Int. J. Contemp. Math. Sciences, Vol. 2, 2007, no. 30, 1471 - 1480

On the Existence of ϕ-Solvability of Boundary Value Problems

Rovshan Z. Humbataliev

Institute of Mathematics and Mechanics of NAS of Azerbaijan
Az 1141, F.Agaev str. 9., Baku, Azerbaijan
elshadent@yahoo.com

Abstract

Sufficient conditions are found for ϕ-solvability of boundary value problems for a class of higher order differential equations whose main part contains a multi charachteristics.

Mathematics Subject Classification: 34K10, 32A10, 46E30

Keywords: Boundary value problems, holomorfic functions, spaces of measurable functions

1^0. Introduction

In the present paper, using the method in the paper [1] we study the existence of holomorphic solution of the operator-differential equations

$$P\left(\frac{d}{dz}\right)u(z) \equiv \left(-\frac{d^2}{dz^2}+A^2\right)^m u(z) + \sum_{j=1}^{2m-1} A_j u^{(2m-j)}(z), \qquad z \in S_{(\alpha,\beta)}, \tag{1}$$

with initial-boundary conditions

$$u^{(S\nu)}(0) = 0, \qquad \nu = \overline{0, m-1}, \tag{2}$$

where A is a positive-definite selfadjoint operator, A_j $(j = \overline{0, m-1})$ are linear operators in an abstract separable space H, $u(z)$ and $f(z)$ are $H-$valued holomorphic functions in the domain

$$S_{(\alpha,\beta)} = \{z/\ -\beta < \arg z < \alpha\}, \quad 0 \le \alpha < \frac{\pi}{2}, \quad 0 \le \beta < \frac{\pi}{2}$$

and the integers s_ν $(\nu=\overline{0,m-1})$ satisfy the conditions

$$0<s_0<s_1<\ldots<s_{m-1}\le m-1.$$

Let H be α separable Hilbert space, A be a positive definite selfadjoint operator in H, and H_γ be a scale of Hilbert spaces generated by the operator A, i.e. $H_\gamma=D(A^\gamma)$, $\|x\|_\gamma=\|A^\gamma x\|$, $x\in D(A^\gamma)$, $\gamma\ge 0$. Denote by $L_2(R_+:H)$ Hilbert space of vector-functions $f(t)$ with values from H, defined in $R_+=(0,+\infty)$, measurable, and for which

$$\|f\|_{L_2(R_+:H)}=\left(\int\limits_0^\infty\|f(t)\|^2\,dt\right)^{\frac12}<\infty.$$

Then, denote by $H_2(\alpha,\beta:H)$ a set of vector-functions $f(z)$ with values from H, that are holomorphic in the sector $S_{(\alpha,\beta)}=\{z/\ -\beta<\arg z<\alpha\}$ and for any $\varphi\in[-\beta,\alpha]$ of the function $f(\xi e^{i\varphi})\in L_2(R_+:H)$. Note that for the vector-function $f(z)$ there exist boundary values $f_{-\beta}(\xi)=f(\xi e^{-i\beta})$ and $f_\alpha(\xi)=f(\xi e^{i\alpha})$ from the space $L_2(R_+:H)$ and we can reestablish the vector-function $f(z)$ with their help by Cauchy formula

$$f(z)=\frac{1}{2\pi i}\int\limits_0^\infty\frac{f_{-\beta}(\xi)}{\xi e^{-i\beta}-z}e^{-i\beta}d\xi-\frac{1}{2\pi i}\int\limits_0^\infty\frac{f_\alpha(\xi)}{\xi e^{i\alpha}-z}e^{i\alpha}d\xi.$$

The linear set $H_2(\alpha,\beta:H)$ becomes a Hilbert space with respect to the norm [1]

$$\|f\|_{(\alpha,\beta)}=\frac{1}{\sqrt2}\left(\|f_{-\beta}\|^2_{L_2(R_+:H)}+\|f_\alpha\|^2_{L_2(R_+:H)}\right)^{\frac12}.$$

Now, define the space $W_2^{2m}(\alpha,\beta:H)$

$$W_2^{2m}(\alpha,\beta:H)=\left\{u/\ A^{2m}u\in H_2(\alpha,\beta:H),\ \ u^{(2m)}\in H_2(\alpha,\beta:H),\right\}$$

with norm

$$\||u|\|_{(\alpha,\beta)}=\left(\|\ A^{2m}u\|^2_{(\alpha,\beta)}+\left\|u^{(2m)}\right\|^2_{(\alpha,\beta)}\right)^{\frac12}.$$

Here and in the sequel, the derivatives are understood in the sense of complex analisys in abstract spaces ([2]).

Definition 1. *The vector-function $u(z)\in W_2^{2m}(\alpha,\beta:H)$ is said to be a regular solution of problem (1),(2), if $u(z)$ satisfies equation (1) in $S_{(\alpha,\beta)}$ identically and boundary conditions are fulfilled in the sense*

$$\lim_{\substack{z\to0\\-\beta<\arg z<\alpha}}\left\|u^{(s_j)}(z)\right\|_{2m-s_j-\frac12}=0,\qquad j=\overline{0,m-1}.$$

Definition 2. *Problem (1),(2) is said to be ϕ-solvable, if for any $f(z) \in H_1 \subset H_2(\alpha,\beta : H)$ there exists $u(z) \in W_1 \subset W_2^{2m}(\alpha,\beta : H)$, which is a regular solution of boundary value problem (1),(2) and satisfies the inequality*

$$|||u|||_{(\alpha,\beta)} \leq const \, \|f\|_{(\alpha,\beta)} ,$$

moreover, the spaces H_1 and W_1 have finite-dimensional orthogonal complements in the spaces $H_2(\alpha,\beta : H)$ and $W_2^{2m}(\alpha,\beta : H)$, respectively.

In the present paper we study the ϕ-solvability of problem (1), (2). The similar problem was investigated in general form in [1], when the principal part doesn't contain a multiple characterics. In the author's paper [3] for $\alpha = \beta = \pi/4$ the one valued and correct solvability conditions of problem (1), (2) are found in the case when the principal part of equation (1) is biharmonic. For simplicity we consider equation (1) with boundary conditions

$$u^{(j)}(0) = 0, \qquad j = \overline{0, m-1}. \tag{3}$$

The general case is considered similarly.

2^0. Some auxiliary facts

First, let's prove some lemmas.

Lemma 1. *The boundary-value problem*

$$P_0\left(\frac{d}{dz}\right) u(z) \equiv \left(-\frac{d^2}{dz^2} + A^2\right)^m u(z) = v(z), \qquad z \in S_{(\alpha,\beta)} \tag{4}$$

$$u^{(j)}(0) = 0, \qquad j = \overline{0, m-1} \tag{5}$$

is regularly solvable.

Proof. It is easily seen that ([1]) the vector function

$$u_0(z) = \frac{1}{2\pi i} \int\limits_{\Gamma} P_0^{-1}(\lambda) \widehat{v}(\lambda) e^{\lambda z} d\lambda \tag{6}$$

satisfies equation (4) identically in $S_{(\alpha,\beta)}$ where $\widehat{v}(\lambda)$ is a Laplace transform of the vector-function $v(z)$:

$$\widehat{v}(z) = \int\limits_0^\infty v(t) e^{-\lambda t} dt,$$

that is an analytic vector-function in the domain

$$\widetilde{S}_{(\alpha,\beta)} = \left\{ \lambda / \ -\frac{\pi}{2} - \alpha < \arg \lambda < \frac{\pi}{2} + \beta \right\}$$

and for $\lambda \in \widetilde{S}_{(\alpha,\beta)}$

$$\|\widehat{v}(\lambda)\| \to 0, \qquad |\lambda| \to \infty, \quad ([4])$$

in formula (6) the integration contour $\Gamma = \Gamma_1 \cup \Gamma_2$, where $\Gamma_1 = \left\{ \lambda / \ \arg \lambda = \frac{\pi}{2} + \beta \right\}$, $\Gamma_2 = \left\{ \lambda / \ \arg \lambda = -\frac{\pi}{2} - \alpha \right\}$. Thus,

$$u_0(z) = \frac{1}{2\pi i} \int_{\Gamma_1} P_0^{-1}(\lambda) \widehat{v}(\lambda) e^{\lambda z} d\lambda - \frac{1}{2\pi i} \int_{\Gamma_2} P_0^{-1}(\lambda) \widehat{v}(\lambda) e^{\lambda z} d\lambda, \qquad z \in S_{(\alpha,\beta)}.$$

On the other hand, it is easy to check that on the rays Γ_1 and Γ_2 it holds the estimate

$$\left\| \lambda^{2m} P_0^{-1}(\lambda) \right\| + \left\| A^{2m} P_0^{-1}(\lambda) \right\| \le const.$$

Then using the analogies of Plancherel formula for a Laplace transform we get $u_0(z) \in W_2^{2m}(\alpha, \beta : H)$. Further, a general regular solution of the equation we seek in the form

$$u(z) = u_0(z) + \sum_{p=0}^{m-1} (zA)^p \, e^{-zA} C_p, \tag{7}$$

where $C_p \in H_{2m-\frac{1}{2}}$, and e^{-zA} is a holomorphic in $S_{(\alpha,\beta)}$ group of bounded operators generated by the operator $(-A)$. Now, let's define the vectors C_p $(p = \overline{0, m-1})$ from condition (5). Then, obviously, for the vectors C_p $(p = \overline{0, m-1})$ we get the following system of equations:

$$\begin{cases} c_0 = -u_0(0), \\ -c_0 + c_1 = -A^{-1} u'(0), \\ c_0 - 2c_1 + 2c_2 = -A^{-2} u''(0), \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ (-1)^{m-1} c_0 + (-1)^{m-2} \begin{pmatrix} 1 \\ m-1 \end{pmatrix} c_1 + \cdots + c_{m-1} = -A^{-m+1} u^{(m-1)}(0). \end{cases}$$

It is evident that the main matrix differs from zero, since it is triangle. Therefore, we can define all the vectors C_p $(p = \overline{0, m-1})$ by a unique way. On the other hand, $u_0^{(j)}(z) \in H_{2m-j-\frac{1}{2}}$, since $u_0^{(j)}(z) \in W_2^{2m}(\alpha, \beta : H)$, therefore the vectors $C_p \in H_{2m-j-\frac{1}{2}}$.

Thus,

$$u(z) = u_0(z) + \sum_{p=0}^{m-1} (zA)^p e^{-zA} \sum_{q=0}^{p} \alpha_{pq} A^{-q} u_0^{(q)}(0). \tag{8}$$

The form $u(z)$ and the trace theorem implies that the inequality

$$\|u\|_{W_2^{2m}(\alpha,\beta:H)} \leq const\, \|v\|_{H_2(\alpha,\beta:H)}$$

holds. The lemma is proved.

For the further study we transform the form of the vector function $u_0(z)$. From formula (6) after simple transformations we get

$$u_0(z) = \int_0^\infty \left(\frac{1}{2\pi i} \int_0^{i\infty} P_0^{-1}(\lambda e^{i\beta}) e^{\lambda(ze^{i\beta}-\xi)} d\lambda \right) v_{-\beta}(\xi) d\xi -$$

$$- \int_0^\infty \left(\frac{1}{2\pi i} \int_0^{-i\infty} P_0^{-1}(\lambda e^{-i\alpha}) e^{\lambda(ze^{-i\alpha}-\xi)} d\lambda \right) v_\alpha(\xi) d\xi =$$

$$= \int_0^\infty G_1(ze^{i\beta} - \xi) v_{-\beta}(\xi) d\xi - \int_0^\infty G_2(ze^{-i\alpha} - \xi) v_\alpha(\xi) d\xi, \tag{9}$$

where

$$v_\alpha(t) = v(te^{i\alpha}), \qquad v_\beta(t) = v(te^{-i\beta})$$

and

$$\left. \begin{aligned} G_1(s) &= \tfrac{1}{2\pi i} \int_0^{i\infty} P_0^{-1}(\lambda e^{i\beta}) e^{\lambda s} d\lambda \\ G_1(s) &= \tfrac{1}{2\pi i} \int_0^{-i\infty} P_0^{-1}(\lambda e^{-i\alpha}) e^{\lambda s} d\lambda \end{aligned} \right\}. \tag{10}$$

Now, let's prove the main result of the paper.

3^0. **The main results**

Theorem 1. *Let A be a positive selfadjoint operator with completely continuous inverse A^{-1}. The resolvent $P^{-1}(\lambda)$ exist on the rays $\Gamma_1 = \left\{\lambda / \ \arg\lambda = \frac{\pi}{2} + \beta\right\}$, $\Gamma_2 = \left\{\lambda / \ \arg\lambda = -\frac{\pi}{2} - \alpha\right\}$ and be iniformly bounded, the operators $B_j = A_j \times A^{-j}$ $(j = \overline{1, 2m-1})$ be completely continuous in H. Then, problem (1),(3) is ϕ-solvable.*

Proof. Write $P(d/dz)$ in the form

$$P(d/dz)u(z) = P_0(d/dz)u(z) + P_1(d/dz)u(z),$$

where

$$P_0(d/dz)u(z) = \left(-\frac{d^2}{dz^2} + A^2\right)^m u(z),$$

$$P_1(d/dz)u(z) = \sum_{j=1}^{2m-1} A_j u^{(2m-j)}(z).$$

Having applied the operator $P(d/dz)$ to both sides of equality (8) we get

$$v(z) = P_0(d/dz)u_0(z) + P_1(d/dz)\sum_{p=0}^{m-1}(zA)^p e^{-zA}\sum_{q=0}^{p}\alpha_{pq}A^{-q}u_0^{(q)}(0). \qquad (11)$$

Passing in equality (11) to the limit as $z \to te^{i\alpha}$ and $z \to te^{-i\beta}$ ($t \in R_+ = = (0,\infty)$) and using for $u_0^{(q)}(0)$ the expressions found from equality (9) allowing for (10) we get the following system of integral equations in the space $L_2(R_+ : H)$

$$\left.\begin{array}{l} v_\alpha(t) + \int\limits_0^\infty (K_2(t-\xi) + K_4(te^{i\alpha},\xi))\, v_\alpha(\xi)d\xi + \\ + \int\limits_0^\infty \left(K_1(te^{i(\alpha+\beta)}-\xi) + K_3(te^{i\alpha},\xi)\right) v_{-\beta}(\xi)d\xi = f_\alpha(t) \\ v_{-\beta}(t) + \int\limits_0^\infty \left(K_1(t-\xi) + K_3(te^{-i\beta},\xi)\right) v_{-\beta}(\xi)d\xi + \\ + \int\limits_0^\infty \left(K_2(te^{-i(\alpha+\beta)}-\xi) + K_4(te^{-i\beta}-\xi)\right) v_\alpha(\xi)d\xi = f_{-\beta}(t) \end{array}\right\} \qquad (12)$$

where

$$K_1(te^{i\beta}-\xi) = P_1(e^{i\beta}d/dt)G_1(te^{i\beta}-\xi);$$

$$K_2(te^{-i\alpha}-\xi) = P_1(e^{-i\alpha}\frac{d}{dt})G_2(te^{-i\alpha}-\xi);$$

$$K_3(t,\xi) = -P_1(e^{i\beta}d/dt)\sum_{p=0}^{m-1}(te^{-i\beta}A)^p e^{-te^{-i\beta}A}\sum_{q=0}^{p}\alpha_{pq}A^{-q}G_1^{(q)}(-\xi),$$

$$K_4(t,\xi) = P_1(e^{-i\alpha}d/dt)\sum_{p=0}^{m-1}(te^{-i\beta}A)^p e^{-te^{i\alpha}A}\sum_{q=0}^{p}\alpha_{pq}A^{-q}G_2^{(q)}(-\xi).$$

Since the operator $P_0(d/dz)$ maps isomorphically the domain $\overset{0}{W}{}_2^{2m}(\alpha,\beta : H)$ onto $H_2(\alpha,\beta : H)$ where $\overset{0}{W}{}_2^{2m}(\alpha,\beta : H) = \{u(z)/\ u(z) \in W_2^{2m}(\alpha,\beta : H),\ u^{(j)}(0) = 0,\ j = \overline{0, m-1}\}$, then the ϕ-solvability of problem (1), (3) is equivalent to the ϕ-solvability of a system of integral equations (12) in $L_2(R_+ : H)$. Therefore, we study the ϕ-solvability of the system of integral equations (12) in $L_2(R_+ : H)$. Since $P^{-1}(\lambda)$ exists on the rays Γ_1 and Γ_2, then each equation

$$\widetilde{v}(t) + \int_{-\infty}^{+\infty} K_j(t-\xi)\widetilde{v}(\xi)d\xi = \widetilde{f}(\xi), \qquad j = 1, 2$$

is correctly and uniquely solvable in the space

$$L_2(R : H) = L_2(R : H) \oplus L_2(R : H)$$

where $\widetilde{f}(t) \in L_2(R : H)$, $\widetilde{v}(t) \in L_2(R : H)$. Therefore, for ϕ-solvability of the system of integral equations,

$$\left.\begin{array}{r} v_\alpha(t) + \int_0^{+\infty} K_2(t-\xi)v_\alpha(\xi)d\xi = f_\alpha(\xi) \\ v_{-\beta}(t) + \int_0^{+\infty} K_1(t-\xi)v_{-\beta}(\xi)d\xi = f_{-\beta}(\xi) \end{array}\right\}$$

in the space $L_2(R_+ : H)$ it suffices to prove that the kernels $K_1(t+\xi)$ and $K_2(t+\xi)$ generate completely continuous operators in $L_2(R : H)$. Then, to prove the ϕ-solvability of the system of integral equations (12) in the space $L_2(R_+ : H)$, we have to prove that the kernels $K_1(te^{i(\alpha+\beta)}-\xi)$, $K_2(te^{-i(\alpha+\beta)}-\xi)$, $K_3(te^{i\alpha},\xi)$, $K_4(te^{i\alpha},\xi)$, $K_3(te^{-i\beta},\xi)$, $K_4(te^{-i\beta},\xi)$ also generate completely continuous operators in $L_2(R_+ : H)$. The proof of complete continuity of these operators is similar. Therefore, following [1] we shall prove the complete continuity of the operator generated by the kernel $K_1(t+\xi)$. Since

$$K_1(t+\xi) = \sum_{j=1}^{2m-1} A_{2m-j}e^{ij\beta}\frac{d^j}{dt^j}\left(\frac{1}{2\pi i}\int_0^{i\infty}(-\lambda^2 e^{2i\beta}E + A^2)^{-m}e^{\lambda(t+\xi)}d\lambda\right),$$

and taking into account that for $\lambda \in (0, i\infty)$ and for sufficiently small sector adherent to the axis $i\infty$ it holds the estimation

$$\left\|(-\lambda^2 e^{2i\beta}E + A^2)^{-m}\right\| \leq const(1+|\lambda|)^{-2m},$$

we can represent $K_1(t+\xi)$ in the form

$$K_1(t+\xi) = \sum_{j=1}^{2m-1} A_{2m-j}e^{ij\beta}\frac{d^j}{dt^j}\left(\frac{1}{2\pi i}\int_0^{(i-\xi)\infty}(-\lambda^2 e^{2i\beta}E + A^2)^{-m}e^{\lambda(t+\xi)}\right)d\lambda =$$

$$= \sum_{j=1}^{2m-1} \frac{B_{2m-j}}{2\pi i} \int\limits_{0}^{(i-\xi)\infty} \lambda^{2m-j} A^j (-\lambda^2 e^{2i\beta} E + A^2)^{-m} e^{\lambda(t+\xi)} d\lambda$$

$$\equiv \frac{1}{2\pi i} \sum_{j=1}^{2m-1} B_{2m-j} K_{1,j}(t+\xi),$$

where $\varepsilon > 0$ is sufficiently small number, and

$$K_{1,j}(t+\xi) = \int\limits_{t,\varepsilon>0}^{(i-\xi)\infty} \lambda^{2m-j} A^j (-\lambda^2 e^{2i\beta} E + A^2)^{-m} e^{\lambda(t+\xi)} d\lambda.$$

Then

$$\|K_{1,j}(t+\xi)\|_{H\to H} = \left\| \int\limits_{0}^{(i-\xi)\infty} \lambda^{2m-j} A^j (-\lambda^2 e^{2i\beta} E + A^2)^{-m} e^{\lambda(t+\xi)} d\lambda \right\| =$$

$$= \left\| \int\limits_{0}^{\infty} (i-\varepsilon)^{2m+1-j} \lambda^{2m-j} A^j (-\lambda^2 (i-\varepsilon)^2 e^{2i\beta} E + A^2)^{-m} e^{-\varepsilon\lambda(t+\xi)} e^{i\lambda(t+\xi)} d\lambda \right\| \leq$$

$$\leq |(i-\varepsilon)| \int\limits_{0}^{\infty} \left\| ((i-\varepsilon)\lambda)^{2m-j} A^j (-\lambda^2 (i-\varepsilon)^2 e^{2i\beta} E + A^2)^{-m} e^{-\varepsilon\lambda(t+\xi)} \right\| d(\lambda\varepsilon) \leq$$

$$\leq C_\varepsilon \int\limits_{0}^{\infty} e^{-\varepsilon\lambda(t+\xi)} d(\lambda\varepsilon) \leq \frac{C_\varepsilon}{t+\varepsilon}.$$

Using Hilbert's inequality [5] we get from the last inequality that $K_{1,j}(t+\xi)$ generates a continuous operator in $L_2(R_+ : H)$. To prove the complete continuity of the operator generated by the operator $B_{2m-j}K_{1,j}(t+\xi)$ we act as follows. Let $\{e_n\}$ be an orthonormal system of eigen vector of the operator A responding to $\{\mu_n\} : Ae_n = \mu_n e_n$, $0 < \mu_1 < \ldots < \mu_n < \ldots$ and let $L_m = \sum\limits_{i=1}^{m} (\cdot, e_i) e_i$ be an orthogonal projector on a sub-space generated by the first m vectors. Since B_{2m-j} is a completely continuous operator, then as $m \to \infty$

$$\|Q_{m,j}\|_{H\to H} = \|B_j - B_j L_m\|_{H\to H} \to 0.$$

On the other hand

$$\|B_j L_m K_{1,j}(t+\xi)\| =$$

$$= \left\| \sum_{n=1}^{m} \int_0^{(i-\xi)\lambda} \lambda^{2m-j}(i-\varepsilon)^{2m+1-j}\mu_n^j(-\lambda^2(i-\varepsilon)^2 e^{2i\beta} + \mu_n^2)^{-m}(\cdot, e_n)B_j e_n e^{i\lambda(t+\xi)} e^{-\lambda\varepsilon(t+\xi)} d\lambda \right\| \leq$$

$$\leq C_\varepsilon \sum_{n=1}^{m} \int_0^{\infty} \frac{|(\varepsilon\lambda)^{2n-j}|\mu_n^j}{|-\lambda^2(i-\varepsilon)^2 e^{2i\beta} + \mu_n^2|} e^{-\varepsilon\lambda(t+\xi)} d(\lambda\varepsilon) \leq C_\varepsilon(m) \int_0^{\infty} \frac{\lambda^{2m-j}}{1+\lambda^{2m}} e^{-\lambda(t+\xi)} d\lambda.$$

Hence, it follows that the kernel $B_{2m-j} L_m K_{1,j}(t+\xi)$ generates a Hilbert-Schmidt operator, since for $j = \overline{1, 2m-1}$ the following inequality holds

$$\int_0^{\infty}\int_0^{\infty} \|B_j L_m K_{1,j}(t+\xi)\|^2 d\xi dt \leq$$

$$\leq \int_0^{\infty}\int_0^{\infty} \left(\int_0^{\infty} \frac{\lambda^{2m-j}}{1+\lambda^{2m}} e^{-\lambda(t+\xi)} \int_0^{\infty} \frac{s^{2m-j}}{1+s^{2ml}} e^{-s(t+\xi)} ds \right) dt d\xi =$$

$$= 2\int_0^{\infty}\int_0^{\infty} \frac{\lambda^{2m-j} s^{2m-j}}{(1+\lambda^{2m})(1+s^{2m})(1+s)^2} d\lambda ds \leq 2\int_0^{\infty}\int_0^{\infty} \frac{\lambda^{2m-1-j} s^{2m-1-j}}{(1+\lambda^{2m})(1+s^{2m})} d\lambda ds =$$

$$= 2\int_0^{\infty} \frac{\lambda^{2m-1-j}}{1+\lambda^{2m}} d\lambda \int_0^{\infty} \frac{s^{2m-1-j}}{1+s^{2m}} ds < 0 \qquad (j = \overline{1, 2m-1}).$$

On the other hand

$$B_{2m-j} K_{1,j}(t+\xi) = Q_{m,j} K_{1,j}(t+\xi) + B_{2m-j} L_m K_{1,j}(t+\xi)$$

then the boundedness of the operator $\widetilde{K_{1,j}}$ generated by the kernel implies that the operator $\widetilde{T_{1,j}}$ generated by the kernel $B_{2n-j} K_{1,j}(t+\xi)$ is the limit of completely continuous operators $T_{1,j,m}$ generated by the kernels $B_j L_m K_{1,j}(t+\xi)$. In fact the difference operators

$$\left\| \widetilde{T_{1,j}} - T_{1,j,m} \right\|_{L_2(R_+:H)\to L_2(R_+:H)} \leq \|Q_{m,j}\| \left\| \widetilde{K_{1,j}} \right\|_{L_2(R_+:H)\to L_2(R_+:H)} \to 0 \ \ (m \to \infty)$$

Thus, $B_{2m-j}K_{1,j}(t+\xi)$ generates a completely continuous operator in $L_2(R_+ : H)$. Since $K_1(t+\xi)$ generates a completely continuous operator in $L_2(R_+ : H)$. The theorem is proved.

References

[1] M.G.Gasymov. On solvabilty of boundary value problems for a class of operator differential equations. Soviet Nat. Dokl., 1977, v.235, No3, pp.505-508. (Russian)

[2] J.-L.Lions, E.Majenes. Inhomogeneous boundary value problems and their applications. M. Mir, 1971, 371 p.

[3] R.Z.Humbataliyev. On holomorphic solutions for a class of operator differential biharmonic equations. Izv. AN Azerb. ser. fiz.-math. nauk, v. XVIII, No 4-5, 1997, pp. 63-73. (Russian)

[4] Jn.V.Sidirov, M.V.Federjuk, M.I.Shabunin. Lectures on thory of complex variable functions. M. Nauka, 1989, 477p.

[5] G.T.Hardy, D.E.Littlewood, G.Polia. Inequalities. M. IL, 1948, 456p.

Received: August 21, 2007