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Intuitionistic Fuzzy Bi-Ideals of Γ-Semigroups

S. Lekkoksung

Rajamangala University of Technology Isan
Khon Kaen Campus, Thailand
Lekkoksung_somsak@hotmail.com

Abstract

In this paper we investigate some properties of a Q-fuzzy interior ideal of a semigroup. We also consider Q-fuzzy characteristic interior ideals.

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1 Introduction

The notion of a fuzzy set in a set was introduced by L. A. Zadeh[8], and since then this concept has been applied to various algebraic structures. K. T. Atanassov[1] defined the notion of an intuitionistic fuzzy set, as a concept more general than a fuzzy set. Using fuzzy ideals, N. Kuroki [5] discussed characterizations of semigroups. K. H. Kim and Y. B. Jun[3] considered the intuitionistic fuzzification of the notion of several ideals in a semigroup, and investigated some properties of such ideals. M. K. Sen and N. K. Saha [7] defined the concept of a Γ-semigroup, and established a relation between regular Γ-semigroup and Γ-group. In this paper, we introduce the notion of an intuitionistic fuzzy bi-ideal of a Γ-semigroup, and we investigate some properties connected with intuitionistic fuzzy bi-ideals in a Γ-semigroup.

2 Preliminary Notes

We include some elementary aspects of Γ-semigroups that are necessary for this paper.

Definition 2.1 *Let S and Γ be two non-empty sets. Then S is called a Γ-semigroup if it satisfies:*

(i) $x\gamma y \in S$,
(ii) $(x\beta y)\gamma z = x\beta(y\gamma z)$,
for all $x, y, z \in S$ *and* $\beta, \gamma \in \Gamma$.

Let S be a Γ-semigroup. By a Γ-*subsemigroup* of S we mean a non-empty subset A of S such that $A\Gamma A \subseteq A$, and by a *left (right) ideal* of S we mean a non-empty subset A of S such that $T\Gamma A \subseteq A$ ($A\Gamma S \subseteq A$). By *two-sided ideal* or simply *ideal*, we mean a non-empty subset of S which is both a left and a right ideal of S. A Γ-subsemigroup A of a Γ-semigroup S is called a *bi-ideal* of S if $A\Gamma S\Gamma A \subseteq A$. A Γ-semigroup S is called *intra-regular* if for every $a \in S$ there exist $x, y \in S, \alpha, \beta, \gamma \in \Gamma$ such that $a = x\alpha a\gamma a\beta y$.

By a *fuzzy set* μ in a non-emptyset S we mean a function $\mu : S \to [0; 1]$, and the complement of μ, denoted by $\bar{\mu}$, is the fuzzy set in T given by $\bar{\mu}(x) = 1 - \mu(x)$ for all $x \in T$.

Definition 2.2 *A fuzzy set* μ *of* S *is called a fuzzy bi-ideal of* S *if*
(i) $(\forall x, y \in S, \gamma \in \Gamma)(\mu(x\gamma y) \geq \min\{\mu(x), \mu(y)\})$,
(ii) $(\forall x, y, z \in S, \alpha, \beta \in \Gamma)(\mu(x\alpha y\beta z) \geq \min\{\mu(x), \mu(z)\})$.

An intuitionistic fuzzy set (briefly, IFS) A in a non-empty set X is an object having the form

$$A = \{(x; \mu_A(x); \gamma_A(x)) \mid x \in X\}$$

where the functions $\mu_A : X \to [0; 1]$ and $\gamma_A : X \to [0; 1]$ denote the degree of membership and the degree of nonmembership, respectively, and

$$0 \leq \mu_A(x) + \gamma_A(x) \leq 1$$

for all $x \in X$.

An intuitionistic fuzzy set $A = \{(x; \mu_A(x); \gamma_A(x)) \mid x \in X\}$ in X can be identified to an ordered pair $(\mu_A; \gamma_A)$ in $I^X \times I^X$. For the sake of simplicity, we shall use the symbol $A = (\mu_A; \gamma_A)$ for the IFS $A = \{(x; \mu_A(x); \gamma_A(x)) \mid x \in X\}$.

3 Main Results

In what follows, let S denote a Γ-semigroup unless otherwise specified.

Definition 3.1 *An IFS* $A = (\mu_A; \gamma_A)$ *in* T *is called an intuitionistic fuzzy* Γ-*subsemigroup of* S *if*
(i) $\mu_A(x\alpha y) \geq \min\{\mu_A(x), \mu_A(y)\}$;
(ii) $\gamma_A(x\alpha y) \leq \max\{\gamma_A(x), \gamma_A(y)\}$.
for all $x, y \in S, \alpha \in \Gamma$.

Definition 3.2 *An IFS* $A = (\mu_A; \gamma_A)$ *in* S *is called an intuitionistic fuzzy left ideal of* S *if* $\mu_A(x\alpha y) \geq \mu_A(y)$ *and* $\gamma_A(x\alpha y) \leq \gamma_A(y)$ *for all* $x, y \in S, \alpha \in \Gamma$. *An intuitionistic fuzzy right ideal of* S *is defined in an analogous way. An IFS* $A = (\mu_A; \gamma_A)$ *in* S *is called an intuitionistic fuzzy ideal of* S *if it is both an intuitionistic fuzzy right and an intuitionistic fuzzy left ideal of* S.

It is clear that any intuitionistic fuzzy left (right) ideal of S is an intuitionistic fuzzy Γ-subsemigroup of S.

Definition 3.3 *An intuitionistic fuzzy* Γ*-subsemigroup* $A = (\mu_A; \gamma_A)$ *of* S *is called an intuitionistic fuzzy bi-ideal of* S *if*
(i) $\mu_A(x\alpha w\beta y) \geq \min\{\mu_A(x), \mu_A(y)\}$.
(ii) $\gamma_A(x\alpha w\beta y) \leq \max\{\gamma_A(x), \gamma_A(y)\}$.
for all $w, x, y \in S, \alpha, \beta \in \Gamma$.

Theorem 3.4 *Let* $A = (\mu_A; \gamma_A)$ *be an intuitionistic fuzzy ideal of* S. *If* S *is an intra-regular, then* $A(a) = A(a\beta a)$ *for all* $a \in S, \beta \in \Gamma$.

Proof. Let a be any element of S. Then since S is intra-regular, there exist $x, y \in S$ and $\alpha, \beta, \gamma \in \Gamma$ such that $a = x\alpha a\beta a\gamma y$. Hence $A = (\mu_A; \gamma_A)$ is an intuitionistic fuzzy ideal,

$$\begin{aligned} \mu_A(a) &= \mu_A(x\alpha a\beta a\gamma y) \\ &\geq \mu_A(x\alpha a\beta a) \\ &\geq \mu_A(a\beta a) \\ &\geq \mu_A(a) \end{aligned}$$

and

$$\begin{aligned} \gamma_A(a) &= \gamma_A(x\alpha a\beta a\gamma y) \\ &\leq \gamma_A(x\alpha a\beta a) \\ &\leq \gamma_A(a\beta a) \\ &\leq \gamma_A(a) \end{aligned}$$

Hence we have $\mu_A(a) = \mu_A(a\beta a)$ and $\gamma_A(a) = \gamma_A(a\beta a)$. Therefore $A(a) = A(a\beta a)$ for all $a \in S, \beta \in \Gamma$. ■

Theorem 3.5 *Let* $A = (\mu_A; \gamma_A)$ *be an intuitionistic fuzzy ideal of* S. *If* S *is an intra-regular, then* $A(a\beta b) = A(b\beta a)$ *for all* $a, b \in S, \beta \in \Gamma$.

Proof. Let $a, b \in S$ and $\beta \in \Gamma$. Then by Theorem 3.4, we have

$$\begin{aligned} \mu_A(a\beta b) &= \mu_A(a\beta b\beta a\beta b) = \mu_A(a\beta(b\beta a)\beta b) \\ &\geq \mu_A(b\beta a) \\ &= \mu_A(b\beta a\beta b\beta a) = \mu_A(b\beta(a\beta b)\beta a) \geq \mu_A(a\beta b) \end{aligned}$$

and

$$\begin{aligned} \gamma_A(a\beta b) &= \gamma_A(a\beta b\beta a\beta b) = \gamma_A(a\beta(b\beta a)\beta b) \\ &\leq \mu_A(b\beta a) = \gamma_A(b\beta a\beta b\beta a) \\ &= \gamma_A(b\beta(a\beta b)\beta a) \leq \mu_A(a\beta b). \end{aligned}$$

Hence we have $\mu_A(a\beta b) = \mu_A(b\beta a)$ and $\gamma_A(a\beta b) = \gamma_A(b\beta a)$. Therefore $A(a\beta b) = A(b\beta a)$ for all $a, b \in S, \beta \in \Gamma$. ■

Theorem 3.6 *Let $A = (\mu_A; \gamma_A)$ be an intuitionistic fuzzy bi-ideal of S if and only if the fuzzy set μ_A and $\bar{\gamma_A}$ are fuzzy bi-ideals of S.*

Proof. Let $A = (\mu_A; \gamma_A)$ be an intuitionistic fuzzy bi-ideal of S. Then clearly μ_A is a fuzzy bi-ideal of S. Let $x, a, y \in S, \alpha, \beta \in \Gamma$. Then

$$\begin{aligned} \bar{\gamma_A}(x\alpha y) &= 1 - \gamma_A(x\alpha y) \\ &\geq 1 - \max\{\gamma_A(x), \gamma_A(y)\} \\ &= \min\{1 - \gamma_A(x), 1 - \gamma_A(y)\} = \min\{\bar{\gamma_A}(x), \bar{\gamma_A}(y)\} \end{aligned}$$

and

$$\begin{aligned} \bar{\gamma_A}(x\alpha a\beta y) &= 1 - \gamma_A(x\alpha a\beta y) \\ &\geq 1 - \max\{\gamma_A(x), \gamma_A(y)\} \\ &= \min\{1 - \gamma_A(x), 1 - \gamma_A(y)\} = \min\{\bar{\gamma_A}(x), \bar{\gamma_A}(y)\}. \end{aligned}$$

Hence $\bar{\gamma_A}$ is a fuzzy bi-ideal of S.

Conversely, suppose that μ_A and $\bar{\gamma_A}$ are fuzzy bi-ideal of S. Let $a, x, y \in S, \alpha, \beta \in \Gamma$. Then

$$\begin{aligned} 1 - \gamma_A(x\alpha y) &= \bar{\gamma_A}(x\alpha y) \\ &\geq \min\{\bar{\gamma_A}(x), \bar{\gamma_A}(y)\} \\ &= \min\{1 - \gamma_A(x), 1 - \gamma_A(y)\} = \max\{\gamma_A(x), \gamma_A(y)\} \end{aligned}$$

and

$$\begin{aligned} 1 - \gamma_A(x\alpha a\beta y) &= \bar{\gamma_A}(x\alpha a\beta y) \\ &\geq \min\{\bar{\gamma_A}(x), \bar{\gamma_A}(y)\} = 1 - \max\{\gamma_A(x), \gamma_A(y)\}, \end{aligned}$$

which imply that $\gamma_A(x\alpha y) \leq \max\{\gamma_A(x), \gamma_A(y)\}$ and $\gamma_A(x\alpha a\beta y) \leq \max\{\gamma_A(x), \gamma_A(y)\}$. This completes the proof. ■

Let f be a map from a set X to a set Y. If $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ are IFSs in X and Y respectively, then the *preimage* of B under f, denoted by $f^{-1}(B)$, is an IFS in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)).$$

Let S, T be Γ-semigroups and $f : S \to T$ be a function from S into T, f is called a Γ-semigroup homomorphism if the following conditions hold: For $a, x, t \in S, \alpha, \beta \in \Gamma$,
(i) $f(x\alpha y) = f(x)\alpha f(y)$,
(ii) $f(x\alpha a\beta y) = f(x)\alpha f(a)\beta f(y)$.

Theorem 3.7 *Let $f : S \to T$ be a homomorphism of Γ-semigroups. If $B = (\mu_B, \gamma_B)$ is an intuitionistic fuzzy bi-ideal of T, then the preimage $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ of B under f is an intuitionistic fuzzy bi-ideal of S.*

Proof. Assume that $B = (\mu_B, \gamma_B)$ is an intuitionistic fuzzy bi-ideal of T and let $x, y \in S, \alpha \in \Gamma$. Then

$$\begin{aligned} f^{-1}(\mu_B)(x\alpha y) &= \mu_B(f(x\alpha y)) = \mu_B(f(x)\alpha f(y)) \\ &\geq \min\{\mu_B(f(x)), \mu_B(f(y))\} \\ &= \min\{f^{-1}(\mu_B(x)), f^{-1}(\mu_B(y))\}, \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\gamma_B)(x\alpha y) &= \gamma_B(f(x\alpha y)) = \gamma_B(f(x)\alpha f(y)) \\ &\leq \max\{\gamma_B(f(x)), \gamma_B(f(y))\} \\ &= \max\{f^{-1}(\gamma_B(x)), f^{-1}(\gamma_B(y))\}, \end{aligned}$$

Hence $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ is an intuitionistic fuzzy Γ-subsemigroup of S. For any $a, x, y \in S, \alpha, \beta \in \Gamma$ we have

$$\begin{aligned} f^{-1}(\mu_B)(x\alpha a\beta y) &= \mu_B(f(x\alpha a\beta y)) = \mu_B(f(x)\alpha f(a)\beta f(y)) \\ &\geq \min\{\mu_B(f(x)), \mu_B(f(y))\} \\ &= \min\{f^{-1}(\mu_B(x)), f^{-1}(\mu_B(y))\}, \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\gamma_B)(x\alpha a\beta y) &= \gamma_B(f(x\alpha a\beta y)) = \gamma_B(f(x)\alpha f(a)\beta f(y)) \\ &\leq \max\{\gamma_B(f(x)), \gamma_B(f(y))\} \\ &= \max\{f^{-1}(\gamma_B(x)), f^{-1}(\gamma_B(y))\}. \end{aligned}$$

Therefore $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ is an intuitionistic fuzzy bi-ideal of S.

■

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