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On Hypersurface of Special Finsler Spaces Admitting Metric Like Tensor Field

H. Wosoughi

Department of Mathematics
Islamic Azad University, Babol Branch, Iran
hmd_vosogh@yahoo.com

Abstract. In the present work, the hypersurfaces of some special Finsler spaces such as C-reducible, Semi C-reducible, C_2-like, P-reducible, P_2-like, S_3-like, $C^h-$and C^v-Recurrent Finsler spaces, admitting the tensor field $X_{\alpha\delta} = h_{\alpha\delta} + \overline{X}_{00} l_\alpha l_\delta$, which satisfies the condition $C^\alpha_{\beta\gamma} X_{\alpha\delta} = C_{\beta\gamma\delta}$ have been studied.

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1. INTRODUCTION

Consider a non-Riemannian hypersurface F^{n-1} of $F^n (n \geq 2)$ characterized by the equation $x^i = x^i(u^\alpha)$. The fundamental tensor of F^{n-1} is given by

$$g_{\alpha\beta} = g_{ij} B^i_\alpha B^j_\beta, \tag{1.1}$$

where

$$B^i_\alpha = \frac{\partial x^i}{\partial u^\alpha},$$

where u^{α} are Gaussian coordinate on F^{n-1} and Greek indices run from 1 to n-1 . The components h_{ij} of angular metric tensor are given by

$$h_{ij} = g_{ij} - l_i l_j ,$$

where l_i is the normalized element of support. By simple calculation, we can get

$$h_{\alpha\beta} = g_{\alpha\beta} - l_{\alpha} l_{\beta} , \tag{1.2}$$

where

$$g_{\alpha\beta} = g_{ij} B^i_{\alpha} B^j_{\beta} , \qquad h_{\alpha\beta} = h_{ij} B^i_{\alpha} B^j_{\beta} , \qquad l_{\alpha} = l_i B^i_{\alpha} , \qquad l_{\beta} = l_i B^i_{\beta} \ .$$

We use the following notations on Finsler hypersurface [6],

(1.3) (a) $C^{\alpha} l_{\alpha} = 0$, (b) $h^{\alpha}_{\beta} = \delta^{\alpha}_{\beta} - l^{\alpha} l_{\beta}$, (c) $C^{\alpha} h_{\alpha\beta} = C_{\beta}$, (d) $h_{\alpha\beta} h^{\alpha}_{\gamma} = h_{\beta\gamma}$

2. METRIC LIKE FINSLER FIELD IN F^{n-1}

The metric like tensor field is defined by [5]and studied by ([4], [5]) as

(2,1) $$X_{ij} = h_{ij} + X_{00} l_i l_j ,$$

which satisfies the condition $C^h_{ij} X_{hk} = C_{ijk}$.

By a simple calculation with the help of $l_{\alpha} = B^i_{\alpha} l_i$,and the relation (1.1) the relation (2.1) will become

$$X_{\alpha\beta} = h_{\alpha\beta} + \overline{X}_{00} l_{\alpha} l_{\beta} , \tag{2.2}$$

where 0 denote contraction with $l^i = l^{\alpha} B^i_{\alpha}$ and $\overline{X}_{00} = X_{ij} B^i_0 B^j_0$, $B^i_0 = B^i_{\alpha} v^{\alpha}$. Therefore we have

Theorem(2.1). In a hypersurface of F^n , the metric like tensor field is of the form (2.2).

3. THE EXISTENCE OF COVARIENT TENSOR $X_{\alpha\beta}$ IN SPECIAL FINSLER HYPERSURFACES

In this section we study hypersufaces of special Finsler spaces like C-reducible, Semi C-reducible, C_2-like, P-reducible, P_2-like, S_3-like, and S_4-like which are admitting metric like tensor field $X_{\alpha\beta}$.

Definition(3.1). A Finsler space F^n $(n \geq 2)$ is said to be C-reducible, if the (h)hv-torsion tensor C_{ij}^h is given as [1]

$$C_{jk}^i = \frac{1}{n+1}(C^i h_{jk} + C_j h_k^i + C_k h_j^i)\,, \tag{3.1}$$

.

where $C_i = g^{jk} C_{ijk}$.

Contracting (3.1) by $B_{h\beta\gamma}^{\alpha ij}$ and using $C_{\beta\gamma}^{\alpha} = C_{ij}^h B_{h\beta\gamma}^{\alpha ij}$, where $B_{h\beta\gamma}^{\alpha ij} = B_h^{\alpha} B_{\beta}^i B_{\gamma}^j$ we get

$$C_{\beta\gamma}^{\alpha} = \frac{1}{n+1}(C^{\alpha} h_{\beta\gamma} + C_{\beta} h_{\gamma}^{\alpha} + C_{\gamma} h_{\beta}^{\alpha})\,, \tag{3.2}$$

where the matrix of projection factor $B_{\alpha}^i = \dfrac{\partial x^i}{\partial u^{\alpha}}$ is of the rank n-1.

Now contracting (3.2) by $X_{\alpha\delta}$ and using (1.3) in between ,we have

$$C_{\beta\gamma}^{\alpha} X_{\alpha\delta} = \frac{1}{n+1}[X_{\alpha\delta}(C^{\alpha} h_{\beta\gamma} + C_{\beta} h_{\gamma}^{\alpha} + C_{\gamma} h_{\beta}^{\alpha})]$$

$$= \frac{1}{n+1}[C^{\alpha} h_{\beta\gamma}(h_{\alpha\delta} + \overline{X}_{00} l_{\alpha} l_{\delta}) + C_{\beta} h_{\gamma}^{\alpha}(h_{\alpha\delta} + \overline{X}_{00} l_{\alpha} l_{\delta}) + C_{\gamma} h_{\beta}^{\alpha}(h_{\alpha\delta} + \overline{X}_{00} l_{\alpha} l_{\delta})]$$

$$= \frac{1}{n+1}(C_{\delta} h_{\beta\gamma} + C_{\beta} h_{\gamma\delta} + C_{\gamma} h_{\beta\delta} + C_{\beta} h_{\gamma}^{\alpha} \overline{X}_{00} l_{\alpha} l_{\delta} + C_{\gamma} h_{\beta}^{\alpha} \overline{X}_{00} l_{\alpha} l_{\delta})$$

$$=\frac{1}{n+1}[C_\delta h_{\beta\gamma}+C_\beta h_{\gamma\delta}+C_\gamma h_{\beta\delta}+C_\beta \overline{X}_{00} l_\alpha l_\delta(\delta^\alpha_\gamma - l^\alpha l_\gamma)+C_\gamma \overline{X}_{00} l_\alpha l_\delta(\delta^\alpha_\beta - l^\alpha l_\beta)]$$

$$=\frac{1}{n+1}[C_\delta h_{\beta\gamma}+C_\beta h_{\gamma\delta}+C_\gamma h_{\beta\delta}+C_\beta \overline{X}_{00} l_\delta(l_\gamma - l_\gamma)+C_\gamma \overline{X}_{00} l_\delta(l_\beta - l_\beta)]$$

$$=\frac{1}{n+1}(C_\delta h_{\beta\gamma}+C_\beta h_{\gamma\delta}+C_\gamma h_{\beta\delta}),$$

therefore ,

(3.3) $$C^\alpha_{\beta\gamma} X_{\alpha\delta} = C_{\beta\gamma\delta}.$$

Hence we have the following ,

Theorem (3.1). In a hypersurface of C-reducible Finsler space, the covariant tensor field $X_{\alpha\delta}$ satisfies (2.2) is of the form (3.3).

Definition (3.2). A Finsler space F^n $(n \succ 2)$ with non-zero length C of the torsion vector C_i is to be semi C-reducible, if the torsion tensor C_{ijk} is of the form [3]

(3.4) $$C_{hjk} = p(h_{hj}C_k + h_{jk}C_h + h_{kh}C_j)/(n+1) + qC_hC_jC_k/C^2.$$

Contraction of above relation by g^{hi} ,we get

(3.5) $$C^i_{jk} = p(C_k h^i_j + C^i h_{jk} + C_j h^i_k)/(n+1) + qC^iC_jC_k/C^2,$$

where $C^2 = g^{ij}C_iC_j = C_iC^j$ and p+q=1.

Contracting (3.5) by $B^{\alpha jk}_{i\beta\gamma}$ we get,

(3.6) $$C^\alpha_{\beta\gamma} = p(C_\gamma h^\alpha_\beta + C^\alpha h_{\beta\gamma} + C_\beta h^\alpha_\gamma)/(n+1) + qC^\alpha C_\beta C_\gamma/\overline{C}^2,$$

where $\overline{C}^2 = C_\alpha C^\alpha$, $C_\alpha = C_i B^i_\alpha$, $C^\alpha = C^i B^\alpha_i$. Therefore we get

Theorem (3.2). A hypersurface F^{n-1} of semi C-reducible Finsler space F^n is semi C-reducible.

Contracting (3.6) with $X_{\alpha\delta}$ and using (1.3), we have

$$C^{\alpha}_{\beta\gamma} X_{\alpha\delta} = \frac{p}{n+1}(C_\beta h^{\alpha}_{\gamma} + C^{\alpha} h_{\beta\gamma} + C_\gamma h^{\alpha}_{\beta}) X_{\alpha\delta} + qC^{\alpha} C_\beta C_\gamma X_{\alpha\delta} / \overline{C}^2$$

$$= \frac{p}{n+1}[C_\beta h^{\alpha}_{\gamma}(h_{\alpha\delta} + \overline{X}_{00} l_\alpha l_\delta) + C^{\alpha} h_{\beta\gamma}(h_{\alpha\delta} + \overline{X}_{00} l_\alpha l_\delta) + C_\gamma h^{\alpha}_{\beta}(h_{\alpha\delta} + \overline{X}_{00} l_\alpha l_\delta)$$

$$+ C_\gamma h^{\alpha}_{\beta}(h_{\alpha\delta} + \overline{X}_{00} l_\alpha l_\delta)] + qC^{\alpha} C_\beta C_\gamma (h_{\alpha\delta} + \overline{X}_{00} l_\alpha l_\delta) / \overline{C}^2$$

$$= \frac{p}{n+1}(C_\beta h_{\gamma\delta} + C_\delta h_{\beta\gamma} + C_\gamma h_{\beta\delta} + C_\beta h^{\alpha}_{\gamma} \overline{X}_{00} l_\alpha l_\delta + C_\gamma h^{\alpha}_{\beta} \overline{X}_{00} l_\alpha l_\delta)$$

$$+ qC^{\alpha} C_\beta C_\gamma h_{\alpha\delta} / \overline{C}^2$$

$$= \frac{p}{n+1}[C_\beta h_{\gamma\delta} + C_\delta h_{\beta\gamma} + C_\gamma h_{\beta\delta} + C_\beta \overline{X}_{00} l_\alpha l_\delta (\delta^{\alpha}_{\gamma} - l^{\alpha} l_\gamma)$$

$$+ C_\gamma \overline{X}_{00} l_\alpha l_\delta (\delta^{\alpha}_{\beta} - l^{\alpha} l_\beta)] + qC^{\alpha} C_\beta C_\gamma h_{\alpha\delta} / \overline{C}^2$$

$$= \frac{p}{n+1}[C_\beta h_{\gamma\delta} + C_\delta h_{\beta\gamma} + C_\gamma h_{\beta\delta} + C_\beta \overline{X}_{00} l_\delta (l_\gamma - l_\gamma) + C_\gamma \overline{X}_{00} l_\delta (l_\beta - l_\beta)]$$

$$+ qC^{\alpha} C_\beta C_\gamma h_{\alpha\delta} / \overline{C}^2$$

$$= \frac{p}{n+1}(C_\beta h_{\gamma\delta} + C_\delta h_{\beta\gamma} + C_\gamma h_{\beta\delta}) + qC_\beta C_\gamma C_\delta / \overline{C}^2,$$

therefore

$$(3.7) \qquad C^{\alpha}_{\beta\gamma} X_{\alpha\delta} = C_{\beta\gamma\delta},$$

Therefore we have

Theorem (3.3). In a hypersurface of semi C-reducible Finsler space ,the covariant tensor field $X_{\alpha\delta}$ satisfies (2.2) is of the form (3.7).

Again the hypersurface F^{n-1} of a C_2-like Finsler space is given by [7] as

$$(3.8) \qquad C^{\alpha}_{\beta\gamma} = \frac{1}{\overline{\overline{C}}^2} C^{\alpha} C_\beta C_\gamma,$$

where $\overline{\overline{C}}^2$ stands for $C^{\alpha} C_\alpha$ and is non-zero.

Contracting (3.8) by $X_{\alpha\delta}$, we get

$$C^{\alpha}_{\beta\gamma} X_{\alpha\delta} = X_{\alpha\delta}(C^{\alpha} C_\beta C_\gamma) \frac{1}{\overline{\overline{C}}^2}$$

$$=\frac{1}{\overline{\overline{C}}^{2}}[C^{\alpha}C_{\beta}C_{\gamma}(h_{\alpha\delta}+\overline{X}_{00}l_{\alpha}l_{\delta})],$$

Which according to (3.1)a ,

$$=\frac{1}{\overline{\overline{C}}^{2}}C_{\beta}C_{\gamma}C_{\delta},$$

Therefore

$$C^{\alpha}_{\beta\gamma}X_{\alpha\delta}=C_{\beta\gamma\delta}, \tag{3.9}$$

Therefore we have

Theorem (3.4). In a hypersurface of a C_2-like Finsler space ,the covariant tensor field $X_{\alpha\delta}$ which satisfies (2.2) is of the form (3.9).

Definition (3.3).A Finsler space is called P-reducible, if the torsion tensor p_{ijk} is written as

$$P_{hjk}=(h_{hj}P_{k}+h_{jk}P_{h}+h_{kh}P_{j})/(n+1), \tag{3.10}$$

where $P_i=P^l_{il}=C_{i|0}$.

Contraction of (3.10) with g^{hi}, we get

$$P^{i}_{jk}=(h_{jk}P^{i}+h^{i}_{j}P_{k}+h^{i}_{h}P_{j})/(n+1). \tag{3.11}$$

Contracting (3.11) with $B^{\alpha jk}_{i\beta\gamma}$, we get

$$P^{\alpha}_{\beta\gamma}=(h_{\beta\gamma}P^{\alpha}+h^{\alpha}_{\beta}P_{\gamma}+h^{\alpha}_{\gamma}P_{\beta})/(n+1) \tag{3.12}$$

Therefore we have

Theorem (3.5). A hypersurface F^{n-1} of P-reducible Finsler space F^n is P-reducible .

Contracting (3.12) by $X_{\alpha\delta}$,we get

$$P^{\alpha}_{\beta\gamma}X_{\alpha\delta}=\frac{1}{n+1}(h_{\beta\gamma}P^{\alpha}+h^{\alpha}_{\beta}P_{\gamma}+h^{\alpha}_{\gamma}P_{\beta})X_{\alpha\delta}$$

$$=\frac{1}{n+1}[h_{\beta\gamma}P^{\alpha}(h_{\alpha\delta}+\overline{X}_{00}l_{\alpha}l_{\delta})+h^{\alpha}_{\beta}P_{\gamma}(h_{\alpha\delta}+\overline{X}_{00}l_{\alpha}l_{\delta})+$$

$$h^{\alpha}_{\gamma}P_{\beta}(h_{\alpha\delta}+\overline{X}_{00}l_{\alpha}l_{\delta})]$$

$$=\frac{1}{n+1}(h_{\beta\gamma}P_{\delta}+h_{\beta\delta}P_{\gamma}+h_{\gamma\delta}P_{\beta}),$$

where we have made use of (1.3) .Therefore

$$P^{\alpha}_{\beta\gamma}X_{\alpha\delta}=P_{\beta\gamma\delta}. \tag{3.13}$$

Hence we get

Theorem (3.6). In a hypersuface of P-reducible Finsler space, the covariant tensor field $X_{\alpha\delta}$ which satisfies (2.2) is of the form (3.13).

Definition (3.4). A Finsler space F^n $(n \succ 2)$ is P_2 -like, if it is characterized by

(3.14) $$P_{hjkl} = K_h C_{jkl} - K_j C_{hkl},$$

Where $K_h = K_h(x, y)$ is a covariant vector field.

Contraction of the above relation with g^{il} ,it will be characterized as follows

(3.15) $$P^i_{hjk} = K_h C^i_{jk} - K_j C^i_{hk}$$

Contracting (3.15) by $B^{\alpha hjk}_{i\lambda\beta\gamma}$, we get

(3.16) $$P^\alpha_{\lambda\beta\gamma} = K_\lambda C^\alpha_{\beta\gamma} - K_\beta C^\alpha_{\lambda\gamma}$$

Therefore we have

Theorem (3.7). A hypersurface F^{n-1} of P_2 -like, is P_2 -like.

Contracting (3.16) by $X_{\alpha\delta}$, we get

$$\begin{aligned} P^\alpha_{\lambda\beta\gamma} X_{\alpha\delta} &= (K_\lambda C^\alpha_{\beta\gamma} - K_\beta C^\alpha_{\lambda\gamma}) X_{\alpha\delta} \\ &= K_\lambda C^\alpha_{\beta\gamma}(h_{\alpha\delta} + \overline{X}_{00} l_\alpha l_\delta) - K_\beta C^\alpha_{\lambda\gamma}(h_{\alpha\delta} + \overline{X}_{00} l_\alpha l_\delta) \\ &= K_\lambda C_{\beta\gamma\delta} - K_\beta C_{\lambda\gamma\delta} + K_\lambda C^\alpha_{\beta\gamma} h_{\alpha\delta} \overline{X}_{00} l_\alpha l_\delta - K_\beta C^\alpha_{\lambda\gamma} h_{\alpha\delta} \overline{X}_{00} l_\alpha l_\delta \\ &= K_\lambda C_{\beta\gamma\delta} - K_\beta C_{\lambda\gamma\delta}, \end{aligned}$$

Therefore,

(3.17) $$P^\alpha_{\lambda\beta\gamma} X_{\alpha\delta} = P_{\lambda\beta\gamma\delta}$$

Where we have used relation (1.3). Therefore we have

Theorem(3.8). In a hypersrface of P_2 -like , the covariant tensor field $X_{\alpha\delta}$ which satisfies (2.2) is of the form (3.17).

Definition (3.5). A Finsler space F^n $(n \succ 3)$ is called S_3 -like, if the curvature tensor S_{hijk} is written in the form [1]

(3.18) $$L^2 S_{hijk} = S(h_{hk} h_{jl} - h_{hl} h_{jk}),$$

where the scalar curvature $S(= S_{hijk} g^{hj} g^{ik})$ is a function of position alone.

Contraction of (3.18) with g^{hi}, we get S_3 -like in the form

(3.19) $$L^2 S^i_{jkl} = S(h_{jl} h^i_k - h_{jk} h^i_l).$$

Again contracting (3.19) by $B^{\alpha jkl}_{\lambda\beta\gamma}$, we get

(3.20) $$L^2 S^\alpha_{\lambda\beta\gamma} = S(h_{\lambda\gamma} h^\alpha_\beta - h_{\lambda\beta} h^\alpha_\gamma).$$

Therefore we have

Theorem (3.9). A hypersuface of a S_3 -like is a S_3 -like.

Contracting (3.20) by $X_{\alpha\delta}$, we get

$$\begin{aligned} L^2 S^{\alpha}_{\lambda\beta\gamma} X_{\alpha\delta} &= L^2 S^{\alpha}_{\lambda\beta\gamma}(h_{\alpha\delta} + \overline{X}_{00} l_\alpha l_\delta) \\ &= L^2 S[h_{\lambda\gamma} h^{\alpha}_{\beta}(h_{\alpha\delta} + \overline{X}_{00} l_\alpha l_\delta) - h_{\lambda\beta} h^{\alpha}_{\gamma}(h_{\alpha\delta} + \overline{X}_{00} l_\alpha l_\delta)] \\ &= L^2 S(h_{\lambda\gamma} h_{\beta\delta} - h_{\lambda\beta} h_{\gamma\delta} + h_{\lambda\gamma} h^{\alpha}_{\beta} \overline{X}_{00} l_\alpha l_\delta - h_{\lambda\beta} h^{\alpha}_{\gamma} \overline{X}_{00} l_\alpha l_\delta) \\ &= L^2 S(h_{\lambda\gamma} h_{\beta\delta} - h_{\lambda\beta} h_{\gamma\delta} + h_{\lambda\gamma} \overline{X}_{00}(\delta^{\alpha}_{\beta} - l^{\alpha} l_\beta) l_\alpha l_\delta \\ &- h_{\lambda\beta} \overline{X}_{00}(\delta^{\alpha}_{\gamma} - l^{\alpha} l_\gamma) l_\alpha l_\delta, \\ &= L^2 S(h_{\lambda\gamma} h_{\beta\delta} - h_{\lambda\beta} h_{\gamma\delta} + h_{\lambda\gamma} \overline{X}_{00}(l_\beta - l_\beta) l_\delta - h_{\lambda\beta} \overline{X}_{00}(l_\gamma - l_\gamma) l_\delta \\ &= L^2 S(h_{\lambda\gamma} h_{\beta\delta} - h_{\lambda\beta} h_{\gamma\delta}), \end{aligned}$$

therefore, we get

(3.21) $$L^2 S^{\alpha}_{\lambda\beta\gamma} \overline{X}_{00} = S_{\lambda\alpha\beta\delta} \; .$$

Therefore we have

Theorem(3.10). In a hypersurface of S_3-like Finsler space, the covariant tensor field $X_{\alpha\delta}$ satisfies (2.2) is of the form (3.21).

Definition (3.6). A Finsler space F^n is called C^h-recurrent, if torsion tensor C^i_{jk} satisfies the equation [2]

(3.22) $$C^i_{jk|l} = k_l C^i_{jk} ,$$

Where k_l is some vector field, and '|' stands for h-covariant differentiation.

Contracting (3.22) by $B^{\alpha jkl}_{i\lambda\beta\gamma}$, we get

(3.23) $$C^{\alpha}_{\lambda\beta|\gamma} = k_\gamma C^{\alpha}_{\lambda\beta} ,$$

Therefore, we get

Theorem (3.11). A hypersurface of a C^h-recurrent Finsler space is a C^h-recurrent.

Again contraction (3.23) with $X_{\alpha\delta}$, we get

$$\begin{aligned} C^{\alpha}_{\lambda\beta|\gamma} X_{\alpha\delta} &= k_\gamma C^{\alpha}_{\lambda\beta} X_{\alpha\delta} \\ &= k_\gamma C^{\alpha}_{\lambda\beta}(h_{\alpha\delta} + \overline{X}_{00} l_\alpha l_\delta) \\ &= k_\gamma C_{\lambda\beta\delta} , \end{aligned}$$

Hence

(3.24) $$C^{\alpha}_{\lambda\beta|\gamma} X_{\alpha\delta} = k_\gamma C_{\lambda\beta\delta} ,$$

Therefore, we have

Theorem (3.12). In a hypersurface of C^h-recurrent Finsler space, the covariant tensor field $X_{\alpha\delta}$ satisfies (2.2) is of the form (3.24).

Definition (3.7). A Finsler space F^n is called C^v-recurrent, if the torsion tensor C^i_{jk} satisfies the relation [2]

(3.25) $$C^i_{jk|l} = a_l C^i_{jk} ,$$

Where a_l is some covariant vector field and $|$ is $v-$covariant differentiation.

Contraction of (3.25) with $B^{\alpha jkl}_{i\lambda\beta\gamma}$, we obtain

(3.26) $$C^\alpha_{\lambda\beta|\gamma} = a_\gamma C^\alpha_{\lambda\beta} ,$$

Therefore, we have

Theorem (3.13). A hypersurface of C^v-recurrent Finsler space is a C^v-recurrent.

Again contracting (3.26) by $X_{\alpha\delta}$,we get

$$\begin{aligned} C^\alpha_{\lambda\beta|\gamma} X_{\alpha\delta} &= a_\gamma C^\alpha_{\lambda\beta} X_{\alpha\delta} \\ &= a_\gamma C^\alpha_{\lambda\beta} (h_{\alpha\delta} + \overline{X}_{00} l_\alpha l_\delta) \\ &= a_\gamma (C_{\lambda\beta\delta} + C^\alpha_{\lambda\beta} \overline{X}_{00} l_\alpha l_\delta) \\ &= a_\gamma C_{\lambda\beta\delta} , \end{aligned}$$

Therefore, we obtain

(3.27) $$C^\alpha_{\lambda\beta|\gamma} X_{\alpha\delta} = a_\gamma C_{\lambda\beta\delta}$$

Therefore,we have

Theorem (3.14). In a hypersurface of C^v-recurrent Finsler space,the covariant tensor field $X_{\alpha\delta}$ satisfies (2.2) is of the form (3.27).

A T-tensor is introduced simultaneously but independently by Kawaguchi and Matsumoto in 1972. In fact the $v-$covariant differentiation of C_{ijk} defines the T-tensor T_{hijk} in the form

$$T_{lhjk} = LC_{lhj|k} + l_l C_{hjk} + l_h C_{ljk} + l_j C_{lhk} + l_k C_{lhj} ,$$

Contraction of the above relation with g^{il}, we obtain T-tensor as follows,

(3.28) $$T_{lhjk} g^{il} = T^i_{hjk} = LC^i_{hj|k} + l^i C_{hjk} + l_h C^i_{jk} + l_j C^i_{hk} + l_k C^i_{hj}$$

Contracting (3.28) by $B^{\alpha hjk}_{i\lambda\beta\gamma}$, we get

(3.29) $$T^\alpha_{\lambda\beta\gamma} = LC^\alpha_{\lambda\beta|\gamma} + l^\alpha C_{\lambda\beta\gamma} + l_\lambda C^\alpha_{\beta\gamma} + l_\beta C^\alpha_{\lambda\gamma} + l_\gamma C^\alpha_{\lambda\beta} ,$$

Hence, we have

Theorem (3.15). A hypersurface of Finsler space with T-tensor is with T-tensor.

Contracting the above equation (3.29) by $X_{\alpha\delta}$,we obtain

$$T^{\alpha}_{\lambda\beta\gamma}X_{\alpha\delta}=(LC^{\alpha}_{\lambda\beta|\gamma}+l^{\alpha}C_{\lambda\beta\gamma}+l_{\lambda}C^{\alpha}_{\beta\gamma}+l_{\beta}C^{\alpha}_{\lambda\gamma}+l_{\gamma}C^{\alpha}_{\lambda\beta})(h_{\alpha\delta}+\overline{X}_{00}l_{\alpha}l_{\delta})$$

$$=LC_{\delta\lambda\beta|\gamma}+l_{\delta}C_{\lambda\beta\gamma}+l_{\lambda}C_{\delta\beta\gamma}+l_{\beta}C_{\delta\lambda\gamma}+l_{\gamma}C_{\delta\lambda\beta}+LC^{\alpha}_{\lambda\beta|\gamma}\overline{X}_{00}l_{\alpha}l_{\delta}$$

$$+l^{\alpha}C_{\lambda\beta\gamma}\overline{X}_{00}l_{\alpha}l_{\delta}+l_{\lambda}C^{\alpha}_{\beta\gamma}\overline{X}_{00}l_{\alpha}l_{\delta}+l_{\beta}C^{\alpha}_{\lambda\gamma}\overline{X}_{00}l_{\alpha}l_{\delta}+l_{\gamma}C^{\alpha}_{\lambda\beta}\overline{X}_{00}l_{\alpha}l_{\delta}$$

$$=T_{\delta\lambda\beta\gamma}+C_{\lambda\beta\gamma}\overline{X}_{00}l_{\delta}$$

Therefore, if $C_{\lambda\beta\gamma}\overline{X}_{00}l_{\delta}=0$, we get

$$T^{\alpha}_{\lambda\beta\gamma}X_{\alpha\delta}=T_{\delta\lambda\beta\gamma} \tag{3.30}$$

Hence, we have the following

Theorem (3.16). In a hypersurface of Finsler space with T-tensor , the covariant tensor field $X_{\alpha\delta}$ satisfies (2.2) is of the form (3.30) provided that $C_{\lambda\beta\gamma}\overline{X}_{00}l_{\delta}=0$.

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