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Canonical Sine Transform and their Unitary Representation

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Abstract

This paper studies different properties of canonical sine transform. Modulation theorem is also proved. We have proved some important results about the Kernel of canonical sine transform.

Keywords: Generalized function, Canonical transform, Canonical Sine transforms, Modulation theorem, Fourier transform

1) Introduction

The Fourier transform is certainly one of the best known of integral transforms,since its introduction by Fourier in early 1800's. Torre [11] had discussed the case when the entities in the characteristic matrix are complex. He had defined the Bargmann transform and Laplace transform as special case of the complex linear canonical transform.

A much more general integral transform namely linear canonical transform was introduced in 1970 by Moshinsky and Quesne [5]. In 1980 Namias [6] introduced the fractional Fourier transform using eigen value and eigen functions.

Almeida [1], [2] had introduce it and proved many of its properties. Zayed [10] had discussed about product and convolution concerning that transform.

Bhosale and Choudhary [3] had studied it as a tempered distribution, number of applications of fractional Fourier transform in signal processing, image processing filtering optics, etc are studied. Optical implementation of it was explained by Ozaktas and Medlovic [7].

Pei and Ding [8] [9] had studies linear canonical transforms. The definition of canonical sine transforms as follow,

$$\{CST\ f(t)\}(s) = -i\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} f(t)\,dt \qquad \text{for } b \neq 0$$

$$= \sqrt{d}.e^{\frac{i}{2}cds^2} f(d.s) \qquad \text{for } b = 0$$

Notation and terminology of this paper is as per zemanian [12], [13] The paper is organized as follows. Section two gives the definition of canonical sine transform on the space of generalized function and states the property of the kernel of the canonical sine transform. In section three, modulation theorem and in section four, some properties of canonical sine transform are proved. In section five, some operation transforms in terms of parameter are discussed. Lastly the conclusion is stated.

2. Generalized Canonical Sine Transform

2.1 . Definition : The canonical sine transform of generalized function is defined as

$$\{CST\ f(t)\}(s) = \left\langle f(t), K_{(a,b,c,d)}(t,s)\right\rangle$$

Where kernel $K_{(a,b,c,d)}(t,s) = (-i)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\ e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\sin\left(\frac{s}{b}t\right)$ …….(1)

Clearly Kernel $K_{(a,b,c,d)}(t,s) \in E$ and $K_{(a,b,c,d)}(t,s) \in E(R^1)$ the kernel (1) Satisfies the following properties.

2.2. Properties of Kernel :

$K_{(a,b,c,d)}(t,s) \neq K_{(a,b,c,d)}(s,t)$ if $a \neq d$

$K_{(a,b,c,d)}(t,s) = K_{(a,b,c,d)}(s,t)$ if a = d

$K_{(a,b,c,d)}(t,s) = K_{(a,-b,-c,d)}(t,s)$

$K_{(a,b,c,d)}(-t,s) = K_{(a,b,c,d)}(t,-s)$

2.3. Definition of canonical cosine transform as follows,

$$\{CCT\ f(t)\}(s) = \frac{1}{\sqrt{2\pi ib}} e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \int_{-\infty}^{\infty} \cos\left(\frac{s}{b}t\right) e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} f(t)\,dt \qquad \text{for } b \neq 0$$

……….(2)

$$= \sqrt{d}\,.e^{\frac{i}{2}cds^2} f(d.s) \qquad \text{for } b = 0$$

As we have obtained few properties of canonical cosine transform in Chavhan.S.B. and Borkar.V.C.[4],hence we establish similar properties of canonical sine transform.

(3) Modulation Theorem for Canonical Sine Transform

Theorem 3.1 : If $\{CST\ f(t)\}(s)$ is canonical sine transform of f(t), $f(t) \in E^1(R^1)$ then

$$\{CST \cos(ut)\, f(t)\}(s) = \frac{e^{-\frac{i}{2}(du^2b)}}{2}\left[(CST\ f(t))\{(s+bu)e^{-idus} + \{CST\ f(t)\}(s-bu)e^{idus}\right]$$ …..(3)

Proof: Using definition of canonical sine transform

$$\{CST\ f(t)\}(s) = (-i)\frac{1}{\sqrt{2\pi ib}} e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \int_{\infty}^{\infty} \sin\left(\frac{s}{b}t\right) e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} f(t)\,dt$$

$$\{CST \cos(ut)\, f(t)\}(s) = (-i)\frac{1}{\sqrt{2\pi ib}} e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \int_{\infty}^{\infty} \sin\left(\frac{s}{b}t\right) e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} \cos(ut)\, f(t)\,dt$$

$$= (-i)\frac{1}{\sqrt{2\pi ib}} e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \int_{-\infty}^{\infty} \left[\frac{2\sin\left(\frac{s}{b}t\right)\cos(ut)}{2}\right] e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} f(t)\,dt$$

$$= (-i)\frac{1}{2\sqrt{2\pi ib}} e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \int_{-\infty}^{\infty} \left[\sin\left(\frac{s}{b}t + ut\right) + \sin\left(\frac{s}{b}t - ut\right)\right] e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} f(t)\,dt$$

$$= (-i)\frac{1}{2}\left[\frac{1}{\sqrt{2\pi ib}} e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \int_{-\infty}^{\infty} \sin\left(\frac{(s+ub)}{b}\right) t\, e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} f(t)\,dt + \frac{1}{\sqrt{2\pi ib}} e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \int_{-\infty}^{\infty} \sin\left(\frac{s-ub}{b}\right) t\, e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} f(t)\right]$$

$$= \frac{e^{-\frac{i}{2}(du^2b)}}{2}\left\{\left[(-i)\frac{1}{\sqrt{2\pi ib}} d^{\frac{i}{2}\left(\frac{d}{b}\right)(s+ub)^2} e^{-i(dus)} \int_{-\infty}^{\infty} \sin\left(\frac{s+ub}{b}\right) t\, e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} f(t)\,dt\right]\right.$$

$$\left. + \left[(-i)\frac{1}{\sqrt{2\pi ib}} e^{\frac{i}{2}\left(\frac{d}{b}\right)(s-ub)^2} e^{iuds} \int_{-\infty}^{\infty} \sin\left(\frac{s-ub}{b}\right) t\, e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} f(t)\,dt\right]\right\}$$

$$=\frac{e^{-\frac{i}{2}\left(du^2b\right)}}{2}\left[\left\{CST\ f\left(t\right)\right\}\left(s+bu\right)e^{-idus}+\left\{CST\ f\left(t\right)\right\}\left(s-bu\right)e^{idus}\right]$$

$$\{CST\cos ut\ \ f(t)\}(s)=\frac{e^{-\frac{i}{2}\left(du^2b\right)}}{2}\left[\left\{CST\ f\left(t\right)\right\}\left(s+bu\right)e^{-idus}+\left\{CST\ f\left(t\right)\right\}\left(s-bu\right)e^{idus}\right]$$

Theorem 3.2 : If $\left\{CST\ f\left(t\right)\right\}\left(s\right)$ is canonical sine transform of $f\left(t\right),\ f\left(t\right)\in\left(R^1\right)$

Then

$$\left\{CST\sin ut\ f\left(t\right)\right\}\left(s\right)=\frac{-i}{2}e^{-\frac{i}{2}\left(du^2b\right)}\left[\left\{CCT\ f\left(t\right)\right\}\left(s-ub\right)e^{idsu}-\left\{CCT\ f\left(t\right)\right\}\left(s+ub\right)e^{-idsu}\right]\quad\ldots(4)$$

Proof : By using definition canonical sine transform

$$\left\{CST\ f\left(t\right)\right\}\left(s\right)=(-i)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{\infty}^{\infty}\sin\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\ f\left(t\right)dt$$

$$\left\{CST\sin ut\ f\left(t\right)\right\}\left(s\right)=(-i)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right).e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}.\sin ut\ f\left(t\right)dt$$

$$=(-i)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty}\left(\frac{2\sin\left(\frac{s}{b}t\right)\sin ut}{2}\right)e^{\frac{i}{2}\left(\frac{d}{b}\right)t^2}\ f\left(t\right)dt$$

$$=(-i)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty}\left(\frac{\cos\left(\frac{s}{b}t-ut\right)-\cos\left(\frac{s}{b}t+ut\right)}{2}\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\ f\left(t\right)dt$$

$$=\frac{-i}{2}\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty}\left[\cos\left(\frac{s-ub}{b}\right)t-\cos\left(\frac{s+ub}{b}\right)t\right]e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\ f(t)\,dt$$

$$=\frac{-ie^{\frac{-i}{2}\left(du^2b\right)}}{2}\left[\left(\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)\left(s-ub\right)^2}e^{i\left(dus\right)}.\int_{-\infty}^{\infty}\cos\left(\frac{s-ub}{b}\right)t\,e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\ f\left(t\right)dt\right)\right.$$

$$\left.-\left(\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)\left(s+ub\right)}.e^{-idub}.\int_{-\infty}^{\infty}\cos\left(\frac{s+ub}{b}\right)t\,e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\ f\left(t\right)dt\right)\right]$$

$$=\frac{-ie^{\frac{i}{2}\left(du^2b\right)}}{2}\left[\left\{CCT\ f\left(t\right)\right\}\left(s-bu\right)e^{idsu}-\left\{CCT\ f\left(t\right)\right\}\left(s+ub\right)e^{-idus}\right]$$

Theorem 3.3 : If $\left\{CST\ f\left(t\right)\right\}\left(s\right)$ is canonical sine transform of $f\left(t\right),\ f\left(t\right)\in E^1\left(R^1\right)$

Then

$$\{CST\, e^{iut} f(t)\}(s) =$$
$$\frac{1}{2}e^{-\frac{i}{2}(du^2b)}\left\{e^{-idsu}\left[\{CST\ f(t)\}(s+ub)-\{CCT\ f(t)\}(s+ub)\right]+e^{idus}\left[\{CST\ f(t)\}(s-ub)-\{CCTf(t)\}(s-ub)\right]\right\}$$

…..(5)

4. Properties of Canonical Sine Transform

4.1 Time Reverse :

If $\{CST\ f(t)\}$ is canonical sine transforms of $f(t)$, $f(t)\in E^1(R^1)$ then

$$\{CST\ f(-t)\}(s) = -\{CST\ f(t)\}(s) \qquad \text{…….(6)}$$

4.2. Linearity Property :

If C_1, C_2 are constant and f_1, f_2 are functions of t then

$$\{CST[C_1 f_1(t)+C_2 f_2(t)]\}(s) = C_1\{CST\ f_1(t)\}(s)+C_2\{CST f_2(t)\}(s)$$
………….(7)

4.3 Parity :

If $\{CST\ f(t)\}(s)$ is canonical sine transform of $f(t)\in E^1(R^1)$ then

$$\{CST\ f(-t)\}(s) = \{CST\ f(t)\}(-s)$$
……………(8)

OR

$$\{CST\ f(t)\}(s) = \frac{-1}{2}\left[\{CST\ f(-t)\}(s)+\{CST\ f(t)\}(-s)\right]$$
…………….(9)

4.4. Shifting Property :

It *{CST f(t)}(s)* denotes generalized canonical sine transform of *f(t)* and τ is any real number then,

$$\{CST\ (f(t+\tau)\}(s)=e^{\frac{i}{2}\left(\frac{a}{b}\right)\tau^2}\{\cos(s/b)\tau\left[CST\quad f(t)\cdot e^{-it\tau\left(\frac{a}{b}\right)}\right](s)$$

$$+i\sin(s/b)\tau\left[CCT\ f(t)\cdot e^{-it\tau\left(\frac{a}{b}\right)}\right](s)\}$$

………

(10)

4.5. Differentiation Property :

If *{CST (f(t))}(s)* denotes generalized canonical sine transform of *f(t)*, then

$$\{CST\,[\,f^{1}(t)]\}(s)=i\{(s/b)[CCT\ f(t)](s)+(a/b)[CCT\ f(t)](s)\}$$

………(11)

4.6. Scaling Property :

If *{CST f(t)}(s)* denotes generalized canonical sine transform of *f(t)*, then,

$$\{CST\ f(kt)\}(s)=\frac{1}{k}e^{\left(1-\frac{1}{k}\right)\frac{i}{2}\frac{d}{bk}s^2}\left[CST\ f(t)e^{\left(\frac{1}{k}-1\right)\frac{i}{2}\frac{a}{bk}t^2}\right](s)$$

…………(12)

5. Operation Transform in Terms of Parameter

This section present the operational formulae for canonical sine transform with shifted parameter S, and differential of canonical sine transform with respective s.

Result 5.1 : If $\{CST\ f(t)\}(s)$ is canonical sine transform of $f(t)$ in terms of parameter S, then

$$\{CST\ f(t)\}(s+\alpha)=e^{\frac{i}{2}\left(\frac{d}{b}\right)\left(2s\alpha+\alpha^2\right)}\left[\left\{CST\cos\left(\frac{\alpha}{b}t\right)f(t)\right\}(s)+e^{\frac{i}{2}\left(\frac{d}{b}\right)\left(2s\alpha+\alpha^2\right)}\left\{CST\cos\left(\frac{s}{b}t\right)f(t)\right\}(\alpha)\right]$$

….(13)

Proof : We know that

$$\{CST\ f(t)\}(s)=(-i)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t)\,dt$$

$$\left\{CST\ f\left(t\right)\right\}\left(s+\alpha\right)=(-i)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)(s+\alpha)^2}\int_{-\infty}^{\infty}\sin\frac{(s+\alpha)}{b}te^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f\left(t\right)dt$$

$$=(-i)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)(S+\alpha)^2}\int_{-\infty}^{\infty}\left[\sin\left(\frac{s}{b}t\right).\cos\left(\frac{\alpha}{b}t\right)+\cos\left(\frac{s}{b}t\right).\sin\left(\frac{\alpha}{b}t\right)\right]e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t)\,dt$$

$$=(-i)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)(S+\alpha)^2}\left[\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right).\cos\left(\frac{\alpha}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t)\,dt+\int_{-\infty}^{\infty}\cos\left(\frac{s}{b}t\right).\sin\left(\frac{\alpha}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t)\,dt\right]$$

$$=(-i)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}.e^{\frac{i}{2}\left(\frac{d}{b}\right)(2s\alpha+\alpha^2)}\left[\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right).\cos\left(\frac{\alpha}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t)\,dt+\int_{-\infty}^{\infty}\cos\left(\frac{s}{b}t\right).\sin\left(\frac{\alpha}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t)\,dt\right]$$

$$=(-i)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}.e^{i\left(\frac{d}{b}\right)(s\alpha)}.e^{i\left(\frac{d}{b}\right)(\alpha^2)}\left[\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right).\cos\left(\frac{\alpha}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t)\,dt+\int_{-\infty}^{\infty}\cos\left(\frac{s}{b}t\right).\sin\left(\frac{\alpha}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t)\,dt\right]$$

$$=(-i)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}.e^{i\left(\frac{d}{b}\right)(s\alpha)}.e^{i\left(\frac{d}{b}\right)(\alpha^2)}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right).\cos\left(\frac{\alpha}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t)\,dt+(-i)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}.e^{i\left(\frac{d}{b}\right)(s\alpha)}.e^{i\left(\frac{d}{b}\right)(\alpha^2)}\int_{-\infty}^{\infty}\cos\left(\frac{s}{b}t\right).\sin\left(\frac{\alpha}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t)\,dt$$

$$=(-i)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}.e^{\frac{i}{2}\left(\frac{d}{b}\right)(2s\alpha+\alpha^2)}.\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right).\cos\left(\frac{\alpha}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t)\,dt+(-i)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)\alpha^2}.\,e^{\frac{i}{2}\left(\frac{d}{b}\right)(2s\alpha+s^2)}\int_{-\infty}^{\infty}\cos\left(\frac{s}{b}t\right).\sin\left(\frac{\alpha}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t)\,dt$$

$$=e^{\frac{i}{2}\left(\frac{d}{b}\right)(2s\alpha+\alpha^2)}\left\{CST\cos\left(\frac{\alpha}{b}t\right)f\left(t\right)\right\}\left(s\right)+e^{\frac{i}{2}\left(\frac{d}{b}\right)(2s\alpha+s^2)}\left\{CST\cos\left(\frac{s}{b}t\right)f\left(t\right)\right\}(\alpha)$$

$$=e^{\frac{i}{2}\left(\frac{d}{b}\right)(2s\alpha+\alpha^2)}\left\{CST\cos\left(\frac{\alpha}{b}t\right)f\left(t\right)\right\}\left(s\right)+e^{\frac{i}{2}\left(\frac{d}{b}\right)(2s\alpha+s^2)}\left\{CST\cos\left(\frac{s}{b}t\right)f\left(t\right)\right\}(\alpha)$$

Result 5.2 : If $\left\{CST\ f\left(t\right)\right\}\left(s\right)$ is canonical sine transform of f (t) in terms of parameter s

Then $\frac{d}{ds}\left[CST\ f\left(t\right)\right]\left(s\right)=i\left[\frac{ds}{b}\{CST\ f(t)\}(s)-\{CCT\ \frac{t}{b}f(t)\}(s)\right]$ ………..(14)

Proof: Using definition of canonical sine transform

$$\left\{CST\ f\left(t\right)\right\}\left(s\right)=(-i)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f\left(t\right)dt$$

$$\therefore\frac{d}{ds}\left\{CST\ f\left(t\right)\right\}\left(s\right)=\frac{d}{ds}\left[(-i)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f\left(t\right)dt\right]$$

$$=(-i)\frac{1}{\sqrt{2\pi ib}}\int_{-\infty}^{\infty}e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\left\{\frac{d}{ds}\left[e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\sin\left(\frac{s}{b}t\right)\right]f\left(t\right)dt\right\}$$

$$= (-i)\frac{1}{\sqrt{2\pi ib}}\int_{-\infty}^{\infty} e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\left[e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\sin\left(\frac{s}{b}t\right)i\left(\frac{ds}{b}\right)+\cos\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)s^2}\left(\frac{t}{b}\right)\right]f(t)\,dt$$

$$= (-i)\left(i\frac{ds}{b}\right)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{d}{b}\right)t^2}f(t)\,dt - i\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty}\cos\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}.\left(\frac{t}{b}\right)f(t)\,dt$$

$$= (i)\left[\left(-i\frac{ds}{b}\right)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{d}{b}\right)t^2}f(t)\,dt - \frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty}\cos\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}.\left(\frac{t}{b}\right)f(t)\,dt\right]$$

$$= i\left[\left(\frac{ds}{b}\right).\left\{CST\ f(t)\right\}(s) - \left\{CCT\left(\frac{t}{b}\right)f(t)\right\}(s)\right]$$

$$\frac{d}{ds}\{CST\ f(t)\}(s) = i\left[\left(\frac{ds}{b}\right).\left\{CST\ f(t)\right\}(s) - \left\{CCT\left(\frac{t}{b}\right)f(t)\right\}(s)\right]$$

6. Conclusion

The canonical sine transform which is the generalization of number of transform is itself generalized to the spaces of generalized functions as per Zemanian. Since this transform is used to solve ordinary or partial differential equations. This transform is an important tool in signal processing and many other branches of engineering. In this paper we have proved Modulation theorem, time reverse property and linearity property.

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