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On Intuitionistic Q-Fuzzy K-Ideals of Semiring

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Abstract

In this paper, we apply the concept of intuitionistic Q-fuzzy set to semirings. We introduced the notion of anti Q-fuzzy right ideal, anti Q-fuzzy right k-ideal and intuitionistic Q-fuzzy right k-ideal in semiring. We investigate the their properties and connections with right k-ideal, Q-fuzzy right k-ideal, anti Q-fuzzy right k-ideal.

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1 Introduction

Henriksen defined in Henriksen (1958) a more restricted class of ideals in semiring, which is called the class of k-ideals, with the property that if the semiring R is a ring then a complex in R is a k-ideal if and only if it is a ring ideal.

Atanassov introduced intuitionistic fuzzy set which constitute a generalization of the notion of fuzzy sets [1],[2]. The degree of membership of an element in a given set, while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership. M. Akram and W.A. Dudek introduced the notion of intuitionistic fuzzy left k-ideal in semiring [4]. K.H. Kim[3] studied intuitionistic Q-fuzzy ideals. In this paper, we apply the concept of intuitionistic Q-fuzzy set to semirings. We introduced the notion of anti Q-fuzzy right ideal, anti Q-fuzzy right k-ideal and intuitionistic Q-fuzzy right k-ideal in semiring. We investigate the their properties and connections with right k-ideal, Q-fuzzy right k-ideal, anti Q-fuzzy right k-ideal.

2 Preliminary Notes

Definition 2.1 *A nonempty set R together with two binary operations "$+$" and "$\cdot$" is said to be a semiring if*

1) $(R;+)$ *is a commutative semigroup,*
2) $(R;\cdot)$ *is a semigroup,*
3) $a(b+c)=ab+ac$ *and* $(a+b)c=ac+bc$ *for all* $a,b,c\in R$.

By a subsemiring of R we mean a nonempty subset S of R such that S is closed under the operation of addition and multiplication in R. A subsemiring R is called an right (left) ideal of R if for all $r\in R, x\in I, xr\in I(rx\in I)$. A subsemiring I of a semiring S is called an ideal of R if it is both left and right ideal.

A mapping $\mu: M\times Q\to[0,1]$, where M,Q are arbitrary non-empty sets, is called a Q-fuzzy set of M. An upper level set of a Q-fuzzy set μ denoted by $U(\mu;t)$ is defined as $U(\mu;t)=\{x\in M\mid\mu(x,q)\geq t,\forall q\in Q\}$ and a lower level set of a Q-fuzzy set μ denoted by $L(\mu;t)$ is defined as $L(\mu;t)=\{x\in M\mid \mu(x,q)\leq t,\forall q\in Q\}$, for all $t\in[0,1]$.

An intuitionistic Q-fuzzy set (IQFS for short) defined on non-empty sets M and Q as objects of the form

$$A=\{<x,q,\mu_A(x,q),\lambda_A(x,q)>\mid x\in R,q\in Q\},$$

where the function $\mu_A: M\times Q\to[0,1]$ and $\lambda_A: M\times Q\to[0,1]$ denote the degree of membership (namely $\mu_A(x,q)$) and the degree of non-membership (namely $\lambda_A(x,q)$) for each element $x\in M,q\in Q$ to the set A, respectively, and

$$0\leq\mu_A(x,q)+\lambda_A(x,q)\leq 1$$

for each $x\in M,q\in Q$. Obviously, every Q-fuzzy set μ we can have

$$A=\{<x,q,\mu_A(x,q),\lambda_A(x,q)>\mid x\in M,q\in Q\}.$$

For the sake of simplicity, we shall use the symbol $A=(\mu_A,\lambda_A)$ for the intuitionistic Q-fuzzy set $A=\{<x,q,\mu_A(x,q),\lambda_A(x,q)>\mid x\in R,q\in Q\}$. Obviously for an IQFS $A=(\mu_A,\lambda_A)$ in M, when $\lambda_A(x,q)=1-\mu_A(x,q)$, for every $x\in M,q\in Q$, the IQFS A is a Q-fuzzy set.

3 Main Results

Definition 3.1 *A Q-fuzzy set μ of a semiring R is said to be a Q-fuzzy right (left) ideal if*
1) $\mu(x+y,q)\geq\mu(x,q)\wedge\mu(y,q)$.
2) $\mu(xy,q)\geq\mu(x,q)(\mu(xy,q)\geq\mu(y,q))$
for all $x,y\in R,q\in Q$.

Definition 3.2 *A Q-fuzzy set μ of a semiring R is said to be an anti Q-fuzzy right (left) ideal if*

1) $\mu(x+y,q) \leq \mu(x,q) \wedge \mu(y,q)$.
2) $\mu(xy,q) \leq \mu(x,q)(\mu(xy,q) \leq \mu(y,q))$
for all $x, y \in R, q \in Q$.

Definition 3.3 *An intuitionistic Q-fuzzy set* $A = \{< x, q, \mu_A(x,q), \lambda_A(x,q) >| x \in R, q \in Q\}$ *is called an intuitionistic Q-fuzzy right (left) ideal of* R *if*
1) $\mu_A(x+y,q) \geq \mu_A(x,q) \wedge \mu_A(y,q)$.
2) $\mu_A(xy,q) \geq \mu_A(x,q)(\mu_A(xy,q) \geq \mu_A(y,q))$.
3) $\lambda_A(x+y,q) \leq \lambda_A(x,q) \vee \lambda_A(y,q)$.
4) $\lambda_A(xy,q) \leq \lambda_A(x,q)(\lambda_A(xy,q) \leq \lambda_A(y,q))$,
for all $x, y \in R, q \in Q$.

Theorem 3.4 *If a Q-fuzzy set* μ *is a Q-fuzzy right (left) ideal of a semiring* R *if and only if* $1-\mu$ *is an anti Q-fuzzy right (left) ideal in a semiring* R.

Proof. Let μ be a Q-fuzzy right ideal in a semiring R. Let $x, y \in R, q \in Q$. Then $\mu(x+y,q) \geq \mu(x,q) \wedge \mu(y,q)$ implies $-\mu(x+y,q) \leq -(\mu(x,q) \wedge \mu(y,q))$. Thus $-\mu(x+y,q) \leq -\mu(x,q) \vee -\mu(y,q)$. Then $1-\mu(x+y,q) \leq 1-\mu(x,q) \vee 1-\mu(y,q)$. Therefore $(1-\mu)(x+y,q) \leq (1-\mu)(x,q) \vee (1-\mu)(y,q)$. Similarly, we can prove $(1-\mu)(xy,q) \leq \mu(x,q)$. Hence $1-\mu$ is anti Q-fuzzy right ideal in semiring R. Conversely, we can prove that μ is a Q-fuzzy right ideal in similar manner. ■

Lemma 3.5 *A Q-fuzzy set* μ *in a semiring* R *is a Q-fuzzy right (left) ideal if and only if* $U(\mu;t)$ *is a right (left) ideal of a semiring* R *for all* $t \in [0,1]$ *wherever nonempty.*

Definition 3.6 *A right (left) ideal* I *is called right (left) k-ideal of a semiring* R *if* $x+y, y \in I$ *implies* $x \in I$.

Definition 3.7 *A Q-fuzzy right (left) ideal* μ *of a semiring* R *is called Q-fuzzy right (left) k-ideal if for all* $x, y \in R, q \in Q$,

$$\mu(x,q) \geq \mu(x+y,q) \wedge \mu(y,q)$$

Definition 3.8 *An anti Q-fuzzy right (left) ideal* μ *of a semiring* R *is called anti Q-fuzzy right (left) k-ideal if for all* $x, y \in R, q \in Q$,

$$\lambda(x,q) \leq \lambda(x+y,q) \vee \lambda(y,q)$$

Definition 3.9 *An intuitionistic Q-fuzzy right (left) ideal* $A = \{< x, q, \mu_A(x,q), \lambda_A(x,q) >| x \in R, q \in Q\}$ *in* R *is said to be an intuitionistic Q-fuzzy right (left) k-ideal if*
1) $\mu_A(x,q) \geq \mu_A(x+y,q) \wedge \mu_A(y,q)$
2) $\lambda_A(x,q) \leq \lambda_A(x+y,q) \vee \lambda_A(y,q)$
for all $x, y \in R, q \in Q$.

Theorem 3.10 *If a Q-fuzzy set μ is a Q-fuzzy right k-ideal in a semiring R if and only if $1-\mu$ is an anti Q-fuzzy right (left) k-ideal in a semiring R.*

Proof. Let μ be a Q-fuzzy right k-ideal in a semiring R. Let $x, y \in R, q \in Q$. Then $\mu(x,q) \geq \mu(x+y,q) \wedge \mu(y,q)$ implies $-\mu(x,q) \leq -(\mu(x+y,q) \wedge \mu(y,q))$. Thus $-\mu(x,q) \leq -\mu(x+y,q) \vee -\mu(y,q))$. Therefore $1-\mu(x,q) \leq 1-\mu(x+y,q) \vee 1-\mu(y,q))$. Hence $(1-\mu)(x,q) \leq (1-\mu)(x+y,q) \vee (1\mu)(y,q)$. By Theorem 3.4, $1-\mu$ is an anti Q-fuzzy right k-ideal. By similar argument, we can prove the converse part. ■

Lemma 3.11 *A Q-fuzzy set μ of a semiring R is a Q-fuzzy right k-ideal if and only if $U(\mu;t)$ is a right k-ideal in a semiring R for all $t \in [0,1]$ whenever nonempty.*

Proof. Let μ be a Q-fuzzy right k-ideal of a semiring R. Let $t \in [0,1]$. If there exists $x, y \in R, q \in Q$ such that $x+y, y \in U(\mu;t)$ and $x \notin U(\mu;t)$, then $\mu(x+y,q) \wedge \mu(y,q) \geq t > \mu(x,q)$, is a contradiction. Therefore by Lemma 3.5, $U(\mu;t)$ is a right k-ideal of R.

Conversely, if there exists $x, y \in R, q \in Q$ such that $\mu(x,q) < \mu(x+y,q) \wedge \mu(y,q)$, then $x+y, y \in U(\mu;t)$ and $x \notin U(\mu;t)$, where $t = \mu(x+y,q) \wedge \mu(y,q)$ which is a contradiction. Therefore by Lemma 3.5, μ is a Q-fuzzy right k-ideal of R. ■

Corollary 3.12 *A right (left) ideal I in R is a right (left) k-ideal if and only if χ_I is a Q-fuzzy right (left) k-ideal of a semiring R.*

Lemma 3.13 *A Q-fuzzy set λ of a semiring R is an anti Q-fuzzy right k-ideal if and only if $L(\lambda;t)$ is a right k-ideal in a semiring R for all $t \in [0,1]$ whenever nonempty.*

Proof. Let λ be an anti Q-fuzzy right k-ideal of a semiring R. Let $t \in [0,1]$. If there exists $x, y \in R, q \in Q$ such that $x+y, y \in L(\lambda;t)$ and $x \notin L(\lambda;t)$, then $\lambda(x+y,q) \vee \lambda(y,q) \leq t < \lambda(x,q)$, which is a contradiction. Therefore by Lemma 3.5, $L(\lambda;t)$ is a right k-ideal in a semiring R.

Conversely, if there exists $x, y \in R, q \in Q$ such that $\lambda(x,q) > \lambda(x+y,q) \vee \lambda(y,q)$, then $x+y, y \in L(\lambda;t)$ and $x \notin L(\lambda;t)$, where $t = \lambda(x+y,q) \vee \lambda(y,q)$ which is a contradiction. Therefore by Lemma 3.5, λ is an anti Q-fuzzy right k-ideal of a semiring R. ■

Theorem 3.14 *An intuitionistic Q-fuzzy set $A = \{< x, q, \mu_A(x,q), \lambda_A(x,q) > \mid x \in R, q \in Q\}$ in R is an intuitionistic Q-fuzzy right k-ideal in R if and only if $U(\mu_A;t)$ is a right k-ideal in a semiring R and $L(\lambda_A;t)$ is a right k-ideal in a semiring R for all $t \in R$ whenever nonempty.*

Proof. The proof follows from Lamma 3.5 and Lemma 3.11 ■

Corollary 3.15 *An intuitionistic Q-fuzzy set* $A = \{< x, q, \mu_A(x, q), \lambda_A(x, q) >| x \in R, q \in Q\}$ *in R is an intuitionistic Q-fuzzy right k-ideal in R if and only if* μ_A *is a Q-fuzzy right k-ideal in semiring R and* λ_A *is an anti Q-fuzzy right k-ideal in a semiring R.*

Proof. The proof follows from Theorem 3.14 and Lemmas 3.11 and 3.13. ■

Corollary 3.16 *An intuitionistic Q-fuzzy set* $A = \{< x, q, \mu_A(x, q), (1 - \mu_A)(x, q) >| x \in R, q \in Q\}$ *in R is an intuitionistic Q-fuzzy right k-ideal in R if and only if* μ_A *is a Q-fuzzy right k-ideal in semiring R.*

Proof. The proof follows from Theorem 3.14 and Corollary 3.15. ■

Corollary 3.17 *An intuitionistic Q-fuzzy set* $A = \{< x, q, (1 - \lambda_A)(x, q), \lambda_A(x, q) >| x \in R, q \in Q\}$ *in R is an intuitionistic Q-fuzzy right k-ideal in R if and only if* λ_A *is an anti Q-fuzzy right k-ideal in a semiring R.*

Proof. The proof follows from Theorem 3.14 and Corollary 3.15. ■

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