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Rational curves in grassmannians and their Plücker embeddings: an application

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Abstract. Here we prove a corollary of [1, Thm. 1], providing sufficient condition for the existence of α-stable coherent systems of type (n, d, k) for some $k > n$.

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1. Introduction

Let X be a smooth and connected projective curve. A coherent system on X is a pair (E, V) such that E is a vector bundle on X and $V \subseteq H^0(X, E)$ is a linear subspace. The pair (E, V) is of type (n, d, k) if $\mathrm{rank}(E) = n$, $\deg(E) = d$ and $\dim(V) = k$. Fix $\alpha \in \mathbb{R}$. Let $\mu(E) := d/n$ denote the slope of E. Set $\mu_\alpha(E, V) := \mu(E) + \alpha k/n$. The real number μ_α is called the α-slope of the pair (E, V). A coherent subsystem $(F, W) \subseteq (E, V)$ is a coherent system such that $F \subseteq E$ and $W \subseteq V \cap H^0(X, F)$. The pair (E, V) is said to be α-stable (resp. α-semistable) if $\mu_\alpha(F, W) < \mu_\alpha(E, V)$ (resp. $\mu_\alpha(F, W) \le \mu_\alpha(E, V)$) for all proper coherent subsystems (F, W) of (E, V). For the general theory of coherent systems and several results on the moduli schemes of α-stables coherent systems see [7], [4], [2], [5], [6] and [3]. Here we prove a corollary of [1, Thm. 1], providing sufficient conditions for the existence of α-stable coherent systems of type (n, d, k) for some $k > n$.

Proposition 1. *Fix $\alpha \in \mathbb{R}$ and integers integers $n \ge 2$, $a_1 \ge \cdots \ge a_n > 0$ and k such that $\binom{k}{n} \le 1 + na_n$ and $\alpha > (na_1 - \sum_{i=1}^n a_i)/(k - n)$. Set $E := \oplus_{i=1}^n \mathcal{O}_{\mathbf{P}^1}(a_i)$ and take a general k-dimensional linear subspace V of $H^0(\mathbf{P}^1, E)$. Then the coherent system (E, V) is α-stable. Furthermore, for all coherent subsystems (F, W) of (E, V)*

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such that $1 \leq$ *rank*$(F) < n$ *we have* $\mu_\alpha(E,V) - \mu_\alpha(F,W) \geq (\sum_{i=1}^{n} a_i)/n + (k-n)\alpha/n - a_n$

Proof. By [1] for all integers r such that $1 \leq r < n$ and all rank r subsheaves F of E we have $\dim(V) \cap H^0(\mathbf{P}^1, F)) \leq r$. Since $\mu_+(E) = a_1$, we have $\mu(F) \leq a_1$. Thus $\mu_\alpha(F,W) \leq a_1 + \alpha < (\sum_{i=1}^{n} a_i)/n + (k/n)\alpha$, concluding the proof. □

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