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On the Decomposition of δ^*-β-I-*open Set* and Continuity in the Ideal Topological Spaces

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Abstract

In this paper, we define δ^*-β-I-*open set*, δ^*_β-B-*set* and δ^*_β-t-*set* , and also investigate relationships between these sets and other sets given in literature. By using these sets, we obtained new decomposition of continuity.

Mathematics Subject Classification: 54C08, 54C10

Keywords: δ^*-β-I-*open set*, δ^*_β-B-*set*, δ^*_β-t-*set*

1 Introduction

Mashhour, Abd El-Monsef And El-Deeb [1], in 1982, introduced the notion of pre-open set in topological spaces. Raychaudhuri and Mukherjee [7], in 1993, defined the notions of δ-preopen set and δ-almost continuity in topological spaces. Hatir and Noiri [4], in 2006, described the concepts of δ-β-open, δ_β-t-set, δ_β-B-set, δ-β-continuity and δ_β-B-continuity and obtained decompositions of continuity and comlete continuity. Then, Ekici E. [3], in 2009, defined new classes of sets called β^*-I-open set, pre*-I-open set, strongly t-I-set, β^*-t-I-set, strongly B-I-set and B^*-I-set and obtained new decomposition of continuity in ideal topological spaces.

In this paper, we introduce the notions δ^*-β-I-open set, δ^*_β-B-set and δ^*_β-t-set. Also, we investigate further their important properties. By using these sets, we obtain new decomposition of continuity.

2 Preliminaries

Throughout the present paper, X and Y are always mean topological spaces. Let A be a subset of a topological space (X, τ). A subset A is said to be regular open (resp. regular closed) if $A = Int(Cl(A))$ $(resp. A = Cl(Int(A)))$, where $Cl(A)$ and $Int(A)$ point out the closure and the interior of A, respectively. In [6], a point $x \in X$ is called a δ-cluster point of A if $A \cap V \neq \varnothing$ for every regular open set V containing x. The set of all δ-cluster point of A is called the δ-closure of A and denoted by $Cl_\delta(A)$. If $Cl_\delta(A) = A$, then A is said to be δ-closed. The complement of a δ-closed set is said to be δ-open. The set $\{x \in X : x \in V \subset A$ for some regular open set V of $X\}$ is called the δ-interior of A and is denoted by $Int_\delta(A)$. In [2], Janković and Hamlett is defined an ideal I on a topological space (X, τ), such that I is nonemty collection of subsets of X satisfying the following two conditions:

$$A \in I \text{ and } B \subset A \text{ implies } B \in I,$$
$$A \in I \text{ and } B \in I \text{ implies } A \cup B \in I.$$

if I is an ideal on X, then (X, τ, I) is called an ideal topological space or simply an ideal space. A local function [5] of A with respect to τ and I is defined follows: $A^*(I, \tau) = \{x \in X : U \cap A \notin I, \text{ for every } x \in U \text{ and } U \in \tau\}$, for $A \subset X$. It is well known that $Cl^*(A) = A \cup A^*(I, \tau)$ describes a Kuratowski closure operator $Cl^*(.)$ for a topology $\tau^*(I, \tau)$,[2]. When there is no chance for confisuon, we will simply write A^* for $A^*(I, \tau)$ and τ^* for $\tau^*(I, \tau)$.

A subset A of an ideal space (X, τ, I) is said to be R-I-open [8] if $A = Int(Cl^*(A))$. A point x in ideal space (X, τ, I) is called a δ-I-cluster point of A if $A \cap Int(Cl^*(V)) \neq \varnothing$ for each neighborhood V is of x. The set of all δ-I-cluster points of A is called δ-I-clusure of A and is denoted by $Cl_{\delta I}(A)$. A is said to be δ-I-closed [8] if $Cl_{\delta I}(A) = A$.

Definition 1 *A subset A of in ideal topological space (X, τ, I) is said to be*

a) pre-open [1] if $A \subset Int(Cl(A))$,

b) δ-pre-open [7] if $A \subset Int(Cl_\delta(A))$,

c) pre*-I-open [3] if $A \subset Int(Cl_{\delta I}(A))$,

d) β^*-I-open [3] if $A \subset Cl^*(Int(Cl_\delta(A)))$,

e) δ-β-open [4] if $A \subset Cl(Int(Cl_\delta(A)))$,

f) strongly t-I-set [3] if $Int(A) = Int(Cl_{\delta I}(A))$,

g) strongly B-I-set [3] if there exist a $U \in \tau$ and a strongly t-I-set V in X such that $A = U \cap V$,

h) β^*-t-I-set [3] if $Int\left(A\right) = Cl^*\left(Int\left(Cl_\delta\left(A\right)\right)\right),$

ı) B^*-I-set [3] if there exist a $U \in \tau$ and a β^*-t-I-set V in X such that $A = U \cap V$,

j) δ_β-t-set [4] if $Int\left(A\right) = Cl\left(Int\left(Cl_\delta\left(A\right)\right)\right),$

k) δ_β-B-set [4] if there exist a $U \in \tau$ and a δ_β-t-set V in X such that $A = U \cap V$.

Lemma 1 *(Janković and Hamlett, [2]) Let (X, τ, I) be an ideal topological space and A, B be subset of X.*

a) $If\ A \subset B$, then $Cl^*(A) \subset Cl^*\left(B\right)$

b) $\left(A \cap B\right)^* \subset A^* \cap B^*$

d) $A^* = Cl(A^*) \subset Cl(A)$

c) $\left(A \cup B\right)^* \subset A^* \cup B^*$

e) $If\ U \in \tau,\ then\ U \cap A^* \subset \left(U \cap A\right)^*$

Lemma 2 *([8]) Let (X, τ, I) be an ideal topological space and $A, B \subset X$.*

a) $A \subset Cl_{\delta I}(A)$

b) $if\ A \subset B,\ then\ Cl_{\delta I}(A) \subset Cl_{\delta I}(B).$

Lemma 3 *([9])Let (X, τ, I) be an ideal topological space and $A^* \subset A$, then $A^* = Cl(A^*) = Cl^*\left(A\right) = Cl(A).$*

3 δ^*-β-I-sets

Definition 2 *A subset A of in ideal topological space (X, τ, I) is said to be δ^*-β-I-open if $A \subset Cl^*\left(Int\left(Cl_{\delta I}\left(A\right)\right)\right).$ The complement of a δ^*-β-I-open set is said to be δ^*-β-I-closed set.*

Proposition 4 *For a subset of in ideal topological space the following satisfiys:*

a) Every pre*-I-open set is δ^*-β-I-open.

b) Every δ^*-β-I-open set is β^*-I-open.

c) Every β^*-I-open set is δ-β-open.

Proof.

a) Let A be a pre*-I-open set. Then, we write

$$A \subset Int\left(Cl_{\delta I}\left(A\right)\right) \subset Cl^*\left(Int\left(Cl_{\delta I}\left(A\right)\right)\right).$$

This shows that A is δ^*-β-I-open set.

Proof of (b) and (c) done similar to (a)

■

Remark 1 *For several sets defined above, the following diagram holds for a subset A of an ideal topological space (X, τ, I):*

$$\begin{array}{ccccc} & & \delta^*\text{-}\beta\text{-}I\text{-open} \longrightarrow & \beta^*\text{-}I\text{-open} \longrightarrow & \delta\text{-}\beta\text{-open} \\ & & \uparrow & \uparrow & \\ \text{open} \longrightarrow \text{pre-open} \longrightarrow & & \text{pre}^*\text{-}I\text{-open} \longrightarrow & \delta\text{-pre-open} & \end{array}$$

None of these suggestions is inversible, as exhibited in the following examples.

Example 1 *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$ and $I = \{\emptyset, \{c\}\}$. Then the set $A = \{a, b, d\}$ is both δ^*-β-I-open and β^*-I-open but it is neither pre*-I-open and nor δ-pre-open.*

Example 2 *[3]Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$ and $I = \{\emptyset, \{a\}\}$. Then, the set $A = \{b, c\}$ is both β^*-I-open and δ-pre-open but it is neither δ^*-β-I-open and nor pre*-I-open.*

Example 3 *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$ and $I = \{\emptyset, \{b\}\}$. Then the set $A = \{a, b, d\}$ is δ-β-open but it is not β^*-I-open.*

Proposition 5 *Let (X, τ, I) be ideal topological space.If $A \subset B \subset Cl_{\delta I}(A)$ and B is δ^*-β-I-open, then A is δ^*-β-I-open.*

Proof. Let $A \subset B \subset Cl_{\delta I}(A)$ and B be δ^*-β-I-open set. Then, we write $Cl_{\delta I}(A) = Cl_{\delta I}(B)$ .Thus $A \subset B \subset Cl^*(Int(Cl_{_{\delta I}}(B))) = Cl^*(Int(Cl_{_{\delta I}}(A)))$ and hence A is δ^*-β-I-open. ■

By $\delta^*\beta IO(X)$, we denote the family of all δ^*-β-I-open sets of (X, τ, I).

Proposition 6 *Let (X, τ, I) be ideal topological space and A, B subsets of X. If $U_\alpha \in \delta^*\beta IO(X)$, for each $\alpha \in \Delta$, then $\bigcup \{U_\alpha : \alpha \in \Delta\} \in \delta^*\beta IO(X)$.*

Proof. Since $U_\alpha \in \delta^*\beta IO(X)$, we write $U_\alpha \subset Cl^*(Int\,(Cl_{\delta I}\,(U_\alpha)))$ for each $\alpha \in \Delta$. Thus by using Lemma 1, we obtain

$$\begin{aligned}\bigcup U_\alpha &\subset \bigcup Cl^*(Int\,(Cl_{\delta I}\,(U_\alpha))) \\ &= \bigcup \{(Int\,(Cl_{\delta I}\,(U_\alpha)))^* \cup Int\,(Cl_{\delta I}\,(U_\alpha))\} \\ &\subset (Int\left(Cl_{\delta I}\left(\bigcup U_\alpha\right)\right)^* \cup Int(Cl_{\delta I}\left(\bigcup U_\alpha\right)) \\ &= Cl^*(Int\left(Cl_{\delta I}\left(\bigcup U_\alpha\right)\right)).\end{aligned}$$

This shows that $\bigcup U_\alpha \in \delta^*\beta IO(X)$. ■

Remark 2 *The intersection of any two δ^*-β-I-open sets needn't be a δ^*-β-I-open as given in the following example.*

Example 4 *Let $X = \{a, b, c, d\}$, $\tau = \{\oslash, X, \{b\}, \{c\}, \{b, c\}\}$ and $I = \{\oslash, \{d\}\}$. Then the set $A = \{a, b, d\}$ and $B = \{a, c\}$ are δ^*-β-I-open sets but the set $A \cap B = \{a\}$ is not δ^*-β-I-open.*

- Consequently, the family of sets $\delta^*\beta IO(X)$ doesn't form a topological space.

Lemma 7 *(Jankovič and Hamlett [2]). Let (X, τ, I) be an ideal topological space and B, A subsets of X such that $B \subset A$. Then $B^*(\tau_{|A}, I_{|A}) = B^*(\tau, I) \cap A$.*

Proposition 8 *Let (X, τ, I) be an ideal topological space. If $U \in \tau$ and $A \in \delta^*\beta IO(X)$, then $U \cap A \in \delta^*\beta IO(U, \tau_{|U}, I_{|U})$.*

Proof. If U is open, we write $Int_U(A) = Int(A)$ for any subset A of U.By using this fact and Lemma 7, we write

$$\begin{aligned}U \cap A &\subset U \cap Cl^*(Int\,(Cl_{\delta I}\,(A))) = U \cap [(Int\,(Cl_{\delta I}\,(A)))^* \cup Int\,(Cl_{\delta I}\,(A))] \\ &= [U \cap (Int\,(Cl_{\delta I}\,(A)))^*] \cup [U \cap Int(Cl_{\delta I}\,(A))] \\ &\subset [U \cap (Int(U \cap Cl_{\delta I}\,(A)))^*] \cup [U \cap (Int(U \cap Cl_{\delta I}\,(A)))] \\ &= (Int_U(U \cap Cl_{\delta I}\,(A)))^* \cup (Int_U(U \cap (Cl_{\delta I}\,(A)))) \\ &\subset [U \cap (Int_U((Cl_{\delta I})_U(U \cap A)))^*] \cup [U \cap (Int_U((Cl_{\delta I})_U\,(U \cap A)))] \\ &= Cl_U^*(Int_U((Cl_{\delta I})_U\,(U \cap A))).\end{aligned}$$

■

4 δ^*-*B*-*sets and* δ^*_β-*t*-*sets*

Definition 3 *A subset A of in ideal topological space (X, τ, I) is said to be*

a) δ^*_β-t-set if $Int\,(A) = Cl^*\,(Int\,(Cl_{\delta I}\,(A)))\,,$

b) δ^*_β-B-set if there exist a $U \in \tau$ and a δ^*_β-t-set V in X such that $A = U \cap V$.

Proposition 9 *Let (X, τ, I) be an ideal topological space and A a subset of X is δ^*_β-t-set . Then the following hold:*

a) Let $I = \{\emptyset\}$.Then A is both δ_β-t-set and β^*-t-I-set.

b) Let $I = P(X)$. Then A is strongly t-I-set.

Proof.

a) Let $I = \{\emptyset\}$. For $\forall A \subset X$, we write $Cl_{\delta I}\,(A) = Cl_\delta\,(A)$ and $A^* = Cl(A)$. That is, $Cl^*(A) = A \cup A^* = Cl(A)$. Since A is a δ^*_β-t-set, then

$$\begin{aligned} Int\,(A) &= Cl^*\,(Int\,(Cl_{\delta_I}\,(A))) \\ &= Cl^*\,(Int\,(Cl_\delta\,(A))) \\ &= Cl\,(Int\,(Cl_\delta\,(A)))\,. \end{aligned}$$

Hence, A is both δ_β-t-set and β^*-t-I-set.

b) Let $I = P(X)$. For $\forall A \subset X$, then $A^* = \emptyset$. Since A is a δ^*_β-t-set, then we have

$$\begin{aligned} Int\,(A) &= Cl^*\,(Int\,(Cl_{\delta I}\,(A))) \\ &= Int\,(Cl_{\delta_I}\,(A)) \cup (Int\,(Cl_{\delta_I}\,(A)))^* \\ &= Int\,(Cl_{\delta_I}\,(A))\,. \end{aligned}$$

Thus, A is strongly t-I-set.

■

Proposition 10 *For a subset of in ideal topological space the following hold:*

a) Every δ^*_β-t-set is δ^*_β-B-set.

b) Every β^*-t-I-set is δ^*_β-t-set.

c) Every δ^*_β-t-set is strongly t-I-set.

d) Every δ_β-t-set is β^*-t-I-set.

Proof.

a) Since A is δ^*_β-t-set and $X \in \tau$, proof is obvious.

b) Let A be a β^*-t-I-set. Then, we write $Int\,(A) = Cl^*\,(Int\,(Cl_\delta\,(A)))$. By using $Cl_{\delta I}\,(A) \subset Cl_\delta\,(A)$, we have

$$Int\,(A) \subset Cl^*\,(Int\,(Cl_{\delta I}\,(A))) \subset Cl^*\,(Int\,(Cl_\delta\,(A))) = Int\,(A).$$

Hence, A is δ^*_β-t-set.

Proof of (c) and (d) are done similar to (b).

■

Proposition 11 *For a subset of in ideal topological space the following hold:*

a) Every B^*-I-set is δ^*_β-B-set.

b) Every δ^*_β-B-set is strongly B-I-set.

c) Every δ_β-B-set is B^*-I-set .

Proof. From Definitions 1, 3 and Proposition 10, proofs of (a), (b) and (c) are obvious. ■

Remark 3 *For several sets defined above, the following diagram holds for a subset A of an ideal topological space (X, τ, I):*

$$\begin{array}{ccccccc}
\delta_\beta\text{-}B\text{-set} & \longrightarrow & B^*\text{-}I\text{-set} & \longrightarrow & \delta^*_\beta\text{-}B\text{-set} & \longrightarrow & \text{strongly } B\text{-}I\text{-set} \\
\uparrow & & \uparrow & & \uparrow & & \uparrow \\
\delta_\beta\text{-}t\text{-set} & \longrightarrow & \beta^*\text{-}t\text{-}I\text{-set} & \longrightarrow & \delta^*_\beta\text{-}t\text{-set} & \longrightarrow & \text{strongly } t\text{-}I\text{-set}
\end{array}$$

Remark 4 *The converse of Propositions 10 and 11 needn't be true, as exhibited in the following examples.*

Example 5 *[3]Let $X = \{a, b, c\}, \tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$ and $I = \{\emptyset, \{a\}\}$. Then the set $A = \{b, c\}$ is both δ^*_β-t-set and δ^*_β-B-set but it is neither β^*-t-I-set nor B^*-I-set.*

Example 6 *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$ and $I = \{\emptyset, \{d\}\}$. Then the set $A = \{a, b, d\}$ is both strongly t-I-set and strongly B-I-set but it is neither δ^*_β-t-set nor δ^*_β-B-set.*

Example 7 *Let* $X = \{a,b,c,d\}$, $\tau = \{\oslash, X, \{a\}, \{b,d\}, \{a,b,d\}\}$ *and* $I = \{\oslash, \{a\}\}$. *Then the set* $A = \{a,c\}$ *is both* β^**-t-I-set and* B^**-I-set but it is neither* δ_β*-t-set and nor* δ_β*-B-set.*

Proposition 12 *Every open set in ideal topological space* (X,τ,I) *is* δ^*_β*-B-set.*

Proof. Proof is clearly seen by using Propositions 10, 11 and Definition 3. ■

By δ^*_β-t-(I) , we denote the family of all δ^*_β-t-set of (X,τ,I).

Proposition 13 *Let* $A, B \subset X$ *in ideal topological space* (X,τ,I). *If* A *and* B *are* δ^*_β*-t-set, then* $A \cap B$ *is a* δ^*_β*-t-set.*

Proof. Let A and B be δ^*_β-t-set. Then, we have

$$\begin{aligned} Int(A\cap B) &\subset Int(Cl_{\delta I}(A\cap B) \subset Cl^*(Int(Cl_{\delta I}(A\cap B)) \\ &\subset Cl^*(Int(Cl_{\delta I}(A)\cap Cl_{\delta I}(B))) \\ &= Cl^*(Int(Cl_{\delta I}(A)\cap Int(Cl_{\delta I}(A))) \\ &\subset Cl^*(Int(Cl_{\delta I}(A)\cap Cl^*(Int(Cl_{\delta I}(B))) \\ &= Int(A)\cap Int(B) = Int(A\cap B). \end{aligned}$$

■

Remark 5 *The union of any two* δ^*_β*-t-set needn't be a* δ^*_β*-t-set as given in the following example.*

Example 8 *Let* $X = \{a,b,c\}$, $\tau = \{\oslash, X, \{a\}, \{b\}, \{a,b\}\}$ *and* $I = \{\oslash, \{a\}, \{b\}, \{a,b\}\}$. *Then, the sets* $A = \{a\}$ *and* $B = \{b\}$ *are* δ^*_β*-t-set but the set* $A \cup B = \{a,b\}$ *is not* δ^*_β*-t-set.*

- Consequently, the family of sets δ^*_β-t-(I) doesn't form a topological space.

Remark 6 *In ideal topological space* (X,τ,I), δ^*_β*-B-set needn't be* δ^**-*β*-I-open set as given in the following example.*

Example 9 *In Example 2, the set* $A = \{b,c\}$ *is* δ^*_β*-B-set but it is not* δ^**-*β*-I-open.*

Remark 7 *In ideal topological space* (X,τ,I), δ^**-*β*-I-open set needn't be* δ^*_β*-B-set as seen in the following example.*

Example 10 *In Example 4, the set* $A = \{a,b,d\}$ *is* δ^**-*β*-I-open but it is not* δ^*_β*-B-set .*

Theorem 14 *A subset A of space (X,τ,I) is open set if and only if A is both δ^*-β-I-open and δ^*_β-B-set .*

Proof. $\Rightarrow$: Let A is open set, then necessary condition is obvious from Propositions 10, 11 and Definition 3

$\Leftarrow$: Let A is both δ^*-β-I-open and δ^*_β-B-set, then there exist a $U \in \tau$ and a δ^*_β-t-set V in X such that $A = U \cap V$. Therefore, we write

$$\begin{aligned} A &= U \cap A \subset U \cap Cl^*\left(Int\left(Cl_{\delta I}\left(A\right)\right)\right) \\ &= U \cap Cl^*\left(Int\left(Cl_{\delta I}\left(U \cap V\right)\right)\right) \\ &\subset U \cap Cl^*\left(Int\left(Cl_{\delta I}\left(U\right)\right)\right) \cap Cl^*\left(Int\left(Cl_{\delta I}\left(V\right)\right)\right) \\ &= U \cap Cl^*\left(Int\left(Cl_{\delta I}\left(V\right)\right)\right) = U \cap Int\left(V\right) \\ &= Int\left(U \cap V\right) \\ &= Int\left(A\right). \end{aligned}$$

Thus, A is an open set. ■

5 Decomposition of Continuity

Definition 4 *A function $f : (X,\tau,I) \longrightarrow (Y,\sigma)$ is said to be pre-continuous[1] (resp.δ-almost continuous[7], β^*-I-continuous[3], pre*-I-continuous[3] and δ_β-B-continuous[4]) if for every $V \in \sigma$, $f^{-1}(V)$ is a pre-open (resp. δ-pre-open, β^*-I-open, pre*-I-open and δ_β-B-open) of (X,τ,I).*

Definition 5 *A function $f : (X,\tau,I) \longrightarrow (Y,\sigma)$ is called δ^*-β-I-continuous if for every $V \in \sigma$, $f^{-1}(V)$ is a δ^*-β-I-open of (X,τ,I).*

Remark 8 *For several continuous function defined above, we can give the following diagram.*

$$\begin{array}{ccccc} & & \delta^*\text{-}\beta\text{-}I\text{-continuous} \longrightarrow & \beta^*\text{-}I\text{-continuous} \longrightarrow & \delta\text{-}\beta\text{-continuous} \\ & & \uparrow & \uparrow & \\ \text{pre-continuous} \longrightarrow & & \text{pre}^*\text{-}I\text{-continuous} \longrightarrow & \delta\text{-almost-continuous} & \end{array}$$

Remark 9 *None of these suggestions is inversible, as exhibited in the following examples.*

Example 11 *Let $Y = X = \{a,b,c,d\}$, $\sigma = \tau = \{\emptyset, X, \{b\}, \{c\}, \{b,c\}\}$ and $I = \{\emptyset, \{c\}\}$.
Let $f : (X,\tau,I) \longrightarrow (Y,\sigma)$ be a function defined as follows: $f(a) = b$, $f(b) = b$, $f(c) = c$, $f(d) = b$. Then f is both δ^*-β-I-continuous and β^*-I-continuous but f is neither pre*-I-continuous nor δ-almost continuous.*

Example 12 *[3]Let* $Y = X = \{a, b, c\}$, $\sigma = \tau = \{\oslash, X, \{a\}, \{a, b\}, \{a, c\}\}$ *and* $I = \{\oslash, \{a\}\}$.*Let* $f : (X, \tau, I) \longrightarrow (Y, \sigma)$ *be a function defined as follows:* $f(a) = c$, $f(b) = c$, $f(c) = a$. *Then* f *is both* δ*-almost continuous and* β^**-*I*-continuous but* f *is neither* δ^**-*β*-*I*-continuous nor pre***-*I*-continuous.*

Example 13 *Let* $Y = X = \{a, b, c, d\}$, $\sigma = \tau = \{\oslash, X, \{b\}, \{c\}, \{b, c\}\}$ *and* $I = \{\oslash, \{b\}\}$. *Let* $f : (X, \tau, I) \longrightarrow (Y, \sigma)$ *be a function defined as follows:* $f(a) = b$, $f(b) = b$, $f(c) = c$, $f(d) = b$. *Then* f *is* δ*-*β*-continuous but* f *is not* β^**-*I*-continuous*

Proposition 15 *Let* $f : (X, \tau, I) \longrightarrow (Y, \sigma)$ *be* δ^**-*β*-*I*-continuous and* $U \in \tau$. *Then the restriction* $f_{|U} : (U, \tau_{|U}, I_{|U}) \longrightarrow (Y, \sigma)$ *is* δ^**-*β*-*I*-continuous.*

Proof. Let $V \in \sigma$. Since f is δ^*-β-I-continuous, $f^{-1}(V) \in \delta^*\beta IO(X)$ and Proposition 8 $f_{|U}^{-1} = U \cap f^{-1}(V) \in \delta^*\beta IO(U, \tau_{|U}, I_{|U})$. It is clearly seen that $f_{|U} : (U, \tau_{|U}, I_{|U}) \longrightarrow (Y, \sigma)$ is δ^*-β-I-continuous. ■

Definition 6 *A function* $f : (X, \tau, I) \longrightarrow (Y, \sigma)$ *is said to be*

a) strongly B-I-continuous if $f^{-1}(V)$ is a strongly B-I-set in (X, τ, I) for every $V \in \sigma$ [3].

b) B^*-I-continuous if $f^{-1}(V)$ is a B^*-I-set in (X, τ, I) for every $V \in \sigma$ [3].

c) δ_β-B-continuous if $f^{-1}(V)$ is a δ_β-B-set in (X, τ, I) for every $V \in \sigma$ [4].

Definition 7 *A function* $f : (X, \tau, I) \longrightarrow (Y, \sigma)$ *is called* δ^*_β*-*B*-continuous if* $f^{-1}(V)$ *is a* δ^*_β*-*B*-set in* (X, τ, I) *for every* $V \in \sigma$.

Remark 10 *For several continuous function defined above, we can give the following diagram.*

δ_β-B-continuous → B^*-I-continuous→ δ^*_β-B-continuous→strongly B-I-continuous

Remark 11 *None of these suggestions is inversible, as exhibited in the following examples.*

Example 14 *Let* $Y = X = \{a, b, c, d\}$, $\sigma = \tau = \{\oslash, X, \{a\}, \{b, d\}, \{a, b, d\}\}$ *and* $I = \{\oslash, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$. *Let* $f : (X, \tau, I) \longrightarrow (Y, \sigma)$ *be a function defined as follows:* $f(a) = a$, $f(b) = b$, $f(c) = a$, $f(d) = b$. *Then* f *is* B^**-*I*-continuous but* f *is not* δ_β*-*B*-continuous.*

Example 15 *[3]Let* $Y = X = \{a, b, c\}$, $\sigma = \tau = \{\oslash, X, \{a\}, \{a, b\}, \{a, c\}\}$ *and* $I = \{\oslash, \{a\}\}$. *Let* $f : (X, \tau, I) \longrightarrow (Y, \sigma)$ *be a function defined as follows:* $f(a) = c$, $f(b) = c$, $f(c) = a$. *Then* f *is* δ^*_β*-*B*-continuous but* f *is not* B^**-*I*-continuous*

Example 16 *Let $Y = X = \{a, b, c, d\}$, $\sigma = \tau = \{\oslash, X, \{b\}, \{c\}, \{b, c\}\}$ and $I = \{\oslash, \{d\}\}$. Let $f : (X, \tau, I) \longrightarrow (Y, \sigma)$ be a function defined as follows: $f(a) = b$, $f(b) = b$, $f(c) = c$, $f(d) = b$. Then f is strongly B-I-continuous but f is not δ^*_β-B-continuous.*

Theorem 16 *Let $f : (X, \tau, I) \longrightarrow (Y, \sigma)$ be a function. f function is continuous if and only if f function is both δ^*-β-I-continuous and δ^*_β-B-continuous .*

Proof. Proof is obvious from Theorem 14 and Definition 5. ■

References

[1] A. S. Mashhour, M. E. Abd El-Monsef And S. N. El-Deeb, On pre-continuous and weak pre-continuous mappings, Proc. Math. Phys. Soc. Egypt, vol. 53 (1982), 47-53.

[2] D. Janković and T.R. Hamlett, New topologies from old via ideals, Amer. Math. Monthly, 97 (1990), 295-310.

[3] E. Ekici and T. Noiri, On subsets and decompositions of continuity in ideal topological spaces,The arabian journal for science and engineering, vol(34) (2009), number 1A, 165.

[4] E. Hatir and T. Noiri, Decomposition of continuity and complete continuity, Acta. Math. Hungar., 113 no.(4) (2006), 281-287.

[5] K. Kuratowski, Topology, Vol. 1 (transl.), Academic Press, New York, 1966.

[6] N.V. Velićko, H-closed topological spaces, Am. Math. Soc. Transl. 1968; 2 (78):103–18.

[7] S. Raychaudhuri and M.N. Mukherjee, On δ-almost continuity and δ-preopen sets, Bull. Inst. Math. Acad. Sinica, 21 (1993), 357-366.

[8] S. Yuksel, A. Acıkgoz and T. Noiri, On δ-I-continuous functions, Turk J Math., 29 (2005), 39-51.

[9] V.R. Devi, D. Sivaraj, and T.T. Chelvam, "Codense and Completely Codense Ideals", Acta Math. Hungar.,108 (2005), pp. 195–204.

[10] Y. Yazlik, On the Distribution of δ^*-β-I-Open Set and Continuity in the Ideal Topological Spaces, Master Thesis, Selcuk University, (2009).

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