

Certain Generating Functions for Partial Mock Theta Functions of Order Two and Order Ten

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Abstract

The present paper is focused on the development of generating function for partial mock theta functions of order two and order ten.

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1 Introduction

The Mock theta functions appeared in Ramanujan's last letter to G.H. Hardy in January 1920 and he quoted "I discovered very interesting functions recently which I call 'Mock' θ -functions. Unlike the 'false' θ -functions they enter into mathematics as beautifully as the ordinary θ -functions". The first detailed description of these functions was given by G.N. Watson [5] in 1935, in his presidential address to the London mathematical society. Many facet of these Mock theta functions are being investigated by the mathematicians working in the field of basic hypergeometric functions, specially works of R.P. Agarwal [13, 14], G.E. Andrews [6], R.Y. Denis and S.N. Singh [15], Remy Y. Denis, S.N. Singh and S. Ahmad Ali [16], Remy Y. Denis, S.N. Singh and S.P. Singh

[17, 18], A.K. Srivastava [1], B.Srivastava [2, 3], M. Pathak and P. Srivastava [9], Pankaj Srivastava and Anwar Jahan Wahidi [10], and etc have shown that the theory of Mock theta function can be best studied through the basic hypergeometric functions.

Generating functions were first introduced by 'Abraham de moivre' [4] in 1730. According to Herbert Wilf [7] "A generating function is a clothesline on which we hang up a sequence of numbers for display". Generating functions play an important role in the investigation of various useful properties and can also be used with good effect for the determination of asymptotic behavior.

Some q - generating functions connected with basic multiple hypergeometric series have been given by Themistocles M.Rassias, S.N.Singh and H.M.Srivastava [20]. In view of the importance and usefulness of the generating functions, we have extended the idea of generating functions for the Mock theta functions. The generating functions for Mock theta functions will provide a new platform for further investigation.

Recently, B. Srivastava [2, 3] has defined generalized functions for mock theta functions of order two and order ten and authors [11] have also introduced generalized function for mock theta function of order two given by Hikami.

In the present article, we have established certain generating functions for partial mock theta functions of order two and order ten by making use of identity due to Srivastava [1].

2 Definitions and Notations

We shall use the following q -symbols:

For $|q| < 1$ and $|q^r| < 1$,

$$(a; q)_n = \prod_{s=0}^{n-1} (1 - aq^s), \quad n \geq 1.$$

$$(a; q^r)_n = \prod_{s=0}^{n-1} (1 - aq^{rs}), \quad n \geq 1.$$

$$(a; q)_0 = 1, \quad (a; q^r)_0 = 1.$$

$$(a; q^r)_\infty = \prod_{s=0}^{\infty} (1 - aq^{rs}).$$

A generalized basic hypergeometric function with base q is defined as:

$${}_r\phi_s \left[\begin{matrix} a_1, a_2, \dots, a_r; q, z \\ b_1, b_2, \dots, b_s; q^i \end{matrix} \right] = \sum_{n=0}^{\infty} \frac{(a_1; q)_n \dots (a_r; q)_n}{(b_1; q)_n \dots (b_s; q)_n (q; q)_n} q^{in(n-1)/2} z^n, \quad (1)$$

and the series on the right hand side of (1) converges for $|q| < 1$, $|z| < \infty$ and $|q| < 1, |z| < 1$, when $i = 0$.

Also,

$${}_r\phi_s \left[\begin{matrix} a_1, a_2, \dots, a_r; q, z \\ b_1, b_2, \dots, b_s; q^i \end{matrix} \right]_m = \sum_{n=0}^m \frac{(a_1; q)_n \dots (a_r; q)_n}{(b_1; q)_n \dots (b_s; q)_n (q; q)_n} q^{in(n-1)/2} z^n,$$

denotes partial sum of the generalized basic hypergeometric series.

Definitions and notations of mock theta functions that shall be used in our analysis are as:

Mock theta function of order two:

McIntosh [12] defined following mock theta functions of order two:

$$\begin{aligned} A(q) &= \sum_{n=0}^{\infty} \frac{q^{(n+1)^2} (-q; q^2)_n}{(q; q^2)_{n+1}^2} = \sum_{n=0}^{\infty} \frac{q^{(n+1)} (-q^2; q^2)_n}{(q; q^2)_{n+1}}, \\ B(q) &= \sum_{n=0}^{\infty} \frac{q^{n(n+1)} (-q^2; q^2)_n}{(q; q^2)_{n+1}^2} = \sum_{n=0}^{\infty} \frac{q^n (-q; q^2)_n}{(q; q^2)_{n+1}}, \\ \mu(q) &= \sum_{n=0}^{\infty} \frac{(-1)^n q^{n^2} (q; q^2)_n}{(-q^2; q^2)_n^2}, \end{aligned}$$

and Hikami [8] gave the following Mock theta function of order two:

$$D_5(q) = \sum_{n=0}^{\infty} \frac{q^n (-q; q)_n}{(q; q^2)_{n+1}}.$$

Mock theta functions of order ten:

Mock theta functions of order ten defined by Ramanujan [19] as:

$$\begin{aligned} \phi(q) &= \sum_{n=0}^{\infty} \frac{q^{n(n+1)/2}}{(q; q^2)_{n+1}}, \\ \psi(q) &= \sum_{n=0}^{\infty} \frac{q^{(n+1)(n+2)/2}}{(q; q^2)_{n+1}}, \\ X(q) &= \sum_{n=0}^{\infty} \frac{(-1)^n q^{n^2}}{(-q; q)_{2n}}, \\ \chi(q) &= \sum_{n=0}^{\infty} \frac{(-1)^n q^{(n+1)^2}}{(-q; q)_{2n+1}}. \end{aligned}$$

If $F(q) = \sum_{n=0}^{\infty} f(q; n)$ is a mock theta function, then the corresponding partial mock theta function is denoted by the truncated series $F_p(q) = \sum_{n=0}^p f(q; n)$.

3 Known Results

In order to establish our main results, we shall use the following known results: Srivastava [1] has established the following identity:

$$\sum_{r=0}^n \alpha_r \sum_{m=0}^n \delta_m = \sum_{m=0}^n \delta_m \sum_{r=0}^m \alpha_r + \sum_{r=0}^{n-1} \alpha_{r+1} \sum_{m=0}^r \delta_m. \tag{2}$$

B. Srivastava [2] defined the following generalized functions for mock theta functions of order two:

$$A(k, \alpha) = \frac{1}{(k; q)_\infty} \sum_{n=0}^{\infty} \frac{(k; q)_n (-q; q^2)_n q^{n^2+n(\alpha+1)+1}}{(q; q^2)_{n+1}^2}. \quad (3)$$

$$B(k, \alpha) = \frac{1}{(k; q)_\infty} \sum_{n=0}^{\infty} \frac{(k; q)_n (-q^2; q^2)_n q^{n^2+n\alpha}}{(q; q^2)_{n+1}^2}. \quad (4)$$

$$\mu(k, \alpha) = \frac{1}{(k; q)_\infty} \sum_{n=0}^{\infty} \frac{(k; q)_n (q; q^2)_n (-1)^n q^{n^2-n+n\alpha}}{(-q^2; q^2)_n^2}. \quad (5)$$

Recently, authors [11] have introduced generalized function for mock theta function of order two given by Hikami and developed its properties. The generalized function is as:

$$D_5(k, \alpha) = \frac{1}{(k; q)_\infty} \sum_{n=0}^{\infty} \frac{(k; q)_n (-q; q)_n q^{n\alpha}}{(q; q^2)_{n+1}}. \quad (6)$$

For $k = 0$ and $\alpha = 1$, these functions reduce to mock theta functions of order two viz. $A(q)$, $B(q)$, $\mu(q)$ and $D_5(q)$ respectively.

We have used the following generalized functions for mock theta functions of order ten given by B. Srivastava [3] as:

$$\phi(k, \alpha) = \frac{1}{(k; q)_\infty} \sum_{n=0}^{\infty} \frac{(k; q)_n q^{n(n-1)/2+n\alpha}}{(q; q^2)_{n+1}}. \quad (7)$$

$$\psi(k, \alpha) = \frac{1}{(k; q)_\infty} \sum_{n=0}^{\infty} \frac{(k; q)_n q^{n(n+1)/2+n\alpha}}{(q; q^2)_{n+1}}. \quad (8)$$

$$X(k, \alpha) = \frac{1}{(k; q)_\infty} \sum_{n=0}^{\infty} \frac{(k; q)_n (-1)^n q^{n^2-n+n\alpha}}{(-q; q)_{2n}}. \quad (9)$$

$$\chi(k, \alpha) = \frac{1}{(k; q)_\infty} \sum_{n=0}^{\infty} \frac{(k; q)_n (-1)^n q^{n^2+n+n\alpha}}{(-q; q)_{2n+1}}. \quad (10)$$

For $k = 0$ and $\alpha = 1$, these functions reduce to mock theta functions of order ten viz. $\phi(q)$, $\psi(q)$, $X(q)$ and $\chi(q)$ respectively.

Taking the partial sum of the above functions from 0 to r , we get generalized functions for partial mock theta functions of order two $A_r(k, \alpha)$, $B_r(k, \alpha)$, $\mu_r(k, \alpha)$, $D_{5r}(k, \alpha)$ and order ten $\phi_r(k, \alpha)$, $\psi_r(k, \alpha)$, $X_r(k, \alpha)$, $\chi_r(k, \alpha)$ respectively.

4 Main Results

In this section, we shall establish the generating functions for the partial mock theta functions of order two and order ten respectively.

The identity (2) has been used to develop our main results and for $\alpha_r = z^r$, identity (2) yields

$$z^n \sum_{m=0}^n \delta_m + (1 - z) \sum_{r=0}^{n-1} z^r \sum_{m=0}^r \delta_m = \sum_{m=0}^n \delta_m z^m, \quad |z| < 1. \tag{11}$$

4.1 Generating function for partial mock theta functions of order two:

(a) If we take $\delta_m = \frac{1}{(k;q)_\infty} \frac{(k;q)_m (-q;q^2)_m q^{m^2+m(\alpha+1)+1}}{(q;q^2)_{m+1}^2}$ in (11) and after simplification, we get

$$z^n A_n(k, \alpha) + (1 - z) \sum_{r=0}^{n-1} A_r(k, \alpha) z^r = \frac{q}{(1 - q)^2 (k;q)_\infty} {}_4\phi_4 \left[\begin{matrix} k, i\sqrt{q}, -i\sqrt{q}, q; q, q, q^{(\alpha+2)}z \\ q^{3/2}, -q^{3/2}, q^{3/2}, -q^{3/2}; q^2 \end{matrix} \right]_n. \tag{12}$$

Taking the limit $n \rightarrow \infty$, (12) yields the following generating function for partial generalized mock theta function of order two $A_r(k, \alpha)$.

$$\sum_{r=0}^{\infty} A_r(k, \alpha) z^r = \frac{q}{(1 - q)^2 (k;q)_\infty} {}_4\phi_4 \left[\begin{matrix} k, i\sqrt{q}, -i\sqrt{q}, q; q, q, q^{(\alpha+2)}z \\ q^{3/2}, -q^{3/2}, q^{3/2}, -q^{3/2}; q^2 \end{matrix} \right] \sum_{r=0}^{\infty} z^r.$$

Taking $k = 0$ and $\alpha = 1$, we get

$$\sum_{r=0}^{\infty} A_r(q) z^r = \frac{q}{(1 - q)^2} {}_3\phi_4 \left[\begin{matrix} i\sqrt{q}, -i\sqrt{q}, q; q, q, q^3z \\ q^{3/2}, -q^{3/2}, q^{3/2}, -q^{3/2}; q^2 \end{matrix} \right] \sum_{r=0}^{\infty} z^r. \tag{13}$$

Where $A_r(q)$ is the partial mock theta function of order two and (13) is the generating function for $A_r(q)$.

(b) If we take $\delta_m = \frac{1}{(k;q)_\infty} \frac{(k;q)_m (-q^2;q^2)_m q^{m^2+m\alpha}}{(q;q^2)_{m+1}^2}$ in (11) and after simplification, we get

$$z^n B_n(k, \alpha) + (1 - z) \sum_{r=0}^{n-1} B_r(k, \alpha) z^r = \frac{1}{(1 - q)^2 (k;q)_\infty} {}_4\phi_4 \left[\begin{matrix} k, iq, -iq, q; q, q, q^{(\alpha+1)}z \\ q^{3/2}, -q^{3/2}, q^{3/2}, -q^{3/2}; q^2 \end{matrix} \right]_n. \tag{14}$$

Taking the limit $n \rightarrow \infty$, (14) yields the following generating function for partial generalized mock theta function of order two $B_r(k, \alpha)$.

$$\sum_{r=0}^{\infty} B_r(k, \alpha)z^r = \frac{1}{(1-q)^2(k; q)_{\infty}} {}_4\phi_4 \left[\begin{matrix} k, iq, -iq, q; q, q^2z \\ q^{3/2}, -q^{3/2}, q^{3/2}, -q^{3/2}; q^2 \end{matrix} \right] \sum_{r=0}^{\infty} z^r.$$

Taking $k = 0$ and $\alpha = 1$, we get

$$\sum_{r=0}^{\infty} B_r(q)z^r = \frac{1}{(1-q)^2} {}_3\phi_4 \left[\begin{matrix} iq, -iq, q; q, q^2z \\ q^{3/2}, -q^{3/2}, q^{3/2}, -q^{3/2}; q^2 \end{matrix} \right] \sum_{r=0}^{\infty} z^r. \tag{15}$$

Where $B_r(q)$ is the partial mock theta function of order two and (15) is the generating function for $B_r(q)$.

(c) If we take $\delta_m = \frac{1}{(k; q)_{\infty}} \frac{(k; q)_m (q; q^2)_m (-1)^m q^{m^2 - m + m\alpha}}{(-q^2; q^2)_m^2}$ in (11) and after simplification, we get

$$z^n \mu_n(k, \alpha) + (1-z) \sum_{r=0}^{n-1} \mu_r(k, \alpha)z^r = \frac{1}{(k; q)_{\infty}} {}_4\phi_4 \left[\begin{matrix} k, i\sqrt{q}, -i\sqrt{q}, q; q, -q^{\alpha}z \\ iq, -iq, iq, -iq; q^2 \end{matrix} \right]_n. \tag{16}$$

Taking the limit $n \rightarrow \infty$, (16) yields the following generating function for partial generalized mock theta function of order two $\mu_r(k, \alpha)$.

$$\sum_{r=0}^{\infty} \mu_r(k, \alpha)z^r = \frac{1}{(k; q)_{\infty}} {}_4\phi_4 \left[\begin{matrix} k, i\sqrt{q}, -i\sqrt{q}, q; q, -q^{\alpha}z \\ iq, -iq, iq, -iq; q^2 \end{matrix} \right] \sum_{r=0}^{\infty} z^r.$$

Taking $k = 0$ and $\alpha = 1$, we get

$$\sum_{r=0}^{\infty} \mu_r(q)z^r = {}_3\phi_4 \left[\begin{matrix} i\sqrt{q}, -i\sqrt{q}, q; q, -qz \\ iq, -iq, iq, -iq; q^2 \end{matrix} \right] \sum_{r=0}^{\infty} z^r. \tag{17}$$

Where $\mu_r(q)$ is the partial mock theta function of order two and (17) is the generating function for $\mu_r(q)$.

(d) If we take $\delta_m = \frac{1}{(k; q)_{\infty}} \frac{(k; q)_m (-q; q)_m q^{m\alpha}}{(q; q^2)_{m+1}}$ in (11) and after simplification, we get

$$z^n D_{5n}(k, \alpha) + (1-z) \sum_{r=0}^{n-1} D_{5r}(k, \alpha)z^r = \frac{1}{(1-q)(k; q)_{\infty}} {}_3\phi_2 \left[\begin{matrix} k, q, -q; q, q^{\alpha}z \\ q^{3/2}, -q^{3/2} \end{matrix} \right]_n. \tag{18}$$

Taking the limit $n \rightarrow \infty$, (18) yields the following generating function for partial generalized mock theta function of order two $D_{5r}(k, \alpha)$.

$$\sum_{r=0}^{\infty} D_{5r}(k, \alpha)z^r = \frac{1}{(1-q)(k; q)_{\infty}} {}_3\phi_2 \left[\begin{matrix} k, q, -q; q, q^{\alpha}z \\ q^{3/2}, -q^{3/2} \end{matrix} \right] \sum_{r=0}^{\infty} z^r.$$

Taking $k = 0$ and $\alpha = 1$, we get

$$\sum_{r=0}^{\infty} D_{5r}(q)z^r = \frac{1}{(1-q)} {}_2\phi_2 \left[\begin{matrix} q, -q; q, qz \\ q^{3/2}, -q^{3/2} \end{matrix} \right] \sum_{r=0}^{\infty} z^r. \tag{19}$$

Where $D_{5r}(q)$ is the partial mock theta function of order two and (19) is the generating function for $D_{5r}(q)$.

4.2 Generating function for partial mock theta functions of order ten:

(a) If we take $\delta_m = \frac{1}{(k;q)_{\infty}} \frac{(k;q)_m q^{m(m-1)/2+m\alpha}}{(q;q^2)_{m+1}}$ in (11) and after simplification, we get

$$z^n \phi_n(k, \alpha) + (1-z) \sum_{r=0}^{n-1} \phi_r(k, \alpha)z^r = \frac{1}{(1-q)(k;q)_{\infty}} {}_2\phi_2 \left[\begin{matrix} k, q; q, q^{\alpha}z \\ q^{3/2}, -q^{3/2}; q \end{matrix} \right]_n. \tag{20}$$

Taking the limit $n \rightarrow \infty$, (20) yields the following generating function for partial generalized mock theta function of order ten $\phi_r(k, \alpha)$.

$$\sum_{r=0}^{\infty} \phi_r(k, \alpha)z^r = \frac{1}{(1-q)(k;q)_{\infty}} {}_2\phi_2 \left[\begin{matrix} k, q; q, q^{\alpha}z \\ q^{3/2}, -q^{3/2}; q \end{matrix} \right] \sum_{r=0}^{\infty} z^r.$$

Taking $k = 0$ and $\alpha = 1$, we get

$$\sum_{r=0}^{\infty} \phi_r(q)z^r = \frac{1}{(1-q)} {}_1\phi_2 \left[\begin{matrix} q; q, qz \\ q^{3/2}, -q^{3/2}; q \end{matrix} \right] \sum_{r=0}^{\infty} z^r. \tag{21}$$

Where $\phi_r(q)$ is the partial mock theta function of order ten and (21) is the generating function for $\phi_r(q)$.

(b) If we take $\delta_m = \frac{1}{(k;q)_{\infty}} \frac{(k;q)_m q^{m(m+1)/2+m\alpha}}{(q;q^2)_{m+1}}$ in (11) and after simplification, we get

$$z^n \psi_n(k, \alpha) + (1-z) \sum_{r=0}^{n-1} \psi_r(k, \alpha)z^r = \frac{1}{(1-q)(k;q)_{\infty}} {}_2\phi_2 \left[\begin{matrix} k, q; q, q^{(\alpha+1)}z \\ q^{3/2}, -q^{3/2}; q \end{matrix} \right]_n. \tag{22}$$

Taking the limit $n \rightarrow \infty$, (22) yields the following generating function for partial generalized mock theta function of order ten $\psi_r(k, \alpha)$.

$$\sum_{r=0}^{\infty} \psi_r(k, \alpha)z^r = \frac{1}{(1-q)(k;q)_{\infty}} {}_2\phi_2 \left[\begin{matrix} k, q; q, q^{(\alpha+1)}z \\ q^{3/2}, -q^{3/2}; q \end{matrix} \right] \sum_{r=0}^{\infty} z^r.$$

Taking $k = 0$ and $\alpha = 1$, we get

$$\sum_{r=0}^{\infty} \psi_r(q)z^r = \frac{1}{(1-q)} {}_1\phi_2 \left[\begin{matrix} q; q, q^2z \\ q^{3/2}, -q^{3/2}; q \end{matrix} \right] \sum_{r=0}^{\infty} z^r. \tag{23}$$

Where $\psi_r(q)$ is the partial mock theta function of order ten and (23) is the generating function for $\psi_r(q)$.

(c) If we take $\delta_m = \frac{1}{(k;q)_\infty} \frac{(k;q)_m (-1)^m q^{m^2 - m + m\alpha}}{(-q;q)_{2m}}$ in (11) and after simplification, we get

$$z^n X_n(k, \alpha) + (1 - z) \sum_{r=0}^{n-1} X_r(k, \alpha) z^r = \frac{1}{(k;q)_\infty} {}_2\phi_4 \left[\begin{matrix} k, q; q, -q^\alpha z \\ i\sqrt{q}, -i\sqrt{q}, iq, -iq; q^2 \end{matrix} \right]_n. \tag{24}$$

Taking the limit $n \rightarrow \infty$, (24) yields the following generating function for partial generalized mock theta function of order ten $X_r(k, \alpha)$.

$$\sum_{r=0}^{\infty} X_r(k, \alpha) z^r = \frac{1}{(k;q)_\infty} {}_2\phi_4 \left[\begin{matrix} k, q; q, -q^\alpha z \\ i\sqrt{q}, -i\sqrt{q}, iq, -iq; q^2 \end{matrix} \right] \sum_{r=0}^{\infty} z^r.$$

Taking $k = 0$ and $\alpha = 1$, we get

$$\sum_{r=0}^{\infty} X_r(q) z^r = {}_1\phi_4 \left[\begin{matrix} q; q, -qz \\ i\sqrt{q}, -i\sqrt{q}, iq, -iq; q^2 \end{matrix} \right] \sum_{r=0}^{\infty} z^r. \tag{25}$$

Where $X_r(q)$ is the partial mock theta function of order ten and (25) is the generating function for $X_r(q)$.

(d) If we take $\delta_m = \frac{1}{(k;q)_\infty} \frac{(k;q)_m (-1)^m q^{m^2 + m + m\alpha}}{(-q;q)_{2m+1}}$ in (11) and after simplification, we get

$$z^n \chi_n(k, \alpha) + (1 - z) \sum_{r=0}^{n-1} \chi_r(k, \alpha) z^r = \frac{1}{(1+q)(k;q)_\infty} {}_2\phi_4 \left[\begin{matrix} k, q; q, -q^{(\alpha+2)} z \\ i\sqrt{q}, -i\sqrt{q}, iq, -iq; q^2 \end{matrix} \right]_n. \tag{26}$$

Taking the limit $n \rightarrow \infty$, (26) yields the following generating function for partial generalized mock theta function of order ten $\chi_r(k, \alpha)$.

$$\sum_{r=0}^{\infty} \chi_r(k, \alpha) z^r = \frac{1}{(k;q)_\infty (1+q)} {}_2\phi_4 \left[\begin{matrix} k, q \\ i\sqrt{q}, -i\sqrt{q}, iq, -iq; q^2 \end{matrix} ; q, -q^{(\alpha+2)} z \right] \sum_{r=0}^{\infty} z^r.$$

Taking $k = 0$ and $\alpha = 1$, we get

$$\sum_{r=0}^{\infty} \chi_r(q) z^r = \frac{1}{(1+q)} {}_1\phi_4 \left[\begin{matrix} q; q, -q^3 z \\ i\sqrt{q}, -i\sqrt{q}, iq, -iq; q^2 \end{matrix} \right] \sum_{r=0}^{\infty} z^r. \tag{27}$$

Where $\chi_r(q)$ is the partial mock theta function of order ten and (27) is the generating function for $\chi_r(q)$.

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