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Mellin Transform Involving the Product of a General Class of Polynomials, H-Function of One and r Variables

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Abstract

The object of this paper is to establish integrals involving the product of general class of polynomials, H-function of one and r variables. Some special cases have also derived.

Keywords: Mellin transform, H-function of one and ’r’ variables

1 Introduction and Preliminaries

Recently, The Mellin transform of the product of Struve’s function, H-function of one and r variables [7] are evaluated. In the present paper we establish the integral transform of product of general class of polynomials, H-function of one and r variales.

We shall utilize the following formulae in the present investigation. The H-function of r variables given by H.M.Srivastava and R. Panda [5]

(1.1) $H[x_1, x_2, ..., x_r] =$

$$H^{0,n:m_1,n_1;....;m_r,n_r}_{p,q:p_1,q_1;....;p_r,q_r}\left[\begin{array}{c|c} x_1 & (a_j;\alpha_{1j},....,\alpha_{rj})_{1,p} : (c_{1j}, C_{1j})_{1,p_1}; ....; (c_{rj}, C_{rj})_{1,p_r} \\ \cdot & \\ \cdot & \\ \cdot & \\ x_r & (b_j;\beta_{1j},....,\beta_{rj})_{1,q} : (d_{1j}, D_{1j})_{1,q_1}; ....; (d_{rj}, D_{rj})_{1,q_r} \end{array}\right]$$

$$= \frac{1}{(2\pi i)^r}\int\limits_{L_1} .... \int\limits_{L_r} \theta(s_1, ..., s_r) \prod_{k=1}^{r} \phi_k(s_k) x_k^{s_k} ds_k, i = \sqrt{-1}$$

Where

$$\theta(s_1, ..., s_r) = \frac{\prod\limits_{j=1}^{n} \Gamma(1 - a_j + \sum\limits_{k=1}^{r} \alpha_j^{(k)} s_k)}{\prod\limits_{j=n+1}^{p} \Gamma(a_j - \sum\limits_{k=1}^{r} \alpha_j^{(k)} s_k) \prod\limits_{j=1}^{q} \Gamma(1 - b_j + \sum\limits_{k=1}^{r} \beta_j^{(k)} s_k)}$$

$$\phi_k(s_k) = \frac{\prod\limits_{j=1}^{m_k} \Gamma(d_j^{(k)} - D_j^{(k)} s_k) \prod\limits_{j=1}^{n_k} \Gamma(1 - c_j^{(k)} + C_j^{(k)} s_k)}{\prod\limits_{j=m_k+1}^{q_k} \Gamma(1 - d_j^{(k)} + D_j^{(k)} s_k) \prod\limits_{j=n_k+1}^{p_k} \Gamma(c_j^{(k)} - C_j^{(k)} s_k)}, k = 1, ..., r$$

where n, p, q, m_k,n_k, p_k, q_k, $k = 1, 2, ..., r$ are non-negative integers such that $0 \leq n \leq p$, $q \geq 0$, $0 \leq m_k \leq q_k$ and $0 \leq n_k \leq p_k$, $k = 1, 2, ..., r$. $\alpha_j^{(k)}$, $\beta_j^{(k)}$, $C_j^{(k)}$, $D_j^{(k)}$ are all positive.

The contour L_k lies in the complex plane s_k is of Mellin-Barnes type which runs from $-i\infty$ to $+i\infty$ with intendations, if necessary to ensure that all poles of $\Gamma(d_j^{(k)} - D_j^{(k)} s_k)$, $j = 1, 2, ..., n_k$ and $\Gamma(1 - a_j^{(k)} + \sum \alpha_j^{(k)} s_k)$,$j = 1, 2, ..., n$ are to the left of L_k.

The H-function of one variable is defined as follows [3]

(1.2) $H^{m,n}_{p,q}\left[x| \begin{array}{c} (a_j, \alpha_j)_{1,p} \\ (b_j, \beta_j)_{1,q} \end{array}\right] = \frac{1}{2\pi i}\int\limits_{L} \theta(s) x^s ds$

where

$$\theta(s) = \frac{\prod\limits_{j=1}^{m} \Gamma(b_j - \beta_j s) \prod\limits_{j=1}^{n} \Gamma(1 - a_j + \alpha_j s)}{\prod\limits_{j=m+1}^{q} \Gamma(1 - b_j + \beta_j s) \prod\limits_{j=n+1}^{p} \Gamma(a_j - \alpha_j s)}$$

with all conditions detailed in [3]. Mellin transform of the H-function is defined as follows [3],

(1.3) $\int\limits_0^\infty z^{s-1} H^{M,N}_{P,Q}\left[az| \begin{array}{c} (c_j, \gamma_j)_{1,P} \\ (d_j, \delta_j)_{1,Q} \end{array}\right] dz = a^{-s}\theta(-s)$

where

$$\theta(-s)=\frac{\prod_{j=1}^{N}\Gamma((1-c_j)-\gamma_j s)\prod_{j=1}^{M}\Gamma(1-(1-d_j)+\delta_j s)}{\prod_{j=N+1}^{P}\Gamma(1-(1-c_j)+\gamma_j s)\prod_{j=M+1}^{Q}\Gamma((1-d_j)-\delta_j s)}$$

Provided the corresponding conditions stated in [3]. According to Prasanna Kumari [1, p.92]

(1.4)

$$\int_0^\infty x^{s-1}H_{p,q}^{m,n}\left[zx^\sigma\Big|\begin{array}{c}(a_j,A_j)_{1,p}\\(b_j,B_j)_{1,q}\end{array}\right]dx=\frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\frac{s}{\sigma})\prod_{j=1}^{n}\Gamma(1-a_j-A_j\frac{s}{\sigma})}{\sigma z^{\frac{s}{\sigma}}\prod_{j=m+1}^{q}\Gamma(1-b_j-B_j\frac{s}{\sigma})\prod_{j=n+1}^{p}\Gamma(a_j+A_j\frac{s}{\sigma})}$$

The class of polynomials [9]

(1.5)

$$S_n^m[x]=\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}x^k, n=0,1,2,3,....$$

where m is an arbitrary positive integer and the co-efficients $A_{n,k}(n,k\geq 0)$ are arbitrary constants.

2.Main Result:

(2.1)

$$\int_0^\infty x^{s-1}S_n^m[ax^h]H_{p,q}^{m,n}\left[\eta x^\rho\middle|\begin{array}{c}(e_j,E_j)_{1,p}\\(f_j,F_j)_{1,q}\end{array}\right]\times$$

$$H_{P,Q:p_1,q_1;...;p_r,q_r}^{o,N:m_1,n_1;...;m_r,n_r}\left[\begin{array}{c}t_1x^{\sigma_1}\\ \cdot\\ \cdot\\ \cdot\\ t_rx^{\sigma_r}\end{array}\middle|\begin{array}{c}(a_j;\alpha_{1j},...,\alpha_{rj})_{1,P}:(c_{1j},C_{1j})_{1,p_1};...;(c_{rj},C_{rj})_{1,p_r}\\(b_j;\beta_{1j},...,\beta_{rj})_{1,Q}:(d_{1j},D_{1j})_{1,q_1};...;(d_{rj},D_{rj})_{1,q_r}\end{array}\right]dx$$

$$=\frac{1}{\rho}\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}a^k\eta^{-\frac{(s+hk)}{\rho}}$$

$$H_{P+q,Q+p:p_1,q_1;...;p_r,q_r}^{n,N+m:m_1,n_1;...;m_r,n_r}\left[\begin{array}{c}\frac{t_1}{\eta^{\frac{\sigma_1}{\rho}}}\\ \cdot\\ \cdot\\ \cdot\\ \frac{t_r}{\eta^{\frac{\sigma_r}{\rho}}}\end{array}\middle|\begin{array}{l}(a_j;\alpha_{1j},...,\alpha_{rj})_{1,N},(1-f_j-F_j(s+hk);\\(1-e_j-E_j(s+hk);E_j\frac{\sigma_1}{\rho},...,E_j\frac{\sigma_r}{\rho})_{1,p},\end{array}\right.$$

$$\left. \begin{matrix} F_j\frac{\sigma_1}{\rho},...,F_j\frac{\sigma_r}{\rho})_{1,q}(a_j;\alpha_{1j},...,\alpha_{rj})_{N+1,P}:(c_{1j},C_{1j})_{1,p_1};...;(c_{rj},C_{rj})_{1,p_r} \\ (b_j;\beta_{1j},...,\beta_{rj})_{1,Q}:(d_{1j},D_{1j})_{1,q_1};...;(d_{rj},D_{rj})_{1,q_r} \end{matrix} \right]$$

Provided

(i) $\sigma_i > 0$, for i=1,2,...,r, $\sigma > 0$, $\delta > 0$, $h > 0$

(ii) $A_i \leq 0$; $U_i > 0$, $\theta > 0$

$A_i + \frac{1}{\rho}\left[\sum_{j=1}^{q} F_j\sigma_i - \sum_{j=1}^{p} F_j\sigma_i\right] \leq 0,$

$U_i - \frac{1}{\rho}\left[\sum_{j=m+1}^{q} F_j\sigma_i + \sum_{j=1}^{p} F_j\sigma_i\right] > 0,$

where

$\theta = \sum_{j=1}^{n} E_j - \sum_{j=n+1}^{p} E_j + \sum_{j=1}^{m} F_j - \sum_{j=m+1}^{q} F_j$,

$A_i = \sum_{j=1}^{P} \alpha_j^{(i)} + \sum_{j=n+1}^{p_1} C_j^{(i)} - \sum_{j=1}^{Q} \beta_j^{(i)} - \sum_{j=m+1}^{q_1} D_j^{(i)}$,

$U_i = -\sum_{j=N+1}^{P} \alpha_j^{(i)} - \sum_{j=1}^{Q} \beta_j^{(i)} + \sum_{j=1}^{ni} C_j^{(i)} - \sum_{j=ni+1}^{pi} C_j^{(i)} + \sum_{j=1}^{mi} D_j^{(i)} - \sum_{j=mi+1}^{qi} D_j^{(i)},$

$i = 1, ..., r.$

Proof of main result:

Express the H-function of r-variables involved in the left hand side of (2.1) as a contour integral using (1.1) and the general class of polynomialas infinite series(1.5). Change the order of integration and summation and evaluate the inner integral using (1.4), the left hand side of (2.1) becomes,

$$\frac{1}{\rho}\sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} a^k \eta^{-\frac{s+hk}{\rho}} \frac{1}{(2\pi i)^r} \int_{L_1} ... \int_{L_r} \theta(s_1,...,s_r)\phi(s_1)...\phi(s_r)$$

$$\frac{\prod_{j=1}^{m}\Gamma(f_j + F_j(\frac{s+hk+\sigma_1 s_1+...+\sigma_r s_r}{\rho}))\prod_{j=1}^{n}\Gamma(1-e_j-E_j(\frac{s+hk+\sigma_1 s_1+...+\sigma_r s_r}{\rho}))}{\prod_{j=m+1}^{q}\Gamma(1-f_j-F_j(\frac{s+hk+\sigma_1 s_1+...+\sigma_r s_r}{\rho}))\prod_{j=n+1}^{p}\Gamma(e_j+E_j(\frac{s+hk+\sigma_1 s_1+...+\sigma_r s_r}{\rho}))}$$

$$\times\left(\frac{t_1}{\eta^{\frac{\sigma_1}{\rho}}}\right)^{s_1} ... \left(\frac{t_r}{\eta^{\frac{\sigma_r}{\rho}}}\right)^{s_r} ds_1...ds_r$$

where $\phi_i(s_i)$ for i=1,2,...r, $\theta(s_1, s_2, ..., s_r)$ are given by (1.1),from which right hand side of (2.1) is obtained by using (1.1). The change of order of integration is justified, when the given conditions are satisfied because of the absolute convergence of the integral involved.

3. Special cases:

Put h=0, a=1 in (2.1) we get Mellin transform of product of H-function of one variable and H-function of r-variables.

(3.1) $$\int_0^\infty x^{s-1} H_{p,q}^{m,n}\left[\eta x^{\rho}\middle|\begin{matrix}(e_j,E_j)_{1,p}\\(f_j,F_j)_{1,q}\end{matrix}\right]\times$$

$$H_{P,Q:p_1,q_1;...;p_r,q_r}^{0,N:m_1,n_1;...;m_r,n_r}\left[\begin{matrix}t_1x^{\sigma_1}\\ \cdot\\ \cdot\\ \cdot\\ t_rx^{\sigma_r}\end{matrix}\middle|\begin{matrix}(a_j;\alpha_{1j},...,\alpha_{rj})_{1,P}:(c_{1j},C_{1j})_{1p_1};...;(c_{rj},C_{rj})_{1p_r}\\ \\ (b_j;\beta_{1j},...,\beta_{rj})_{1,Q}:(d_{1j},D_{1j})_{1q_1};...;(d_{rj},D_{rj})_{1q_r}\end{matrix}\right]dx$$

$$=\frac{\eta^{\frac{-s}{\rho}}}{\rho}\times$$

$$H_{P+q,Q+p:p_1,q_1;....;p_r,q_r}^{n,N+m:m_1,n_1;....;m_r,n_r}\left[\begin{matrix}\frac{t_1}{\eta^{\frac{\sigma_1}{\rho}}}\\ \cdot\\ \cdot\\ \cdot\\ \frac{t_r}{\eta^{\frac{\sigma_r}{\rho}}}\end{matrix}\middle|\begin{matrix}(a_j;\alpha_{1j},...,\alpha_{rj})_{1,N},(1-f_j-F_js);\\ \\ (1-e_j-E_j(s);E_j\frac{\sigma_1}{\rho},...,E_j\frac{\sigma_r}{\rho})_{1,p},\end{matrix}\right.$$

$$\left.\begin{matrix}F_j\frac{\sigma_1}{\rho},...,F_j\frac{\sigma_r}{\rho})_{1,q},(a_j;\alpha_{1j},...,\alpha_{rj})_{N+1,P}:(c_{1j},C_{1j})_{1,p_1};...;(c_{rj},C_{rj})_{1,p_r}\\ (b_j;\beta_{1j},...,\beta_{rj})_{1,Q}:(d_{1j},d_{1j})_{1,p_1};...;(d_{rj},D_{rj})_{1,q_r}\end{matrix}\right]$$

in (2.1)Put N= 0,to get

(3.2) $$\int_0^\infty x^{s-1} S_n^m[ax^h] H_{p,q}^{m,n}\left[\eta x^{\rho}\middle|\begin{matrix}(e_j,E_j)_{1,p}\\(f_j,F_j)_{1,q}\end{matrix}\right]\times$$

$$H_{P,Q:p_1,q_1;...;p_r,q_r}^{0,0:m_1,n_1;...;m_r,n_r}\left[\begin{matrix}t_1x^{\sigma_1}\\ \cdot\\ \cdot\\ \cdot\\ t_rx^{\sigma_r}\end{matrix}\middle|\begin{matrix}(a_j;\alpha_{1j},...,\alpha_{rj})_{1,P}:(c_{1j},C_{1j})_{1p_1};...;(c_{rj},C_{rj})_{1p_r}\\ \\ (b_j;\beta_{1j},...,\beta_{rj})_{1,Q}:(d_{1j},D_{1j})_{1q_1};...;(d_{rj},D_{rj})_{1q_r}\end{matrix}\right]dx$$

$$=\frac{1}{\rho}\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}a^k\eta^{\frac{-(s+hk)}{\rho}}\times$$

$$H_{P+q,Q+p:p_1,q_1;....;p_r,q_r}^{n,m:m_1,n_1;....;m_r,n_r}\left[\begin{matrix}\frac{t_1}{\eta^{\frac{\sigma_1}{\rho}}}\\ \cdot\\ \cdot\\ \cdot\\ \frac{t_r}{\eta^{\frac{\sigma_r}{\rho}}}\end{matrix}\middle|\begin{matrix}(a_j;\alpha_{1j},...,\alpha_{rj})_{1,P},(1-f_j-F_j(s+hk);\\ \\ (1-e_j-E_j(s+hk);E_j\frac{\sigma_1}{\rho},...,E_j\frac{\sigma_r}{\rho})_{1,p},\end{matrix}\right.$$

$$\left.\begin{matrix} F_j\frac{\sigma_1}{\rho},...,F_j\frac{\sigma_r}{\rho})_{1,q},(a_j;\alpha_{1j},...,\alpha_{rj})_{N+1,P}:(c_{1j},C_{1j})_{1,p_1};...;(c_{rj},C_{rj})_{1,p_r} \\ (b_j;\beta_{1j},...,\beta_{rj})_{1,Q}:(d_{1j},D_{1j})_{1,q_1};...;(d_{rj},D_{rj})_{1,q_r} \end{matrix}\right]$$

put N=P=Q=0 in (2.1) , to get

(3.3)

$$\int_0^\infty x^{s-1}S_n^m[ax^h]H_{p,q}^{m,n}\left[\eta x^\rho \middle| \begin{matrix}(e_j,E_j)_{1,p}\\(f_j,F_j)_{1,q}\end{matrix}\right]\prod_{i=1}^{r}H_{p_i,q_i}^{m_i,n_i}\left[t_i x^{\sigma_i}\middle|\begin{matrix}(c_j^{(i)},C_j^{(i)})_{1,p_i}\\(d_j^{(i)},D_j^{(i)})_{1,q_i}\end{matrix}\right]dx$$

$$=\frac{1}{\rho}\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}a^k\eta^{-\frac{(s+hk)}{\rho}}$$

$$H_{q,p:p_1,q_1;...;p_r,q_r}^{n,m:m_1,n_1;...;m_r,n_r}\left[\begin{matrix}\frac{t_1}{\eta^{\frac{\sigma_1}{\rho}}}\\ \cdot\\ \cdot\\ \cdot\\ \frac{t_r}{\eta^{\frac{\sigma_r}{\rho}}}\end{matrix}\middle|\begin{matrix}(1-f_j-F_j(s+hk);\\ \\ \\ \\ (1-e_j-E_j(s+hk);E_j\frac{\sigma_1}{\rho},...,E_j\frac{\sigma_r}{\rho})_{1,p},\end{matrix}\right.$$

$$\left.\begin{matrix}F_j\frac{\sigma_1}{\rho},...,F_j\frac{\sigma_r}{\rho})_{1,q},:(c_{1j},C_{1j})_{1,p_1};...;(c_{rj},C_{rj})_{1,p_r}\\ :(d_{1j},D_{1j})_{1,q_1};...;(d_{rj},D_{rj})_{1,q_r}\end{matrix}\right]$$

in (2.1)put E_j=F_j=0,we get

(3.4)

$$\int_0^\infty x^{s-1}S_n^m[ax^h]G_{p,q}^{m,n}\left[\eta x^\rho\middle|\begin{matrix}(e_j)_{1,p}\\(f_j)_{1,q}\end{matrix}\right]\times$$

$$H_{P,Q:p_1,q_1;...;p_r,q_r}^{o,N:m_1,n_1;...;m_r,n_r}\left[\begin{matrix}t_1x^{\sigma_1}\\ \cdot\\ \cdot\\ \cdot\\ t_rx^{\sigma_r}\end{matrix}\middle|\begin{matrix}(a_j;\alpha_{1j},...,\alpha_{rj})_{1,P}:(c_{1j},C_{1j})_{1,p_1};...;(c_{rj},C_{rj})_{1,p_r}\\(b_j;\beta_{1j},...,\beta_{rj})_{1,Q}:(d_{1j},D_{1j})_{1,q_1};...;(d_{rj},D_{rj})_{1,q_r}\end{matrix}\right]dx$$

$$=\frac{1}{\rho}\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}a^k\eta^{-\frac{s+hk}{\rho}}\times$$

$$H_{P+q,Q+p:p_1,q_1;...;p_r,q_r}^{n,N+m:m_1,n_1;...;m_r,n_r}\left[\begin{matrix}\frac{t_1}{\eta^{\frac{\sigma_1}{\rho}}}\\ \cdot\\ \cdot\\ \cdot\\ \frac{t_r}{\eta^{\frac{\sigma_r}{\rho}}}\end{matrix}\middle|\begin{matrix}(a_j;\alpha_{1j},...,\alpha_{rj})_{1,N},(1-f_j-(s+hk);\\(1-e_j-(s+hk);\frac{\sigma_1}{\rho},...,\frac{\sigma_r}{\rho})_{1,p},\end{matrix}\right.$$

$$\left.\begin{matrix}\frac{\sigma_1}{\rho},...,\frac{\sigma_r}{\rho})_{1,q}(a_j;\alpha_{1j},...,\alpha_{rj})_{N+1,P}:(c_{1j},C_{1j})_{1,p_1};...;(c_{rj},C_{rj})_{1,p_r}\\(b_j;\beta_{1j},...,\beta_{rj})_{1,Q}:(d_{1j},D_{1j})_{1,q_1};...;(d_{rj},D_{rj})_{1,q_r}\end{matrix}\right]$$

References

[1] B.N. Kanth, Jour. Inst. Sci. Nepal, 3(1980), P.99.

[2] E. D. Rainville, Special Functions, Chelsea Publ. Co., Branx, New York.

[3] H.M. Srivastava,K.C. Gupta and S.P. Goyal, The H-function of one and two variables with applications, South Asian publishers, New Delhi(1982).

[4] H.M. Srivastava and R.G. Bushman, Some convolution integral equations, Indag.Math.36(1974),211-216.

[5] H.M.Srivastava and R.Panda, Some bilateral generating functions for a class of generalized hypergeometric polynomials, J. Reine Argew. Math. 283(1976).

[6] K.C.Gupta and U.C. Jain, The H-function II, Proc. Nat. Acad. Sci. India Sect. A, 36(1966), 594-699.

[7] K.P.Sahul Hameed, Investigations in generalized hypergeometric functions, Ph.D Thesis, Kannur University, India,2010.

[8] P.K. Mittal and K.C. Gupta, An integral involving generalized functions of two variables, Proc. Indian Acad. Sci. Sect. A, 75(1972), 117-123.

[9] R.P.Sharma, On a new theorem involving the H-function and general class of polynomials, Kyungpook Math.J,43(2003), 489-494.

[10] Y. Pragathi Kumar and B. Satyanarayana, Integral transform involving the product of general class of polynomials and H-function of two variables, ABJMI, 3(2011)(in press).

[11] Y. Pragathi Kumar and B. Satyanarayana, On Laplace transform of the product of generalized Struve's function and H-function of two variables, International J. Appl. Math. Anal. and Appl. (in press).

[12] Y. Pragathi Kumar and B. Satyanarayana, On Mellin transform involving the product of general class of polynomials, Struve's function and H-function of two variables, (Communicated).

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