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On the Induced Central Extensions and the Schur Multiplier of Topological Groups

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Abstract

Let G be a topological group and A a trivial G-module . In this paper we define the topological induced extension of A by G. We will show that there is a relation between the induced extension and ,$M(G)$, the Schur multiplier of G.

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Introduction

Induced central extensions were first studied by [9]. If $0 \to T \to M \to G \to 0$ is a central extension ,where T is the additive group of rationals mod 1, then [9,theorem 3.3] every central extension of T by any abelian group is an induced central extension. In [13], based on the second cohomology, a necessary and sufficient condition for a central extension to be induced is obtained.
In this paper we define the induced central extension in a topological context and will find its relation to the Schur multiplier of a nilpotent topological group.

Throughout G is a completely regular and Hausdorff nilpotent topological group of class n(see section 1). A *topological extension* of A by G is a short exact sequence $0 \to A \xrightarrow{i} M \xrightarrow{\pi} G \to 0$,where i is a topological embedding onto a closed subgroup ,π an open onto continuous homomorphism . We consider every extension with a continuous section i.e. $u : G \to M$ such that $\pi u = Id$. For example ,if G is a connected locally compact group ,then any topological extension of G by a connected simply connected Lie group has a continuous section [12,theorem 2].

The *free topological* group is in Markov [8] sense. Let X be a Tychonov space,then the (Markov) free topological group on X is the group $F(X)$ equipped with the finest group topology inducing the given topology on X; every continuous map from X to an arbitrary topological group G lifts in a unique fashion to a continuous homomorphism from $F(X)$to G . Such a topology always exists [8].The free abelian topological group is defined the same way by G being an abelian topological group.

The Schur multiplier is a function taking a group to another group. It is used in different branches of mathematics such as the classification of finite groups and homology theory.
Let G be a topological group . A *free topological presentation* of G is a short exact sequence $0 \to R \to F \xrightarrow{\pi} G \to 0$ with a continuous section $u : G \to F$ such that $\pi u = Id_G$ where F is a free topological group. For a subgroup S of F we write $S' = \frac{\overline{S[R,F]}}{\overline{[R,F]}}$.

Consider the abelian group $\frac{R \cap \overline{[F,F]}}{\overline{[R,F]}}$. We call this group *the Schur multiplier of* G denoted by

$$M(G) = \frac{R \cap \overline{[F,F]}}{\overline{[R,F]}}$$

By [1], $M(G)$ is independent of the choice of the presentation. If $\gamma : G \to H$ is a continuous homomorphism then $M(\gamma) : M(G) \to M(H)$ is the induced map by γ. Note that $M(-)$ defined for a continuous homomorphism is a functor from the category of topological groups to the category of abelian topological groups [1].
We make the following notation. For any topological group , $[G, G]$= the commutator of G ; $G^{ab} = G/\overline{[G, G]}$.

In section 1 we recall cohomology and nilpotent topological groups . in section 2 ,we discuss induced central extensions in general as in [10]. In section 3, we will find a relation between the Schur multiplier and the induced central extension of nilpotent group of class n.

1 Cohomology and nilpotent topological groups

In this section we shall recall some definitions and results to be used in the sequel [11].

When G is a topological group the theory of cohomology gets more interesting since we have both algebraic and topological notions of cohomology and there are different ways to combine them ,[4],[5],[6],[2].

Let Q be a topological group and A an abelian topological group on which Q acts continuously.
Let $C^n(G, A)$ be the continuous maps $\phi : G^n \to A$ with the coboundary map

$$C^n(G, A) \xrightarrow{\delta_n} C^{n+1}(G, A))$$

given by
$\delta\phi(g_1, ..., g_n) = g_1.\phi(g_2, ..., g_n)$

$$+ \sum_{i=1}^{n-1} (-1)^i \phi(g_1, ..., g_i g_{i+1}, ..., g_n) + (-1)^n \phi(g_1, ..., g_{n-1})$$

Note that this is analogous to the homogeneous resolution for the discrete case[3].

Definition 1.1. The continuous group cohomology of G with coefficient in A is

$$H^n(G, A) = ker\delta_n / Im\delta_{n-1}$$

Let $Ext_s(G, A)$ be the set of extensions of A by G with a continuous section. It is known [1] ,by the Baer sum, that $Ext_s(G, A)$ is an abelian topological group. By [7], if G is a topological group and A a trivial G-module then there is an isomorphism between the second cohomology of G and the group of extensions of A by G with continuous sections, namely

$$H^2(G, A) \simeq Ext_s(G, A)$$

Note that if the extension $0 \to A \xrightarrow{i} M \xrightarrow{\pi} G \to 0$ has a continuous section then $M \simeq A \times G$, as topological spaces [1].

Definition 1.2. Let G be a topological group and $G_0 = G$, $G_{(n+1)}$ is the closure of the group generated by $[a, b]$, $a \in G_{(n)}$, $b \in G$.

G is called nilpotent of class n if $G_{(n)} = 1$ for some n.

$$G_0 = G \supseteq G_{(1)} \supseteq G_{(2)} \supseteq ... \supseteq G_{(n)} = 1$$

The main examples of nilpotent groups are upper triangular matrix groups.

2 Induced central extension

In this section we define the induced central extension [10]. We will find a necessary and sufficient condition for an extension to be induced. Also we show that every central extension of T , the additive group of rationals mod 1 , by an abelian topological group is induced.

Definition 2.1. Let G be a nilpotent topological group of class $n \geq 1$ and A an abelian group regarded as a trivial G-module. A central extension

$$(E) : 0 \to A \xrightarrow{i} M \xrightarrow{\pi} G \to 0$$

of A by G is called *induced* if there exists a central extension $0 \to L_n \xrightarrow{i} L \xrightarrow{\gamma} G \to 0$ and an open continuous homomorphism $\beta : L \to M$ making the following diagram commutative

$$\begin{array}{ccccccccc} 0 & \to & L_n & \to & L & \xrightarrow{\gamma} & G & \to & 0 \\ & & \downarrow \overline{\beta} & & \downarrow\beta & & \| & & \\ 0 & \to & A & \to & M & \xrightarrow{\alpha} & G & \to & 0 \end{array}$$

where L_k is the kth term of the lower central series of G.

Lemma 2.2 .*Let*

$$\begin{array}{ccccccccc} 0 & \to & A_1 & \xrightarrow{\alpha_1} & B_1 & \xrightarrow{\beta_1} & C & \to & 0 \\ & & \downarrow\alpha & & \downarrow\beta & & \| & & \\ 0 & \to & A_2 & \xrightarrow{\alpha_2} & B_2 & \xrightarrow{\beta_2} & C & \to & 0 \end{array}$$

be a commutative diagram in the category of topological groups such that

(a) the rows are exact

(b) $\alpha_2(A_2)$ *is in the center of* B_2

Define $u : A_1 \to B_1 \oplus A_2$ *and* $v : B_1 \oplus A_2 \to B_2$ *by*

$$u(x) = (\alpha_1(x), \alpha(x)^{-1}) \quad , \quad v(x, y) = \beta(x)\alpha_2(y) \quad , x \in A_1,\ y \in A_2$$

Then u, v *are continuous homomorphisms and the sequence*

$$0 \to A_1 \xrightarrow{u} B_1 \oplus A_2 \xrightarrow{v} B_2 \to 0$$

is exact .

Proof. It is easy to check that u and v are continuous homomorphisms. Let $x \in A_1$ and $u(x) = (e_{B_1}, e_{A_1})$ where e_G denotes the identity of G. Then $\alpha_1(x) = e_{B_1}$ and therefore $x = e_{A_1}$. Thus u is a monomorphism.
Let $x \in B_2$. Then $\beta_2(x) \in C$. Since β_1 is onto , there exists $y \in B_2$ such that $\beta_1(y) = \beta_2(x)$. By the commutativity of the diagram

$$\beta_2(\beta(y)) = \beta_1(y) = \beta_2(x)$$

Hence $\beta(y)^{-1}x \in ker\beta_2$. Thus $\beta(y)^{-1}x = \alpha_2(x)$ for some $x \in A_2$. Therefore, $x = \beta(y)\alpha_2(x)$. This shows that v is onto.
Now we show that the sequence is exact. Let $x \in A_1$.Then $u(x) = (\alpha_2(x), \alpha(x)^{-1})$.

Therefore,$v(u(x)) = \beta(\alpha_1(x), \alpha_2(\alpha(x)^{-1}))$.But $\beta \circ \alpha_1 = \alpha_2 \circ \alpha$. So $vu(x)) = e_{B_1}$ showing that $Imu \subset Kerv$.
Finally let $(x, y) \in Kerv$ i.e. ,$x \in B_1$, $y \in A_2$ and $\beta(x)\alpha_2(y) = e_{B_2}$. Then $\beta_2(\beta(x)) = e_C$. But $\beta_1 = \beta_2 \circ \beta$. So $\beta_1(x) = e_C$. Hence there exists $z \in A_1$ such that $x = \alpha_1(z)$. Now $u(z) = (x, \alpha(z)^{-1})$, $x = \alpha_2(z)$ implies that $\beta(x) = \alpha_2(y)^{-1}$. Therefore, $\alpha_2(y)^{-1} = \alpha_2(\alpha(z)$. Since α_2 is one to one ,we have $y^{-1} = \alpha(z)$ or $\alpha(z)^{-1} = y$. Hence $u(z) = (x, y)$.

Proposition 2.3 .*A central extension* $0 \to A \xrightarrow{\alpha} M \xrightarrow{\beta} G \to 0$ *with* G *nilpotent group of class* n*, is an induced central extension if and only if* M *has an open subgroup* N *such that*

(a) $N_n = M_n$

(b) $N/N_n \simeq G$ *the isomorphism being induced by* β

Proof . Without loss of generality we may assume that α is the inclusion map. Suppose M has an open subgroup N satisfying (a) and (b). Then we have the commutative diagram

$$\begin{array}{ccccccccc} 0 & \to & N_n & \to & N & \xrightarrow{\beta|_N} & G & \to & 0 \\ & & \downarrow & & \downarrow\pi & & \| & & \\ 0 & \to & A & \to & M & \xrightarrow{\beta} & G & \to & 0 \end{array}$$

which shows that the extension is induced.
Conversely suppose that we have a commutative diagram

$$\begin{array}{ccccccccc} 0 & \to & P_n & \to & P & \xrightarrow{\beta_1} & G & \to & 0 \\ & & \downarrow\alpha & & \downarrow\beta & & \| & & \\ 0 & \to & A & \to & M & \xrightarrow{\beta_2} & G & \to & 0 \end{array}$$

where β is open. By lemma 2.2 we have

$$(*) \qquad 0 \to P_n \xrightarrow{u} P \oplus A \xrightarrow{v} M \to 0$$

where $u(x) = (x, \alpha(x)^{-1})$, $x \in P_n$ and $v(x, t) = \beta(x)t$, $x \in \pi, t \in A$. Since β is an open map then so is v . Therefore, $(*)$ is a topological extension. Let $\beta(P) = N$. We assert that N is the required subgroup of M. Let $\tilde{\beta_2} = \beta_2|_N$ the restriction of β_2 to N . It is clear that $\beta_2|_N$ is open. Let $x \in G$. Then $\beta_1(y) = x$,for some $y \in P$. Therefore, $\beta_2(\beta(y) = \beta_1(y) = x$. Now $\beta(y) \in N$. Hence $\tilde{\beta_2}$ is an epimorphism. Let $y \in P$ such that $\beta_2(\beta(y)) = e_G$. Then $\beta_1(y) = e_G$. Therefore , $y \in P_n$ and $\beta(y) \in N_n$. Hence $Ker\tilde{\beta_2} \subseteq N_n$. But G being of class n , $\beta_2|_{M_n} = 0$. Hence $\tilde{\beta_2}|_{N_n} = 0$ and so $N_n = Ker\tilde{\beta_2}$. This proves the extension $0 \to N_n \to N \xrightarrow{\tilde{\beta_2}} G \to 0$ is exact.
Let $x \in M_n$. Since v is onto , there exists an element $(y, t) \in (P \oplus A)_n$ such

that $v(y,t) = x$. Since $(P \oplus A)_n = P_n \oplus (e_G)$ for $n \geq 1$, we have $y \in P_n$ and $t = e_G$. Now $v(y,t) = \beta(y)t = \beta(y) \in N_n$. For $y \in \pi_n$ implies that $\beta(y) \in N_n$. Hence we have proved that $M_n \in N_n$. As $N_n \subseteq M_n$, it follows that $M_n = N_n$.

Remark. Let $0 \to A \to M \to G \to 0$ be a central extension of A by a group G of class n. Then $M_n \subseteq A$ and we have a central extension

$$0 \to A/M_n \to M/M_n \to G \to 0$$

Theorem 2.4 .*A central extension ,*$0 \to A \to M \to G \to 0$*, with a continuous section, of A by a group of class n is an induced central extension if and only if the central extension* $0 \to A/M_n \to M/M_n \to G \to 0$ *is a split exact.*

Proof. Let $(E) : 0 \to A \to M \to G \to 0$ be an induced central extension , with a continuous section, of A by a group G of class n. By proposition 2.3 , M has an open subgroup N such that $N_n = M_n$ and the following diagram is commutative:

$$(**) \qquad \begin{array}{ccccccccc} 0 & \to & N_n & \to & N & \stackrel{\beta|_N}{\to} & G & \to & 0 \\ & & \downarrow i & & \downarrow i & & \| & & \\ 0 & \to & A & \to & M & \stackrel{\beta}{\to} & G & \to & 0 \end{array}$$

Let $[\zeta] \in H^2(G,A)$, which corresponds to the given extension (E) . By diagram (**) , ζ is in the image of $i^* : H^2(G,M_n) \to H^2(G,A)$, i^* being the homomorphism induced by the inclusion $i : M_n \to A$.
Let $p^* : H^2(G,A) \to H^2(G,A/M_n)$ be the homomorphism induced by the natural projection $p : A \to A/M_n$. As $p^* \circ i^* = 0$, $p^*(\zeta) = 0$. But $0 \to A/M_n \to M/M_n \to G \to 0$ is the extension corresponding to $p^*(\zeta)$. Hence this extension splits. Conversely, assume that the extension $0 \to A/M_n \to M/M_n \to G \to 0$ splits . Since $i^* = kerp^*$, then $\zeta \in Imi^*$. Therefore, there exists a group U such that M_n in contained in the center of U , $U/M_n \simeq G$ and there is a commutative diagram

$$\begin{array}{ccccccccc} 0 & \to & M_n & \to & U & \stackrel{\beta_1}{\to} & G & \to & 0 \\ & & \downarrow i & & \downarrow \beta' & & \| & & \\ 0 & \to & A & \to & M & \stackrel{\beta}{\to} & G & \to & 0 \end{array}$$

As G is of class n , $U_n \subseteq M_n$. Let $x \in M_n$. Then by lemma 2.2, there exists $(u,t) \in (U \oplus A)_n$ such that $x = \beta'(u)t$. For $n \geq 1$, $(U \oplus A)_n = U_n \oplus (e_A)$. Hence $t = e_A$ and $u \in U_n$. Since $U_n \subseteq M$, $\beta'(u) = u$. Hence $x \in U_n$. This proves that $M_n = U_n$. Consequently the above diagram shows that the central extension $0 \to A \to M \to G \to 0$ is an induced extension.

Example. Let T be the additive group of rationals mod 1. Then any central extension of T by an abelian group G is an induced central extension. For if $0 \to T \to M \to G \to 0$ is a central extension with a continuous section , then $0 \to T/M_1 \to M/M_1 \to G \to 0$ is a central extension with a section. Each the entries are abelian. As T is divisible abelian group so is T/M_1. So the exact sequence $0 \to T/M_1 \to M/M_1 \to G \to 0$ splits. Now by theorem 2.4 the extension is induced.

3 The Schur multiplier and induced extensions

In this section we find a relation between the Schur multiplier and the induced extension of a topological group.

Let G be a topological group. Notice that corresponding to the central extension

$0 \to A \to M \to G \to 0$, with a continuous section, there is a 5-term exact sequence [1]

$$M(M) \to M(G) \overset{\delta_E}{\to} A \to M/\overline{[M,M]} \to G/\overline{[G,G]} \to 0$$

Let $[\varepsilon] \in H^2(G,A)$ which corresponds to (E): $0 \to A \to M \to G \to 0$ and $\delta_E : M(G) \to A$ be the map associated with (E) and $\tau : M(G) \to M(G/G_n)$ be the continuous homomorphism induced by the natural map $G \to G/G_n$.

In the following G is nilpotent of class n.

Definition 3.1. An extension $0 \to A \to M \to G \to 0$ is of class n if M in nilpotent of class n.

Proposition 3.2 *A central extension* $0 \to A \to M \to G \to 0$ *is of class* n *iff* δ_E *vanishes on* $Ker\ \tau$.

Proof. Let $0 \to R \to F \to G \to 0$ be the free topological presentation of G with a continuous section .We can define a continuous homomorphism $g : F/\overline{[R,F]} \to M$ such that the diagram

$$(3.2) \qquad \begin{array}{ccccccccc} 0 & \to & R/\overline{[R,F]} & \to & F/\overline{[R,F]} & \to & G & \to & 0 \\ & & \downarrow f & & \downarrow g & & \| & & \\ 0 & \to & A & \to & M & \to & G & \to & 0 \end{array}$$

is commutative . Here $f = g_{|R/\overline{[R,F]}}$.

It follows that $M_k = g(F_k^{'})$, $k \geq 2$. Thus M is of class n iff g vanishes on $F_{n+1}^{'} = Ker\ \tau$ [9]. But $g_{|M(G)} = \delta_E$. So the result follows.

Remark. Let $[\varepsilon] \in H^2(G, A)$ which corresponds to
(E): $0 \to A \to M \to G \to 0$. By the universal coefficient theorem [1] we have the exact sequence

$$0 \to Ext(G^{ab}, A) \to H^2(G, A) \xrightarrow{\sigma} Hom(M(G), A) \to 0$$

where the map σ is given by $\sigma(\varepsilon) = f_{|M(G)}$.

Let B be the image of δ_E.

Lemma 3.3 $M(G) = Ker\delta_E + Ker\ \tau$ *iff* $B = M_{n+1}$.

Proof. Consider diagram (3.2). By [V]

$$M_{n+1} = g(F'_{n+1}) = \delta_E(ker\delta_E + Ker\tau)$$

Now if $M(G) = ker\delta_E + Ker\tau$ then M_{n+1} is the image of δ_E. Conversely, suppose $B = M_{n+1}$. The commutative diagram

$$\begin{array}{ccccccccc} 0 & \to & A & \to & M & \to & G & \to & 0 \\ & & \downarrow i & & \| & & \downarrow p & & \\ 0 & \to & M_nA & \to & M & \to & G/G_n & \to & 0 \end{array}$$

where $G \to G/G_n$ is the natural projection,induces a commutative diagram

$$(3.3) \quad \begin{array}{ccccccccccc} M(M) & \to & M(G) & \xrightarrow{\delta_E} & A & \to & M^{ab} & \to & G^{ab} & \to & 0 \\ \| & & \downarrow \tau & & \downarrow \lambda & & \| & & \| & & \\ M(M) & \to & M(G)/G_n & \to & M_nA/M_{n+1} & \to & M^{ab} & \to & G^{ab} & \to & 0 \end{array}$$

The rows are exact. The hypothesis shows that $\lambda\delta_E$ is the zero map, and an easy diagram chasing shows that $M(G) = Ker\delta_E + Ker\tau$.

Proposition 3.4 *If the extension* (*E*) *is induced then* $M(G) = Ker\delta_E + Ker\tau$.

Proof. By theorem 2.4, if (E): $0 \to A \to M \to G \to 0$ is an induced extension then $\lambda^*(\varepsilon) = 0$ where $\lambda^* : H^2(G, A) \to H^2(G, A/M_{n+1})$ is the homomorphism induced by the natural projection $\lambda : A \to A/M_{n+1}$. We have the commutative diagram

$$(3.4) \quad \begin{array}{ccccccccc} 0 & \to & Ext(G^{ab}, A) & \to & H^2(G, A) & \xrightarrow{\sigma} & Hom(M(G), A) & \to & 0 \\ & & \downarrow \lambda^* & & \downarrow \lambda^* & & \downarrow \lambda^* & & \\ 0 & \to & Ext(G^{ab}, A/M_{n+1}) & \to & H^2(G, A/M_{n+1}) & \xrightarrow{\sigma} & Hom(M(G), A) & \to & 0 \end{array}$$

where the vertical maps are all induced by λ . Now $\lambda\delta_E = 0$

Hence in (3.3), $\lambda\delta_E : M(G) \to A \to M_n A/M_{n+1}$ is the zero map. The result follows from the commutativity of (3.2).

Remark. The following example [13] shows that the converse of proposition 3.4 is not true in general.

Example . Let G be a non-abelian group(with discrete topology) of order p^3, p a prime and $A = Z_p$, the cyclic group of order p. Then $EXt(G^{ab}, Z_p) \neq 0$.

$$Z_p \to M \to G^{ab} \to 0$$

Let $\pi : Ext(G^{ab}, A) \to H^2(G, A)$ be as in the universal coefficient theorem and ε be the image of a non-zero element of $Ext(G^{ab}, A)$. Let E_1 be the central extension which corresponds to ε. By the definition of π it follows that E_1 is of class n and hence is not induced. On the other hand $Ker\delta_{E_1} = \sigma(\varepsilon) = 0$ and so $M(G) = Ker\delta_{E_1} + Ker\tau$ is trivially satisfied.

Recall that G is a perfect topological group if $G = \overline{[G, G]}$.

Example [3]. Let $\widetilde{SL_2(R)}$ be the universal covering of SL_2R ,special real linear group. Let $\alpha \in SL_2(R)$ be central element of infinite order. Let α be irrational rotation in the circle group T. Consider the group

$$G = \widetilde{SL_2(R)}/ < \alpha, \alpha >$$

then G is topologically perfect .

If G is a perfect topological group then we have the following result

Theorem 3.5 *Let G be a perfect topological group .Then*
$(E) : 0 \to A \to M \to G \to 0$ *is induced iff* $M(G) = Ker\delta_E + Ker\ \tau$.

Proof. Let $M(G) = Ker\delta_E + Ker\tau$. By lemma 3.3 ,$B = M_{n+1}$ and by (3.3) $\lambda\delta_E = 0$. Hence ,by commutativity of diagram (3.3), $\sigma\lambda^* = 0$. Since G is perfect $Ext(G^{ab}, A) = 0$ and $Ext(G^{ab}, A/B) \to Ext(G^{ab}, A/B)$ is an epimorphism, $\sigma : H^2(G.A/B) \to Hom(M(G), A/B)$ is an isomorphism . Therefore, $\lambda^*(\zeta) = 0$ in $H^2(G, A/M_{n+1})$ and (E) is induced.

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