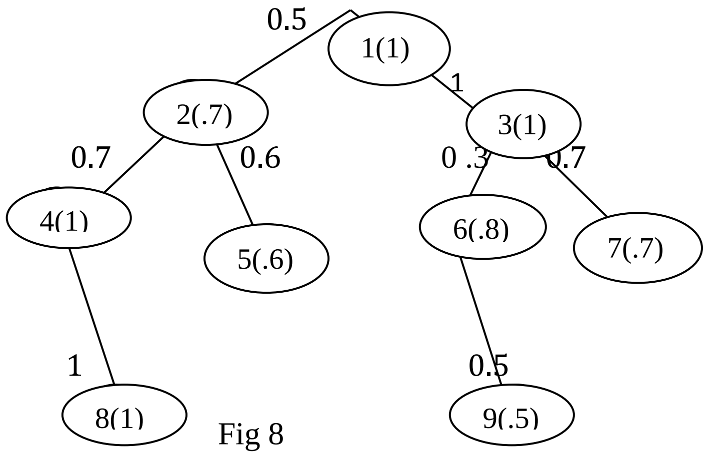


Fig 8

Step 1: Find the end point of the given tree.(i)Compute size(v)

| V | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| size | - | 2.76 | 1.04 | 1 | 0.6 | 0.25 | 0.7 | 1 | 0.5 |

(ii)Compute reduction (r,v) where r=1. Reduction :$(r,u_2)$=1.38 $(r,u_3)$=1.04, $(r,u_4)$=2.08, $(r,u_5)$=1.74, $(r,u_6)$=1.79,$(r,u_7)$=1.53, $(r,u_8)$=3.08, $(r,u_9)$=1.94
(iii)Since the highest reduction is 3.08 the corresponding vertex 8 is the end point, $e_1$=8

Step 2: Re-root the tree at $e_1$

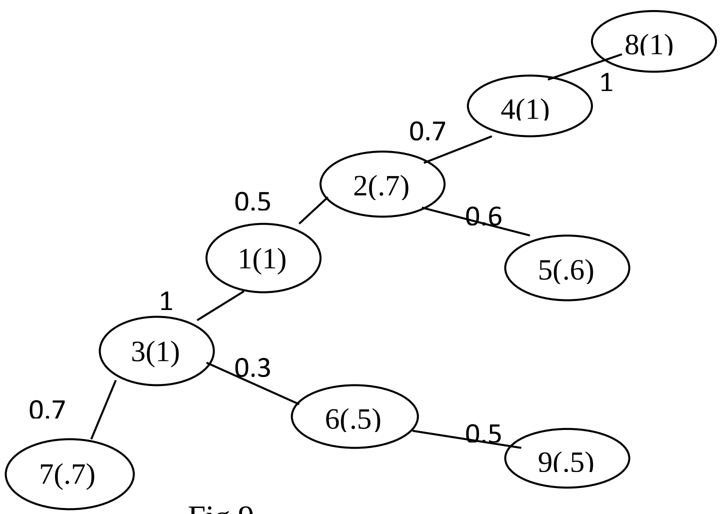


Fig 9

Step: 3: Find the end point of the re-rooted tree (i)Compute sub tree weighted distance (strwd)

| V | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| strwd | 2.79 | 0.36 | 1.04 | 7.8 | 0.6 | 0.25 | 0.7 | - | 0.5 |

(ii)Compute reduction (r,v) where r=8. Reduction: (r,7)=10.537 Since the highest reduction is 10.537 the corresponding vertex 7 is the end point,$e_2$ =7

Step4: The path from $e_1$ to $e_2$ forms the core of the tree

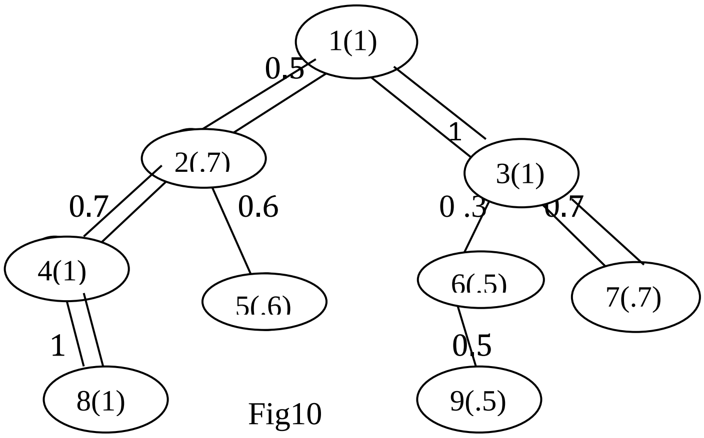


Fig10

## 10. Conclusion

When compare fig7 & fig10 it is clear that the core of a rough tree is different from w-core of the weighted rough tree. Hence the vertex weights have the direct impact on the decision of selecting a facility location.

We make a further study to the combination of rough set and graph theory. Graph theory posses a sophisticated mathematical structure. And it has been widely used in real world. So we hope our work in this paper could be contributive to the theoretical development and applications of rough set.

## References

[1] Abd-El Aziz E. Radwan, Mohammed Shokry Nayle, Ahmed Ibrahem Nasir, New rough set properties on graph theory, *Int. J. Contemp. Math. Sciences*, **7** (2012), no. 25, 1217 – 1232.

[2] E.A. Albacea, Parallel algorithm for finding a core of a tree network, *Information Processing Center*, **51** (1994), 223-226.
http://dx.doi.org/10.1016/0020-0190(94)00102-2

[3] A Rosenfield, Fuzzy Graphs, Chapter in *In Fuzzy Sets and Their Applications to Cognitive and Decision Processes*, L.A. Zadeh, K. S. Fu, M. Shimura, (Eds) Academic Press, Newyork, 1975, 77-95.
http://dx.doi.org/10.1016/b978-0-12-775260-0.50008-6

[4] Fred Bukley, Frank Harary, *Distance in Graphs*, Addision –Wesly, Red Wood city, CA, 1990.

[5] Anjan Mukherjee, S. Bhattacharya Halder, *Fuzzy Sets and Fuzzy Topology*, 2015.

[6] E. Minieka and N.H, Patel, On finding the core of a tree with a specific length, *Journal of Algorithm*, **4** (1983), 345-352.
http://dx.doi.org/10.1016/0196-6774(83)90015-9

[7] Frank Harary, *New Directions in the theory of Graphs*, Academic Press, 1971.

[8] Zdzislaw Pawlak, Rough sets, *Int. Journal of Computer and Information Sciences*, **11** (1982), 341-356. http://dx.doi.org/10.1007/bf01001956

**Received: March 24, 2016; Published: June 1, 2016**