Int. J. Contemp. Math. Sciences, Vol. 7, 2012, no. 5, 209 - 213

On Slightly b-Continuous Functions

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Abstract

The aim of this paper is to introduce a new set of properties of slightly b-continuous functions. Also the relations of slightly b-continuous functions with other weak forms of b-continuous functions have been investigated.

Mathematics Subject Classification: 54C10

Keywords: Slightly b-continuity, almost b-continuity, weakly b-continuity, faintly b-continuity

1 Introduction

Andrijevic [1] introduced the notion of b-open sets in a topological space and obtained their various properties. El-Etik [2] introduced the same concept in the name of γ-open sets. El-Etik also introduced the concept of γ-continuous (b-continuous) functions with the aid of b-open sets. In 2004, Ekici and Caldas [3] introduced the notion of slightly γ-continuity (slightly b-continuity) which is a weakened form of b-continuity. In their paper, the authors have studied basic properties and preservation theorems of slightly b-continuous functions. The relationships of slightly b-continuity with other weaker forms of continuity have also been studied. In the present paper, a new set of conditions which characterize slightly b-continuous functions have been investigated. Also, the relation of slightly b-continuity with other weaker forms of b-continuity, viz. weakly b-continuity [4], somewhat b-continuity [5], almost b-continuity [6] and faintly b-continuity [7] have been studied.

2 Preliminary Notes

Throughout the present paper, X and Y are always topological spaces. Let A be a subset of X.The interior and closure of a set are denoted by $\text{int}(A)$

and cl(A), respectively. A subset of a topological space X is said to be b-open [1](γ -open [2]) if $A \subset$ int(cl(A))$\cup$ cl(int(A)). The complement of a b-open set is called b-closed [1]. The intersection of all b-closed sets of X containing A is called the b-closure [1] of A and is denoted by $b$cl(A). A subset B of X is said to be a b-neighbourhood [1] of a point $x \in X$ if there exists a b-open set containing x and is contained in A. A subset A of X is said to be δ^* -open [8] if for each $x \in A$ there exists a clopen subset G of X such that $x \in G \subset A$. A subset B of X is said to be δ^*-closed [8] if is $X \setminus B$ is δ^*-open. The intersection of all δ^* -closed sets of X containing A is called the δ^*-closure of A and is denoted by δ^*-cl(A). A subset A of X is said to be θ-open [9] if every point of A has an open neighbourhood whose closure is contained in A. A subset A of X is said to be regular open [10] if A=int(cl(A)). The family of all b-open (resp. b-closed, clopen, b-clopen, δ^*-open,δ^*-closed, regular open) sets in X is denoted by $BO(X)$ (resp. $BC(X)$, $CO(X)$, $BCO(X)$, $\delta^*O(X)$, $\delta^*C(X)$, $RO(X)$).

Definition 2.1 *A function $f : X \to Y$ is said to be almost b-continuous (briefly a.b.c.) [6] if for each $x \in X$ and each $V \in RO(Y)$ containing $f(x)$, there exists $U \in BO(X)$ containing x such that $f(U) \subset V$.*

Definition 2.2 *A function $f : X \to Y$ is said to be weakly b-continuous (briefly w.b.c.) [4] if for each $x \in X$ and each open set V in Y containing $f(x)$, there exists $U \in BO(X)$ containing x such that $f(U) \subset cl(V)$.*

Definition 2.3 *A function $f : X \to Y$ is said to be somewhat b-continuous (briefly sw.b.c.) [5] if for each open set V in Y and $f^{-1}(V) \neq \emptyset$ there exists $U \in BO(X)$ such $U \neq \emptyset$ and $U \subset f^{-1}(V)$.*

Definition 2.4 *A function $f : X \to Y$ is said to be faintly b-continuous (briefly f.b.c.) [7] if for each $x \in X$ and each θ-open set V in Y containing $f(x)$, there exists $U \in BO(X)$ containing x such that $f(U) \subset V$.*

Definition 2.5 *A function $f : X \to Y$ is called slightly γ-continuous [3] if for each $x \in X$ and each $V \in CO(X)$ containing $f(x)$, there exists a $U \in BO(X)$ containing x such that $f(U) \subset V$.*

In the present paper a slightly γ-continuous function will be termed as a slightly b-continuous function (briefly s.b.c.).

Theorem 2.6 *For a function $f : X \to Y$ the following are equivalent[3]:*

(a) f is s.b.c.;

(b) $f^{-1}(V) \in BO(X)$ for every $V \in CO(X)$;

(c) $f^{-1}(V) \in BC(X)$ for every $V \in CO(X)$;

(d) $f^{-1}(V) \in BCO(X)$ for every $V \in CO(X)$.

3 Main Results

The following theorem gives a new set of conditions which characterize slightly b-continuous functions.

Theorem 3.1 *For a function $f : X \to Y$ the following are equivalent:*

(a) f is s.b.c.;

(b) $f^{-1}(V) \in BO(X)$ for every δ^*-open set V in Y;

(c) $f^{-1}(V) \in BC(X)$ for every δ^*-closed set V in Y;

(d) $f(bcl(A)) \subset \delta^*\text{-}cl(f(A))$ for every subset A of X;

(e) $bcl(f^{-1}(B)) \subset f^{-1}(\delta^*\text{-}cl(B))$ for every subset B of Y.

Proof. (a)⇒(b): Let V be a δ^*-open set in Y and let $x \in f^{-1}(V)$. Then $f(x) \in V$. The δ^*-openness of V gives a $U \in CO(Y)$ such that $f(x) \in U \subset V$. This implies that $x \in f^{-1}(U) \subset f^{-1}(V)$. Since f is s.b.c., from Theorem 2.5., we have, $f^{-1}(U) \in BO(X)$. Hence $f^{-1}(V)$ is a b-neighbourhood of each of its points. Consequently, $f^{-1}(V) \in BO(X)$.

(b)⇒(c): It is obvious from the fact that the complement of a δ^*-closed set is δ^*-open.

(c)⇒(d): Let A be a subset of X. We have, $\delta^*\text{-cl}(f(A)) = \cap\{F : f(A) \subset F, F \in \delta^*C(Y)\}$ is a δ^*-closed set in Y. Thus $A \subset f^{-1}(\delta^*\text{-cl}(f(A)) = \cap\{f^{-1}(F) : f(A) \subset F, F \in \delta^*C(Y)\} \in BO(X)$. Thus, we obtain $b\text{cl}(A) \subset f^{-1}(\delta^*\text{-cl}(f(A))$. Hence, $f(b\text{cl}(A)) \subset \delta^*\text{-cl}(f(A))$.

(d)⇒ (e): Let B be a subset of Y. We have $f(b\text{cl}(f^{-1}(B))) \subset \delta^*\text{-cl}(f(f^{-1}(B))) \subset \delta^*\text{-cl } (B)$ and hence, we obtain, $b\text{cl}(f^{-1}(B)) \subset f^{-1}(\delta^*\text{-cl } (B))$.

(e)⇒ (a): Let V be a clopen set in Y. Then V is δ^*-closed in Y. Thus $b\text{cl}(f^{-1}(B)) \subset f^{-1}(\delta^*\text{-cl}(B)) = f^{-1}(B)$. Therefore, $f^{-1}(B)$ is closed. Hence, by Theorem 2.6, we obtain f is s.b.c.

Theorem 3.2 *If a function $f : X \to Y$ is w.b.c. then, f is s.b.c.*

Proof. Let $x \in X$ and let V be a clopen set in Y containing $f(x)$. Therefore, by weakly b-continuity of f, there exists $U \in BO(X)$ containing x such that $f(U) \subset \text{cl}(V) = V$. Since, $x \in X$ is arbitrary, hence, f is s.b.c.

Remark 3.3 *The converse of the above result is, however, far from true as shown by the following example.*

Example 3.4 *Let* $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$, $\sigma = \{\emptyset, Y, \{a\}, \{c\}, \{a, c\}\}$. *Then the identity function* $i : (X, \tau) \to (Y, \sigma)$ *is s.b.c. but not w.b.c. at* $b \in X$.

Remark 3.5 *From definition, it is clear that every a.b.c. is w.b.c. and hence s.b.c. The converse is clearly false as shown by Example 3.4.*

Definition 3.6 *A space* X *is said to be extremally disconnected [10] if closure of every open set is open in* X.

Theorem 3.7 *If a function* $f : X \to Y$ *is f.b.c. then,* f *is s.b.c.*

Proof. The result is obvious from the fact that every clopen set is θ-open.

Remark 3.8 *The converse of the above result is however, in general, not true as shown by the following example.*

Example 3.9 *Let* $\tau = \{G \subset \mathbb{R} : 0 \in G\} \cup \{\emptyset\}$ *and let* σ *be the usual topology on* $\mathbb{R}$. *Then the identity function* $i : (\mathbb{R}, \tau) \to (\mathbb{R}, \sigma)$ *is s.b.c. but not f.b.c. at all points of* $\mathbb{R}$ *except 0.*

Theorem 3.10 *A s.b.c.* $f : X \to Y$ *is f.b.c. if* Y *is extremally disconnected.*

Proof. Let $x \in X$ and let V be a θ-open set in Y containing $f(x)$. Thus there exists an open set W such that $f(x) \in \text{cl}(W) \subset V$. By extremally disconnectedness of Y, $\text{cl}(W)$ is open. Thus, $\text{cl}(W) \in CO(Y)$. Since, f is s.b.c., therefore, there exists a b-open set U containing x such that $f(U) \subset \text{cl}(W) \subset V$. Since, $x \in X$ is arbitrary, therefore, f is f.b.c.

Thus we have

Theorem 3.11 *Let* $f : X \to Y$ *be a function, where,* Y *is extremally disconnected. Then* f *is f.b.c. if and only if* f *is s.b.c.*

Proof. It can be directly obtained by using Theorem 3.7 and Theorem 3.10.

Remark 3.12 *Somewhat* b*-continuity and slightly* b*-continuity are independent of each other as shown by*

Example 3.13 *The function defined in Example 3.9 is s.b.c. but not sw.b.c. Again let* $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$, $\sigma = \{\emptyset, Y, \{a\}, \{b, c\}\}$. *Then the identity function* $i : (X, \tau) \to (Y, \sigma)$ *is sw.b.c. but not s.b.c.*

ACKNOWLEDGEMENTS. The author is thankful to Prof. Paritosh Bhattacharyya (Retd.), Department of Mathematics, Kalyani University, West Bengal, India for his valuable suggestions.

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Received: September, 2011