Int. J. Contemp. Math. Sciences, Vol. 6, 2011, no. 35, 1733 - 1747

A Fuzzy Approach on Vendor Managed Inventory Policy

A. Nagoor Gani

PG and Research Department of Mathematics
Jamal Mohamed College (Autonomous)
Tiruchirappalli–620020, Tamilnadu, India
ganijmc@yahoo.co.in

S. Maheswari

Department of Mathematics
Holy Cross College (Autonomous)
Tiruchirappalli-620002, Tamilnadu, India.
mahesraam@yahoo.co.in

Abstract

This paper deals with one-vendor, multi-buyer integrated production inventory model with consignment stock policy, a particular vendor managed inventory policy, in which inspection cost is included in the total cost to supply good quality items to the buyers. An analytical model is developed to obtain optimal replenishment decisions for both vendor and buyers. Fuzzy mathematical model is also developed in which setup costs, holding costs of vendor and buyers and inspection cost of the vendor are taken as triangular fuzzy numbers. Modified integration representation method is used for defuzzifying the total cost. Finally the benefit of CS policy is explained through a numerical example.

Mathematics Subject Classification: 03E72, 90B05

Keywords: Supply chain, consignment stock, inspection cost, triangular fuzzy numbers, defuzzification

1 Introduction

Supply chain management has enabled numerous firms to enjoy great advantages by integrating all activities associated with the flow of materials. The main goal of

supply chain management is to deliver superior quality goods at a lesser cost. For this, the companies have to strengthen their supply agreements and management of their inventories. Vendor managed inventory (VMI) represents an interesting approach to stock monitoring an interesting approach to stock monitoring and control. Consignment Stock (CS) is a particular VMI policy which represents a successful strategy for both the buyer and the supplier. The most radical application of CS may lead to the suppression of the vendor inventory, as this actor uses the buyer's warehouse to stock its finished products. As a counterpart the vendor with guarantee that the quantity stored in the buyer's warehouse will be kept between a maximum level and a minimum one, also supporting the additional costs eventually induced by stock-out conditions. The buyer will pickup from its store the quantity of material needed to meet its production plans and the material itself will be paid to the buyer according to the agreement signed. This strategy spreads out rapidly in different manufacturing environments, confirming its strategic interest for companies and its positive attitude in being implemented in supply chains. When a large company interacts with small or medium sized vendors, the buyer may get maximum advantages from CS agreement. In case when a vendor sells the same device to different customers, he may be doubtful about the real advantages of CS agreement.
Lucio Zavenella, Simone Zanoni [13] investigated single vendor-multibuyer environment where the CS policy may be implemented in supply chains. The ultimate end users will be highly satisfied if they receive all good quality items. This paper introduces inspection cost so that the customers satisfaction is guaranteed with an additional cost which maybe very small.

2. Literature Review

2.1. Single-Vendor – Single–Buyer Models

Goyal [5] suggested a single supplier – single customer problem. Goyal [5] presented a joint economic lotsize model which minimizes the total costs for both the vendor and the buyer. Afterwards the model was generalized by Banerjee [1] and Goyal [7]. These models assume that a perfect balance of power exists between the vendor and the buyer, enforced by contractual agreement. Later Hill [11] formulated a model to minimize the total cost of buyer-vendor system. Goyal [8] improved the model by considering capacity constraint determined by transport equipment. Valentine and Zavenella [20] presented an industrial case and performance analysis of CS and Braglia and Zavanella [4] proposed related analytical approach and some performance evaluation of CS policy Zenoni and Grubbstrom [23] provided a full analytical solution. Hoque and Goyal [12] developed a neuristic solution procedure to minimize the total cost of integrated inventory system under controllable lead time between a vendor and a buyer. Hill and Omar [11] provided an improvement to CS case by offering analytical solution Zhou and Wang [24] presented a model with shortages for the buyer. Finally Sermah et al [18] presented a literature review with buyer-vendor coordination models.

2.2. Single-Vendor Multiple-Buyer Models

Joglekar [14] pointed out that purchase order sizes affect not only the vendor's revenue stream but also the manufacturing cost stream. Joglekar and Tharthare [15] proposed an individually responsible and rational decision approach to lotsizes for one vendor and many buyers. Viswanathan and Piplani [21] proposed a model to study and analyse the benefit of coordinating supply chain by means of common replenishment epochs. Woo et al [22] considered an integrated inventory model where a single vendor purchases and processes raw materials and deliver finished items to multiple buyers. Boyaci and Gallego [3] analysed inventory and pricing policies that jointly maximize the profit of one wholesaler and multi retailers. Siajadi et al [19] proposed a multiple shipment policy for joint economic lotsize which allowed multiple shipments. Kim et al [16] considered three stage supply chain, the third level consists of multiple retailer who interact with the single manufacturer.

In this paper we have taken single vendor-two buyer production inventory model to determine the benefits of CS policy in which inspection cost is included with the total cost. Also setup costs, holding costs of both buyer and vendor and inspection cost are taken as triangular fuzzy numbers. The total cost is defuzzified using modified integration representation method.

3. Notations and Assumptions

A_1 : batch setup cost faced by the vendor (€/setup)
A_{2i} : order emission cost faced by the i^{th} buyer (€/order)
h_1 : vendor holding cost per item and per time unit (€/item time unit)
h_{2i} : i^{th} buyer holding cost per item and per time unit (€/item time unit)
P : vendor production rate (continuous) (item/time unit)
d_i : demand rate seen by the i^{th} buyer (continuous) (item/time unit)
N : number of buyers
T : ordering or production cycle time (time unit)
n_i : i^{th} buyer number of transport operations per production cycle time
C : inspection cost incurred by the vendor (€/item/time unit)
$\tilde{A}_1$: fuzzy batch setup cost faced by the vendor (€/setup)
$\tilde{h}_1$: fuzzy vendor holding cost per item and per time unit (€/item time unit)
$\tilde{h}_{2i}$: fuzzy i^{th} buyer holding cost per item and per time unit (€/item time unit)
$\tilde{A}_{2i}$: fuzzy order emission cost faced by the i^{th} buyer (€/order)
$\tilde{C}$: fuzzy inspection cost incurred by the vendor (€/item/time unit)
TC : average total cost of the system per time unit, function of n_i and T (€/item time unit)
$\tilde{TC}$: fuzzy average total cost of the system per time unit, function of n_i and T (€/item time unit)

A cycle is defined as the period during which the vendor incurs in one setup activity, thus producing the amount of components to be delivered to the Y buyers so as to allow them to satisfy the demand seen by the buyers themselves during the cycle. The cycle is replicated identically within the time horizon. It is also assumed that P > D, where D = $\sum_{i}^{Y} d_i$.

Case $h_{2,i} > h_2$ ∀i

This situation refers to the assumption of items increasing their value while descending the production-distribution chain. As a consequence, goods are preferably kept in the vendor's warehouses until the buyer asks for a further shipment.

Case $h_{2,j} < h_1$ ∀i

The opposite situation can be found in practice, especially as a consequence of the CS inventory parameter settings.

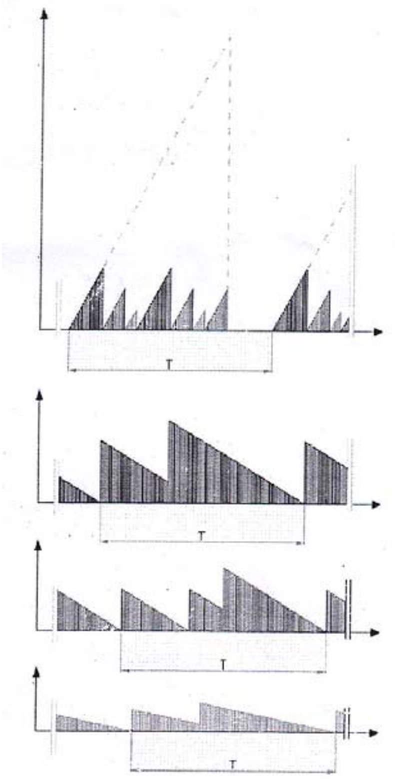

Fig.1. Vendor and buyer stocks against time, with a production cycle time length equal to T

The aim is to minimize the stock held by the vendor, shipping all the stocks available whenever a delivery is ready for transportation. The shipment policy is based on making equal-sized shipments (possibly different for different buyers) while production is taking place, the last shipment being made as soon as production finishes.

According to the given notations, the total cost of the vendor per unit time is the sum of setup cost, holding cost and inspection cost.

$$\therefore \quad TC_{vendor} = \frac{A_1}{T} + \frac{h_1 T}{2P}\sum_{i=1}^{N}\frac{d_i^2}{n_i} + \frac{CP}{T} \qquad \text{. . . (3.1)}$$

The total cost of all the buyers is the sum of order emission cost and holding cost.

$$TC_{\ buyer} = \sum_{i=1}^{N}\left[\frac{n_i A_{2i}}{T} + \frac{T}{2}\ h_{2i} d_i \left(1\text{-}\frac{d_i}{P} + \frac{d_i}{n_i P}\right)\right] \qquad \text{. . . (3.2)}$$

The total average cost for the whole system is given by

$TC = TC_{vendor} + TC_{buyer}$

$$TC\frac{1}{T}\left(A_1 + \sum_{i=1}^{N} n_i A_{2i} + CP\right) + h_1\frac{T}{2}\sum_{i=1}^{N}\left[\frac{d_i^2}{n_i P}\right] + \frac{T}{2}\sum h_{2i} d_i\left(1 - \frac{d_i}{P} + \frac{d_i}{n_i P}\right) \text{ . . (3.3)}$$

when N = 1, the cost function is equal to the cost function of single vendor – single buyer CS case (Braglia and Zavenella, 2003) where TD = nq.

The Joint Optimum: The total cost given in (3) is taken as a function of 2 decision variables T and n_i, i = 1, 2, 3, . . . , N. Whatever the value of n_i, T will be determined by equating $\frac{\partial T_C}{\partial T} = 0$ which gives

$$T^* = \sqrt{\frac{2\left(A_1 + \sum_{i=1}^{N}\left[n_i A_{2i}\right] + PC\right)}{h_1\sum_{i=1}^{N}\frac{d_i^2}{n_i P} + \sum_{i=1}^{N} h_{2i} d_i\left(1 - \frac{d_i}{P} + \frac{d_i}{n_i P}\right)}} \qquad \text{. . . (3.4)}$$

The T^* value leads to the minimum total cost (with respect to T alone)

$$TC^* = \sqrt{2\left(\sum_{i=1}^{N}\left(h_1 + h_{2i}\right)\frac{d_i^2}{n_i P} + h_{2i} d_i\left(1 - \frac{d_i}{P}\right)\right) \text{ x } \left(A_1 + \sum_{j=1}^{N}\left(n_j A_{2j}\right) + PC\right)} \text{ . . .}$$

(3.5)

However minimizing TC^* function requires that

$$\frac{\partial}{\partial n_i}\left(h_1\sum\frac{d_i^2}{n_i P} + \sum h_{2i} d_i\left(1 - \frac{d_i}{P} + \frac{d_i}{n_i P}\right)\right)\left(A_1 + \sum_{j=1}^{N}\left(n_j A_{2j}\right) + PC\right) = 0$$

where i = 1, 2, . . . , N.

$$\text{(ie)} \quad \left(\text{-}h_1\frac{d_i^2}{n_i^2 P} - h_{2i}\frac{d_i^2}{n_i^2 P}\right)\left(A_1 + \sum_{j=1}^{N}\left(n_j A_{2j}\right) + PC\right)$$

$$+A_{2i}\sum_{j=1}^{N}\left(h_1\frac{d_j^2}{n_j P} + h_{2j} d_j\left(1 - \frac{d_j}{P} + \frac{d_j}{n_j P}\right)\right) \quad = 0$$

$$\frac{(A_{2i})n_i^2P}{(h_1+h_{2i})d_i^2} = \frac{A_1+\sum_{j=1}^{N}(n_jA_{2j})+PC}{\sum_{j=1}^{N}\left(\frac{h_1d_j^2}{n_jP}+h_{2j}d_j\left(1-\frac{d_j}{P}+\frac{d_j}{n_jP}\right)\right)} = K$$

and $n_i = \dfrac{\sqrt{K}\sqrt{(h_1+h_{2i})d_1^2}}{\sqrt{(A_{2i})P}}$ where K is independent of individual n_i.

Then $K = \dfrac{A_1+\sum_{j=1}^{N}(n_jA_{2j})+PC}{\sum_{j=1}^{N}\left(\frac{h_1d_j^2}{n_jP}+h_{2j}d_j\left(1-\frac{d_j}{P}+\frac{d_j}{n_jP}\right)\right)}$

After some algebraic steps, we obtain

$$\sqrt{K} = \sqrt{\frac{A_1+PC}{\sum h_{2j}d_j\left(1-\frac{d_j}{P}\right)}} \qquad \text{. . . (3.6)}$$

and so $$n_i^* = \sqrt{\frac{(h_1+h_{2i})d_i^2(A_1+PC)}{(A_{2i})P\sum_{j=1}^{N}\left(h_{2j}d_j\left(1-\frac{d_j}{P}\right)\right)}} \qquad \text{. . . (3.7)}$$

The values determined for n_i^* and T^* minimizes the total cost TC^*.

3.1. The Sequential Solution

Here the total cost of vendor present T as the decision variable while the n_i values are given and buyer's present n_i as decision variable while T is given. Therefore the decisions are taken sequentially so that vendor's optimal choice about T depends on n_i values which are got by differentiating TC_{vendor} with respect to T.

$$\therefore \quad T^{**} = \sqrt{\frac{2P(A_1+PC)}{h_1\sum_{j=1}^{N}\frac{d_j^2}{n_j}}} \qquad \text{. . . (3.1.1)}$$

where the n_i values are chosen by each buyer so as to minimize TC_{buyer} function. Differentiating TC_{buyer} with respect to n_i and equating to zero we have

$$\therefore \quad n_i^{**} = Td_i\sqrt{\frac{h_{2i}}{2P(A_{2i})}} \quad i = 1, 2, . . . N \qquad \text{. . . (3.1.2)}$$

and n_i^{**} depends on T.

The two equations maybe combined thus obtaining the sequential solution.

$$T^{**} = \frac{(A_1 + PC)\sqrt{2P}}{h_1 \sum_{j=1}^{N} d_j \sqrt{\frac{A_{2j}}{h_{2j}}}} \qquad \ldots (3.1.3)$$

$$n_i^{**} = \frac{(A_1 + PC) d_i \sqrt{\frac{h_{2_i}}{A_{2i}}}}{h_1 \sum_{j=1}^{N} d_j \sqrt{\frac{A_{2j}}{h_{2j}}}} \qquad \ldots (3.1.4)$$

3.2. Numerical Illustration

We consider the situation with two buyers with different demands, holding and ordering costs.

$P = 3200$ item/year $d_1 = 500$ item/year $D = 1500$ item/year $d_2 = 1000$ item/year

$A_1 = 400$ ∈/setup $A_{21} = 75$ ∈/order $A_{22} = 25$ ∈/order $h_1 = 5$ ∈/item/year

$h_{21} = 4$ ∈/item/year $h_{22} = 4$ ∈/item/year $C = 0.1$ ∈/item

The application of joint optimum model leads to the following results.

n_1	n_2	T^*	TC_{vendor}	$TC_{buyer,1}$	$TC_{buyer,2}$	TC^*
1	3	.535	1589.61	675.18	987.26	3252.05

The application of sequential solution model leads to the following results

n_1	n_2	T^{**}	TC_{vendor}	$TC_{buyer,1}$	$TC_{buyer,2}$	TC^{**}
4	12	2.46	352.18	2293.67	3632.58	6278.43

From the above table, it is obvious that the buyers are in advantageous position and vendor is in a disadvantageous position when joint optimum policy is adopted instead of sequential solution.

4. Fuzzy Mathematical Model

Let $\tilde{A}_1 = (A_{11}, A_{12}, A_{13})$, $\tilde{A}_{2i} = (\tilde{A}_{2i_1}, \tilde{A}_{2i_2}, \tilde{A}_{2i_3})$, $\tilde{h}_1 = (h_{11}, h_{12}, h_{13})$, $\tilde{h}_{2i} = (\tilde{h}_{2i_1}, \tilde{h}_{2i_2}, \tilde{h}_{2i_3})$ $\tilde{C} = (C_1, C_2, C_3)$ be triangular fuzzy numbers.

4.1. Joint Optimum:

The total average cost of the whole system is given by

$$\tilde{TC} = \tilde{TC}_{vendor} + \tilde{TC}_{buyer} \qquad \ldots (4.1.1)$$

$$\tilde{TC} = \frac{1}{T}\left(\tilde{A}_1 + \sum n_i \tilde{A}_{2i} + \tilde{C}P\right) + \tilde{h}_1 \frac{T}{2} \sum_{i=1}^{N} \frac{d_i^2}{n_i P} + \frac{T}{2} \sum \left(\tilde{h}_{2i} d_i \left(1 - \frac{d_i}{P} + \frac{d_i}{n_i P} \right) \right)$$

$\tilde{TC} =$

$$\left[\left\{\frac{1}{T}\left(A_{11}+\sum n_i A_{2i_1}+C_1P\right)+h_{11}\frac{T}{2}\sum_{i=1}^{N}\frac{d_{i1}^2}{n_iP}+\frac{T}{2}\sum\left(h_{2i_1}d_i\left(1-\frac{d_i}{P}+\frac{d_i}{n_iP}\right)\right)\right\},\right.$$
$$\left\{\frac{1}{T}\left(A_{12}+\sum n_i A_{2i_2}+C_2P\right)+h_{12}\frac{T}{2}\sum_{i=1}^{N}\frac{d_{i2}^2}{n_iP}+\frac{T}{2}\sum\left(h_{2i_2}d_i\left(1-\frac{d_i}{P}+\frac{d_i}{n_iP}\right)\right)\right\},$$
$$\left.\left\{\frac{1}{T}\left(A_{13}+\sum n_i A_{2i_3}+C_3P\right)+h_{13}\frac{T}{2}\sum_{i=1}^{N}\frac{d_{i3}^2}{n_iP}+\frac{T}{2}\sum\left(h_{2i_3}d_i\left(1-\frac{d_i}{P}+\frac{d_i}{n_iP}\right)\right)\right\}\right] \quad \ldots (4.1.2)$$

$P(\tilde{T}C)$ =

$$\frac{1}{6}\left[\left\{\frac{1}{T}\left(A_{11}+\sum n_i A_{2i_1}+C_1P\right)+h_{11}\frac{T}{2}\sum_{i=1}^{N}\frac{d_{i1}^2}{n_iP}+\frac{T}{2}\sum\left(h_{2i_1}d_i\left(1-\frac{d_i}{P}+\frac{d_i}{n_iP}\right)\right)\right\}+\right.$$
$$4\left\{\frac{1}{T}\left(A_{12}+\sum n_i A_{2i_2}+C_2P\right)+h_{12}\frac{T}{2}\sum_{i=1}^{N}\frac{d_{i2}^2}{n_iP}+\frac{T}{2}\sum\left(h_{2i_2}d_i\left(1-\frac{d_i}{P}+\frac{d_i}{n_iP}\right)\right)\right\}+$$
$$\left.\left\{\frac{1}{T}\left(A_{13}+\sum n_i A_{2i_3}+C_3P\right)+h_{13}\frac{T}{2}\sum_{i=1}^{N}\frac{d_{i3}^2}{n_iP}+\frac{T}{2}\sum\left(h_{2i_3}d_i\left(1-\frac{d_i}{P}+\frac{d_i}{n_iP}\right)\right)\right\}\right] \quad \ldots (4.1.3)$$

$\frac{\partial}{\partial T}P(\tilde{T}C)$ = 0 gives

$$T^* = \sqrt{\frac{2\left\{\left(A_{11}+4A_{12}+A_{13}\right)+\sum\left(n_i\left(A_{2i_1}+4A_{2i_2}+A_{2i_3}\right)\right)+P\left(C_1+4C_2+C_3\right)\right\}}{\left(h_{11}+4h_{12}+h_{13}\right)\sum\left(\frac{d_i^2}{n_iP}\right)+\sum\left(h_{2i_1}+4h_{2i_2}+h_{2i_3}\right)d_i\left(1-\frac{d_i}{P}+\frac{d_i}{n_iP}\right)}} \quad \ldots (4.1.4)$$

TC^* =

$$\sqrt{\frac{2\left\{\frac{1}{6}\sum\left(h_{11}+4h_{12}+h_{13}\right)\frac{d_i^2}{n_iP}+4\left(h_{1i_2}+h_{2i_2}A_{11}\right)\frac{d_i^2}{n_iP}+\left(h_{13}+h_{2i_3}\right)\frac{d_i^2}{n_iP}+\frac{1}{6}\left(d_i\left(1-\frac{d_i}{P}\right)\left(h_{2i_1}+4h_{2i_2}+h_{2i_3}\right)\right)\right\}}{\left\{\left(\frac{A_{11}+4A_{12}+A_{13}}{6}\right)+\frac{1}{6}\left(\sum_{j=1}^{N}n_j\left(A_{2j_1}+4A_{2j_2}+A_{2j_3}\right)+P\left(C_1+4C_2+C_3\right)\right)\right\}}} \quad \ldots (4.1.5)$$

The T^* value leads to the minimum total cost (with respect to T alone). But Total cost depends on n_i also. So if we differentiate $P(\tilde{T}C)$ with respect to ni and equating to zero we get after some algebraic steps.

$$n_i^* = \sqrt{\frac{\left(h_{11}+4h_{12}+h_{13}\right)+\left(h_{2i_1}+4h_{2i_2}+h_{2i_3}\right)d_i^2\left(A_{11}+4A_{12}+A_{13}\right)}{\left(A_{2i_1}+4A_{2i_2}+A_{2i_3}\right)P\sum_{j=1}^{N}\left(h_{2j_1}+4h_{2j_2}+h_{2j_3}\right)d_j\left(1-\frac{d_j}{P}\right)}} \quad .4.1.6)$$

4.2. Sequential Solution in Fuzzy Sense

In this case the vendor's total costs present T as the decision variable while n_i values are given. Each buyer's total costs present n_i as the decision variable while T is given. Hence differentiating $\tilde{T}C$ vendor with respect to T and equating to zero we get

$$T^{**} = \sqrt{\frac{2P\left(A_{11}+4A_{12}+A_{13}+C_1P+4C_2P+C_3P\right)}{\left(h_{11}+4h_{12}+h_{13}\right)\sum_{j=1}^{N}\frac{d_j^2}{n_j}}} \quad ...(4.2.1)$$

where the n_i values are chosen by each buyer so as to minimize $\tilde{T}C$ buyer function.

Differentiating $\tilde{T}C$ buyer with respect to n_i, the optimal value is given by

$$n_i^{**} = Td_i\sqrt{\frac{\left(h_{2i_1}+4h_{2i_2}+h_{2i_3}\right)}{2P\left(A_{2i_1}+4A_{2i_2}+A_{2i_3}\right)}} \quad \text{where } i = 1, 2, ..., N. \quad ...(4.2.2)$$

and n_i^{**} depends on T.

The two equations may be combined thus obtaining the sequential solution which is

$$T^{**} = \frac{\left(A_{11}+4A_{12}+A_{13}+C_1P+4C_2P+C_3P\right)\sqrt{2P}}{\left(h_{11}+4h_{12}+h_{13}\right)\sum_{j=1}^{N}d_j\sqrt{\frac{\left(A_{2j_1}+4A_{2j_2}+A_{2j_3}\right)}{\left(h_{2j_1}+4h_{2j_2}+h_{2j_3}\right)}}} \quad ...(4.2.3)$$

$$n_i^{**} = \frac{\left(A_{11}+4A_{12}+A_{13}\right)d_i\sqrt{\frac{\left(h_{2i_1}+4h_{2i_2}+h_{2i_3}\right)}{\left(A_{2i_1}+4A_{2i_2}+A_{2i_3}\right)}}}{\left(h_{11}+4h_{12}+h_{13}\right)\sum_{j=1}^{N}d_j\sqrt{\frac{\left(A_{2j_1}+4A_{2j_2}+A_{2j_3}\right)}{\left(h_{2j_1}+4h_{2j_2}+h_{2j_3}\right)}}} \quad ...(4.2.4)$$

4.3. Numerical Illustration

Let $\tilde{A}_1$ = (390, 400, 410) $\tilde{A}_{21}$ = (74, 75, 76) $\tilde{A}_{22}$ = (24, 25, 26)

$\tilde{h}_1$ = (4.9, 5, 5.1) $\tilde{h}_{21}$ = (3.9, 4, 4.1) $\tilde{h}_{22}$ = (3.9, 4, 4.1) $\tilde{C}$ = (0.09, 0.1, 0.11)

The application of joint optimum model leads to the following result.

n_1	n_2	T^*	TC_{vendor}	$TC_{buyer,1}$	$TC_{buyer,2}$	TC^*
1	3	.535	1589.52	675.22	987.33	3252.07

The application of sequential solution model leads to the following results

n_1	n_2	T^{**}	TC_{vendor}	$TC_{buyer,1}$	$TC_{buyer,2}$	TC^{**}
2	7	2.469	808.33	2337.24	3686.8	6832.37

The adoption of Joint optimum policy instead of sequential solution results in the following economic impact

VENDOR SAVINGS	BUYER 1 SAVINGS	BUYER 2 SAVINGS	TC SAVINGS
-96.64 %	71.11 %	73.22 %	52.4 %

SENSITIVITY ANALYSIS

The aim is to identify the influence of parameters in various costs and savings in costs of members in supply chain

h11	h12	h13	a11	a12	a13	c1	c2	c3	A211	A212	A213	h211	h212	h213	A221	A222	A223	h221
4.9	5	5.1	390	400	410	0.09	0.1	0.11	70	75	80	3.9	4	4.1	24	25	26	3.9
5.4	5.5	5.6	390	400	410	0.09	0.1	0.11	70	75	80	3.9	4	4.1	24	25	26	3.9
4.9	5	5.1	440	450	460	0.09	0.1	0.11	70	75	80	3.9	4	4.1	24	25	26	3.9
4.9	5	5.1	390	400	410	0.09	0.2	0.21	70	75	80	3.9	4	4.1	24	25	26	3.9
4.9	5	5.1	390	400	410	0.09	0.1	0.11	75	80	85	3.9	4	4.1	24	25	26	3.9
4.9	5	5.1	390	400	410	0.09	0.1	0.11	70	75	80	4.4	4.5	4.6	24	25	26	3.9
4.9	5	5.1	390	400	410	0.09	0.1	0.11	70	75	80	3.9	4	4.1	28	30	32	3.9
4.9	5	5.1	390	400	410	0.09	0.1	0.11	70	75	80	3.9	4	4.1	24	25	26	4.3
4.9	5	5.1	390	400	410	0.27	0.3	0.33	70	75	80	3.9	4	4.1	24	25	26	3.9
4.9	5	5.1	390	400	410	0.09	0.1	0.11	70	75	80	3.9	4	4.1	24	25	26	3.9
5.4	5.5	5.6	440	450	460	0.09	0.1	0.11	70	75	80	3.9	4	4.1	24	25	26	3.9
4.9	5	5.1	390	400	410	0.09	0.1	0.11	78	80	82	4.4	4.5	4.6	28	30	32	4.4
5.4	5.5	5.6	440	450	460	0.09	0.1	0.11	78	80	82	4.4	4.5	4.6	28	30	32	4.4
5.4	5.5	5.6	440	450	460	0.09	0.1	0.11	78	80	82	4.4	4.5	4.6	28	30	32	4.4
5.4	5.5	5.6	440	450	460	0.09	0.1	0.11	78	80	82	4.4	4.5	4.6	28	30	32	4.4
5.4	5.5	5.6	440	450	460	0.3	0.35	0.4	78	80	82	4.4	4.5	4.6	28	30	32	4.4

h222	h223	d1	d2	P	T*	TC*	N1*	N2*	T**	N1**	N2**	TC OF VENDOR	TC OF BUYER1	TC OF BUYER2
4	4.1	500	1000	3200	0.535044	861.45	1	3	2.46942	2	7	1589.52	675.22	987.33
4	4.1	500	1000	3200	0.531077	861.45	1	3	2.244927	2	6	1621.97	672.3	982.1
4	4.1	500	1000	3200	0.550204	911.45	1	3	2.640907	2	8	1650.22	686.52	1007.47
4	4.1	500	1000	3200	0.611571	1128.117	1	3	3.56694	2	7	1979.25	734.21	1090.95
4	4.1	500	1000	3200	0.53658	866.45	1	3	2.432398	2	7	1586.374	685.67	989.36
4	4.1	500	1000	3200	0.524369	861.45	1	3	2.536751	2	7	1612.05	732.94	973.28
4	4.1	500	1000	3200	0.539637	875.95	1	3	2.349258	2	6	1580.16	678.62	1021.20
4.5	4.7	500	1000	3200	0.518429	861.65	1	3	2.547497	2	8	1625.07	663.1	1068.12
4	4.1	500	1000	3200	0.704886	1501.45	1	3	4.66446	2	7	2250.63	811.29	1222.47
4	4.1	600	1200	3200	0.506587	883.6	1	4	2.05785	2	7	1706.23	755.95	1071.26
4	4.1	500	1000	3200	0.546125	911.45	1	3	2.400825	2	7	1683.71	683.46	1002.03
4.5	4.6	500	1000	3200	0.514514	881.2	1	3	2.456199	2	7	1633.86	734.32	1091.40
4.5	4.6	500	1000	3200	0.525222	931.2	1	3	2.387971	2	7	1729.34	743.19	1106.90
4.5	4.6	700	1200	3500	0.468462	961.2	1	3	2.011555	2	6	2064.75	908.6	1167.85
4.5	4.6	700	1200	4000	0.481372	1011.2	1	3	2.284846	2	6	2086.8	924.35	1226.72
4.5	4.6	700	1200	4000	0.677418	2011.2	1	3	4.9729	2	6	3182.71	1185.03	1596.08

TC	SEQUENTIAL TC OF VEND	SEQUENTIAL TC OF BUYER1	SEQUENTIAL TC OF BUY2	SEQUENTIAL TC	% OF SAVINGS IN VENDOR TC	% OFSAVINGS FOR BUY1 TC	% OFSAVNGS FOR BUY2 TC	% OF TC SAV
3252.07	808.33	2337.24	3686.80	6832.37	-96.64	71.11	73.22	52.40206
3276.36	883.42	2136.36	3387.44	6407.21	-83.60	68.53	71.01	48.86453
3344.21	807.37	2491.39	3913.3	7212.05	-104.39	72.44	74.26	53.63025
3804.41	1038.	3330.33	5272.08	9640.40	-90.67	77.95	79.13	60.5368
3261.4	805.02	2308.15	3633.67	6746.83	-97.06	70.29	72.77	51.6603
3318.27	814.68	2690.02	3783.52	7288.21	-97.88	72.75	74.28	54.47067
3279.98	841.79	2229.57	3551.56	6622.93	-87.71	69.56	71.25	50.47535
3356.29	780.19	2407.36	4243.06	7430.61	-108.29	72.46	74.83	54.83157
4284.38	1267.67	4332.21	6867.62	12467.49	-77.54	81.27	82.2	65.63557
3533.45	969.99	2310.8	3436.4	6717.19	-75.90	67.28	68.83	47.39694
3369.19	873.37	2275.74	3588.39	6737.49	-92.78	69.97	72.08	49.99337
3459.57	807.13	2612.49	4131.65	7551.26	-102.42	71.89	73.58	54.18551
3579.44	872.14	2543.59	4021. 9	7437.42	-98.28	70.78	72.48	51.87256
4141.21	1164.25	2930.92	3968.91	8064.08	-77.34	69	70.57	48.64625
4237.83	1133.87	3353.78	4705.59	9193.24	-84.04	72.44	73.93	53.90219
5963.82	2030.17	7179.16	10106.32	19315.65	-56.77	83.49	84.20	69.12441

The following are the inferences from Sensitivity analysis

- When holding cost of vendor increases T*, T**, N_2 ** decrease There is an increase in % of savings of vendor, but % of savings for buyers and total cost increase
- When ordering cost of vendor increases T**, T*, N_2** increase. There is an increase in % of savings of vendor decreases, but % of savings for buyers and total cost increase
- When Inspection cost increases, T*,T** increase and % of savings of vendor , buyers and total cost increase
- When holding cost of buyer 1 increases T* decreasws, T** increases. There is decrease in % of savings of vendor, but % of savings for buyers and total cost increase
- When holding cost of buyer 2 increases T*decreases , N_2 ** increases There is decrease in % of savings of vendor, but % of savings for buyers and total cost increase
- When ordering cost of buyer 1 increases T* increases There is decrease in % of savings of vendor, buyers and total cost
- When ordering cost of buyer 2 increases T* ,T**,increase There is an increase in % of savings of vendor, but % of savings for buyers and total cost decrease
- When holding cost ordering cost of vendor increase T*, increases There is an increase in % of savings of vendor, but % of savings for buyers and total cost decrease
- When holding cost ordering cost of buyers 1&2 increase T* decreases There is decrease in % of savings of vendor, but % of savings for buyers and total cost increase

5. Conclusion

The present study aimed at proposing a model for a single-vendor. Multi-buyer integrated production inventory with adoption of CS vendor managed inventory where the inspection cost is included with the total system cost to ensure the supply of good quality items to the buyers. It is proved that total cost is slightly higher in this model than the model given in [13] but quality is guaranteed, which is an essential feature in supply chain. A fuzzy mathematical model is also formulated to find optimal inventory replenishment decisions by taking setup costs, inventory holding cost of buyer and vendors and inspection cost as triangular fuzzy numbers and modified integration representation method is used to defuzzify the total cost. It is observed that there is no difference between the crisp solution and defuzzified solution. The results of sensitivity analysis are summarized above. The cycle time has increased as compared to cycle time in [13] due to the screening time.

References

[1] Banerjee. A, 1986a. On a quantity discount pricing model to increase vendor profit Management Science 32. 1513-1517.

[2] Banerjee, A, 1986b. A joint economic lot-size model for purchaser and vendor. Decision Sciences 17.292-311.

[3] Boyaci, T. Gallego, G., 2002. Coordinating pricing and inventory replenishment policies for one wholesaler and one or more geographically dispersed retailers. International Journal of Production Economics 77(2), 95-111.

[4] Braglia, M., Zavanella, L., 2003. Modelling an industrial strategy for inventory management in supply chains. The ‘Consignment Stock’ case. International Journal of Production Research 41, 3793-3808.

[5] Goyal, S.K., 1976. An integrated inventory model for a single supplier-single customer problem. International Journal of Production Research 15 (1), 107-111.

[6] Goyal, S.K., 1977. Determination of optimal production quantity for a two-stage production system. Operational Research Quarterly 28, 865-870.

[7] Goyal, S.K., 1988. A joint economic-lot-size model for purchaser and vendor: A comment. Decision Sciences. 19.236-241.

[8] Goyal S.K., 2000. On improving the single-vendor single buyer integrated production inventory model with a generalized policy. European Journal of Operational Research 125.429-430.

[9] Goyal, S.K., Gupta, Y.P., 1989. Integrated inventory models; The buyer-vendor coordination. European Journal of Operational Research 41.261-260.

[10] Hill, R.M., 1999. The optimal production and shipment policy for the single-vendor single-buyer integrated production-inventory model. International Journal of Production Research 37. 2463-2475. (Corrigenda. 2002.40. 507 and 2003.41. 1093).

[11] Hill, R.M., Omar, M., 2006. Another look at the single-vendor single-buyer integrated production-inventory problem. International Journal of Production Research 44 (4). 791-800.

[12] Hoque, M., Goyal, S.K., 2006. A heuristic solution procedure for an integrated inventory system under controllable lead-time with equal or unequal sized batch shipments between a vendor and a buyer. International Journal of Production Economics 102 (2), 217-225.

[13] Lucio Zavanella, Simone Zanoni., 2009. A one vendor multibuyer integrated production inventory model: The ‘Consignment Stock’ case. International Journal of Production Economics 118, 225-232.

[14] Joglekar, P.N., 1988. Comments on a quantity discount pricing model to increase vendor profits. Management Science 34, 1391-1398.

[15] Joglekar, P.N., Tharthare, S., 1990. The individually responsible and rational decision approach Co economic lot sizes for one vendor and many purchasers. Decision Sciences 21 (3). 492-506.

[16] Kim, T., Hong, Y., Chang. S.Y., 2006. Joint economic procurement production-delivery policy for multiple items in a single-manufacturer. multiple-retailer system. International Journal of Production Economics 103.199-208.

[17] Lu. L, 1995. A one-vendor multi-buyer integrated inventory model. European Journal of Operational Research 81, 312-323.

[18] Sarmah, S.P., Acharya, D., Goyal, S.K., 2006. Buyer vendor coordination models in supply chain management. European Journal of Operational Research 175,1-15.

[19] Siajadi, H., Ibrahim, R.N., Lochert, P.B. 2006. Joint economic lot size in distribution system with multiple shipment policy. International Journal of Production Economics 102, 302-316.

[20] Valentini, G., Zavanella, L., 2003. The Consignment Stock of inventories: Industrial case study and performance analysis. International Journal of Production Economics 81-82. 215-224.

[21] Viswanathan, S., Piplani, R, 2001. Coordinating supply chain inventories through common replenishment epochs. European Journal of Operational Research 129. 277-286.

[22] Woo, Y.Y., Hsu, S.L., Wu, S., 2001. An integrated inventory model for a single vendor and multiple buyers with ordering cost reduction. International Journal of Production Economics 73. 203-315.

[23] Zanoni, S., Grubbstrom, RW., 2004. A note on an industrial strategy for stock management in supply chains: Modelling and performance evaluation. International Journal of Production Research 42. 4421-4426.

[24] Zhou, Y.W., Wang, S.D., 2007. Optimal production and shipment models for a single-vendor-single-buyer integrated system. European Journal of Operational Research 180 (1), 309-323.

Received: March, 2011