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An Algorithm to Solve Multi-Objective Assignment Problem Using Interactive Fuzzy Goal Programming Approach

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Abstract

This paper proposes an algorithm for solving multi-objective assignment problem (MOAP) through interactive fuzzy goal programming approach. A mathematical model has been established to discuss about multi-objective assignment problem which is characterized by non-linear(exponential) membership function. The approach emphasizes on optimal solution of each objective function by minimizing the worst upper bound, which is close to the best lower bound. To illustrate the algorithm a numerical example is presented.

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1. Introduction

The assignment problem is one of the most-studied, well-known and important problem in mathematical programming. The classical assignment problem, that is, the number of jobs and the number of the machines are equal, has been well studied and many algorithms have been produced to solve these type of problems. But due to the uncertainty of the real life, this problem turns into an uncertain assignment problem. The objective of assignment problem is to assign a number of jobs to an equal number of machines so as to minimize the total assignment cost or to minimize the total consumed time for execution of all the jobs. In the multi-objective assignment problem, the objectives alone are considered as fuzzy. The classical assignment problem refers to a special class of linear programming problems. Linear programming is one of the most widely used decision making tool for solving real world problems. In actual decision-making situations, a major concern is that most decision problems involves multiple criteria (attributes or objectives). However, much of decision making in the real world takes place in an environment where the objectives, constraints or parameters are not precise Liu[15].Therefore, a decision is often made on the basis of vague information or uncertain data. In 1970, Zadeh & Bellmann introduced the concept of fuzzy set theory into the decision-making problem involving uncertainty and imprecision. According to fuzzy set theory, the fuzzy objectives and constraints are represented by associated membership functions. Then Zimmermann [30] first applied suitable membership functions to solve linear programming problem with several objective functions. He showed that solutions obtained by fuzzy linear programming are always efficient. Leberling [10] used a special type non-linear membership functions for the vector maximum linear programming problem. Bit et al. [4] applied the fuzzy programming technique with linear membership function to solve the multi-objective transportation problem. Verma et al.[26] used the fuzzy programming technique with some non-linear (hyperbolic & exponential) membership functions to solve a multi-objective transportation problem. Li et al.[13] used a special type of non-linear (exponential) membership function for the multi-objective linear programming problem. Belacela et al.[2] studied a multi-criteria fuzzy assignment problem. Geetha et al.[8] first expressed the cost-time minimizing assignment as the multi-criteria problem. Yang et al.[29] designed a tabu search algorithm based on fuzzy simulation to achieve an appropriate best solution of fuzzy assignment problem. Lin et al.[14] considered a kind of fuzzy assignment problem and designed a labeling algorithm for it. Wahed et al.[28] presented an interactive fuzzy goal programming approach to determine the preferred compromise solution for the multi-objective transportation problem with linear membership function. Gao et al.[7] developed a two-phase fuzzy goal programming technique for multi-objective transportation problem with linear and non-linear parameters and Pramanik et al. [17] discussed the fuzzy goal programming approach for multi-objective transportation, with crisp and fuzzy coefficients. Pramanik et al.[18] presented the priority based goal programming approach for multi-objective transportation problem, with fuzzy parameters. Shaoyuan et.al.[22] proposed a satisfying optimization method based on goal programming for fuzzy multiple objective optimization problem. Tiwari et al.[25] formulated an additive model to solve fuzzy goal programming. The linear interactive and discrete optimization [LINDO] [21], general interactive optimizer [GINO] [12] and TORA packages [23]as well as many other commercial and academic packages are useful to find the solution of the assignment problem. In this paper we are proposing an algorithm for solving multi-objective assignment problem through interactive fuzzy goal programming approach.

This paper is organized as follows: In section 2, mathematical model of multi-objective assignment problem is described. Section 3 presents interactive fuzzy goal programming approach. The optimization algorithm is provided in section 4. To illustrate the algorithm a numerical example is presented in section 5. Section 6 gives few concluding remarks on the proposed algorithm.

2. Establishment of mathematical model of multi-objective assignment problem

Assume that there are n jobs and n persons. n Jobs must be performed by n persons, where the costs depend on the specific assignments. Each job must be assigned to one and only one person and each person has to perform one and only one job. Let c_{ij} be the cost if the ith person is assigned the jth job, the problem is to find an assignment (which job should be assigned to which person) so that the total cost for performing all jobs is minimum.

Here make a assumption that jth job will be completed by ith person, and let

$$x_{ij} = \begin{cases} 1 & \text{if } i\text{th person is assigned } j\text{th job} \\ 0 & \text{if } i\text{th person is not assigned } j\text{th job} \end{cases}$$

Where x_{ij} denotes that jth job is to be assigned to the ith person.

Then, the mathematical model of multi-objective assignment problem is:

Subject to

$$\sum_{i=1}^{n} x_{ij} = 1,\ j = 1,2,...,n \text{ (only one person should be assigned the } j\text{th job)} \tag{2.2}$$

$$\sum_{j=1}^{n} x_{ij} = 1,\ i = 1,2,...,n \text{ (only one job is done by the } i\text{th person)} \tag{2.3}$$

Where $Z_k(x) = \{Z_1(x), Z_2(x),..., Z_k(x)\}$ is a vector of K objective functions, the subscript on $Z_k(x)$ and superscript on c_{ij}^k are used to identify the number of objective functions $(k = 1,2,..., K)$.

3. Interactive Fuzzy goal programming(IFGP) approach

IFGP approach is the combination of three approaches:

3.1. Interactive approach,
3.2. Fuzzy programming approach,
3.3. Goal programming approach,

Let us briefly discuss about each approach:

3.1. Interactive approach

Ringuest et al.(1987) and climaco et al.(1993) developed two interactive approaches to determine the satisfactory solution of multi-objective transportation problem. Since the solution maker is involved in the solution procedure, the interactive approaches play an important role in deriving the best preferred compromise solution. Interactive approach facilitates efficient solution in large scale problems once it is more effective and suitable.

3.2. Fuzzy programming approach

Fuzzy set theory is useful in solving the interactive multi-objective assignment problem to improve and strengthen the proposed solution techniques. Inspite, this is a tool to treat the incomplete preference information of the decision-maker. Bit et al.(1992) developed a fuzzy approach to get the compromise solution for multi-objective transportation problem. Also, Verma et al.(1997) developed a fuzzy approach to solve multi-objective transportation problem with some non-linear membership functions. Wahed (2001) developed a fuzzy approach to get the compromise solution for multi-objective transportation problem.

3.3. Goal programming approach

The goal programming approach is very useful tool for decision-maker to discuss and find a set of suitable and acceptable solutions to decision problems. The term" Goal programming "was introduced by Charnes et al.(1961). Lee & Moore (1973) and Aenaida and Kwak (1994) applied goal programming to find a satisfactory solution of multi-objective transportation problem. Goal programming is a good decision aid in modeling real world decision problems which involves multiple objectives. Goal programming requires the decision-maker to set definite aspiration values for each objective that he wishes to achieve.

The combination of the fuzzy set theory and the goal programming will defuse the conflict among the objectives and the aspiration levels determination via goal programming. At the same time the fuzzy set theory will take care of uncertainty of the extracted information from the decision-maker. Thus the venture between two approaches is carried out through implementation of the aspiration levels.

Wahed and Lee(2006) combined three approaches, Interactive approach, fuzzy programming approach, and goal programming and developed an Interactive fuzzy goal programming approach to determine the preferred compromise solution for multi-objective transportation problem. Thus, combination of above three approaches produces a powerful method to solve linear multi-objective programming problem. Now, the IFGP approach proposed by wahed and Lee (2006) is applied to solve MOAP.

4. Algorithm

Step 1: Solution Representation

Step 1.1: Solve the multi-objective assignment problem as a single objective assignment problem K times by taking one of the objectives at a time.

Step1.2: According to each solution and value for each objective, we can find a pay-off matrix as follows:

	$Z_1(X)$	$Z_2(X)$	..	$Z_k(X)$
$X^{(1)}$	Z_{11}	Z_{12}	..	Z_{1k}
$X^{(2)}$	Z_{21}	Z_{22}	..	Z_{2k}
:	:	:	:	:
$X^{(k)}$	Z_{k1}	Z_{k2}	..	Z_{kk}

Where $X^{(1)}, X^{(2)}, ..., X^{(k)}$ are the isolated optimal solutions of the K different assignment problems for K different objective functions, $Z_{ij} = Z_j(X^i), (i = 1,2,...,K; j = 1,2,...,K)$ be the ith row and jth column element of the pay-off matrix.

Step 1.3: From step1.2, we find for each objective function the worst (U_k) and the best (L_k) values corresponding to the set of solutions, where,

$U_k = (Z_k)_{\max} = \max(Z_{1k}, Z_{2k},..., Z_{kk})$ and $L_k = (Z_k)_{\min} = \min(Z_{1k}, Z_{2k},..., Z_{kk})$, $k = 1,2,..., K$.

Step 2: Determination of membership function (exponential) for the kth objective function

An exponential membership function for the kth objective function is defined as

$$\mu_k(Z_k(x)) = \begin{cases} 1, & \text{if } Z_k(x) \le L_k \\ \exp\left(\dfrac{\alpha(Z_k(x) \quad L_k)}{L_k \quad U_k}\right), & \text{if } L_k < Z_k(x) < U_k \\ 0, & \text{if } Z_k(x) \ge U_k \text{ and } \alpha \to \infty \end{cases} , \tag{4.1}$$

where α is a non - zero parameter, prescribed by the decision maker.

Step 3: Mathematical model structure

By using the exponential membership function as defined in (4.1), and following the fuzzy decision of Bellmann and Zadeh (1970), the equivalent non-linear programming model is:

$$\text{S2}: \qquad \text{Max } \lambda \tag{4.2}$$

Subject to

$$\lambda \le exp\left(\frac{\alpha \ (Z_k - L_k)}{L_k \quad U_k}\right), \qquad k = 1,2,..., K, \tag{4.3}$$

$$\sum_{i=1}^{n} x_{ij} = 1, \ j = 1,2,...,n \ ; \tag{4.4}$$

$$\sum_{j=1}^{n} x_{ij} = 1, \ i = 1,2,...,n \ ; \qquad \beta \ge 0 \tag{4.5}$$

$$x_{ij} = \begin{cases} 1 & \text{if } i\text{th person is assigned } j\text{th job} \\ 0 & \text{if } i\text{th person is not assigned } j\text{th job} \end{cases} \tag{4.6}$$

The above problem can be transformed into the following linear programming model by substituting

$$\beta = -\ln \lambda \tag{4.7}$$

Now we have:

$$\text{S3}: \quad \text{Min } \beta \tag{4.8}$$

Subject to

$$\beta \geq \left(\frac{\alpha \ (L_k - Z_k)}{L_k - U_k} \right), \qquad k = 1,2,...,K, \tag{4.9}$$

$$\sum_{i=1}^{n} x_{ij} = 1, \ j = 1,2,...,n \,; \tag{4.10}$$

$$\sum_{j=1}^{n} x_{ij} = 1, \ i = 1,2,...,n \,; \qquad \beta \geq 0 \tag{4.11}$$

$$x_{ij} = \begin{cases} 1 & \text{if } i\text{th person is assigned } j\text{th job} \\ 0 & \text{if } i\text{th person is not assigned } j\text{th job} \end{cases} \tag{4.12}$$

Step 4: Formulation of goal programming model

To formulate model (S3) as a goal programming model (Sakawa,1993), let us introduce the following positive and negative deviational variables:

$$Z_k(x) - d_k^+ + d_k^- = G_k \ , \qquad k = 1,2,...,K \tag{4.13}$$

Where G_k is the aspiration level of the objective function K.

Using the IFGP approach presented by Wahed and Lee (2006), model (S3) can be formulated as a mixed integer goal programming as follows:

$$\text{S4}: \ \text{Min} \, \beta \tag{4.14}$$

Subject to

$$\beta \geq \left(\frac{\alpha(L_k - Z_k)}{L_k - U_k} \right), \qquad k = 1,2,...,K, \tag{4.15}$$

$$\sum_{i=1}^{n} x_{ij} = 1, \ j = 1,2,...,n \,; \tag{4.16}$$

$$\sum_{j=1}^{n} x_{ij} = 1, \ i = 1,2,...,n \,; \tag{4.17}$$

$$Z_k(x) - d_k^+ + d_k^- = G_k \ , \tag{4.18}$$

$$x_{ij} = \begin{cases} 1 & \text{if } i\text{th person is assigned } j\text{th job} \\ 0 & \text{if } i\text{th person is not assigned } j\text{th job} \end{cases} \tag{4.19}$$

$$x_{ij}, d_k^+, d_k^- \geq 0, \quad i = 1,2,...,n; \ j = 1,2,...,n; \ k = 1.2,...,K; 0 \leq \beta \leq 1 \tag{4.20}$$

x_{ij} are integers $\forall \ i, j$.

Step 5: Determination of aspiration level

From step 1.3 we know that $(Z_k)_{\min} \le Z_k \le (Z_k)_{\max}$. For the MOAP, we should get the optimal compromise solution that is near to the ideal solution. This can be obtained by setting the aspiration levels in model (S4) equal to the upper bounds $(Z_k)_{\max}, k = 1,2,...,K.$ let us solve model (S4) based on the above described algorithm and the corresponding solution vector be X^*. If the decision maker accepts this solution, then go to step (6). Otherwise, model (S4) is modified as follows:
Let $Z_1^*, Z_2^*,..., Z_k^*$ are the objective functions vectors corresponding to the solution vector X^*. Now compare each Z_k^* with the existing upper bound U_k, $k = 1,2,..., K.$ Now aspiration level can be updated by the following steps:

5.1. If $Z_k^* < U_k$ (means new value of the objective function is less than the upper bound, consider this as a new upper bound, replace U_k by Z_k^*.Repeat this process K times and go to step 2.
5.2. If $Z_k^* = U_k$, no change in aspiration level and algorithm terminates, then go to next step.

Step 6: End.

5. Implementation of the model

In this section, we use numerical example to illustrate the formulation and solution procedure of the MOAP. For this model the proposed algorithm gives the best optimal solution.

Numerical Example

Min $Z_1 = 10x_{11} + 8x_{12} + 15x_{13} + 13x_{21} + 12x_{22} + 13x_{23} + 8x_{31} + 10x_{32} + 9x_{33}$

Min $Z_2 = 13x_{11} + 15x_{12} + 8x_{13} + 10x_{21} + 20x_{22} + 12x_{23} + 15x_{31} + 10x_{32} + 12x_{33}$

subject to

$$\sum_{i=1}^{3} x_{ij} = 1, j = 1,2,3; \quad \sum_{j=1}^{3} x_{ij} = 1, i = 1,2,3.$$

$$x_{ij} = \begin{cases} 1 & \text{if } i\text{th person is assigned } j\text{th job} \\ 0 & \text{if } i\text{th person is not assigned } j\text{th job} \end{cases}$$

As the first step, the solution of each single objective assignment problem is:

$X^1 = (0,1,0,0,0,1,1,0,0)$,

$X^2 = (0,0,1,1,0,0,0,1,0)$,

The objective function values are:

$Z_1(X^1) = 29,\ Z_1(X^2) = 38,\ Z_2(X^1) = 42,\ Z_2(X^2) = 28.$

We can write the pay-off matrix as

	$Z_1(X)$	$Z_2(X)$
$X^{(1)}$	29	42
$X^{(2)}$	38	28

From the pay-off matrix, the upper and lower bounds of each objective function can be written as follows:

$29 \le Z_1 \le 38$ and $28 \le Z_2 \le 42$.

Now, model S3 with the parameter $\alpha = 2$, is developed as follows:

Min β

subject to

$$0.34x_{11} + 0.28x_{12} + .052x_{13} + 0.45x_{21} + 0.41x_{22} + 0.45x_{23} + 0.28x_{31} + 0.34x_{32} + 0.31x_{33} - 0.16\beta \le 1,$$

$$0.46x_{11} + 0.54x_{12} + 0.29x_{13} + 0.36x_{21} + 0.71x_{22} + 0.43x_{23} + 0.54x_{31} + 0.36x_{32} + 0.42x_{33} - 0.25\beta \le 1,$$

$$x_{11} + x_{12} + x_{13} = 1,$$

$$x_{21} + x_{22} + x_{23} = 1,$$

$$x_{31} + x_{32} + x_{33} = 1,$$

$$x_{11} + x_{21} + x_{31} = 1,$$

$$x_{12} + x_{22} + x_{32} = 1,$$

$$x_{13} + x_{23} + x_{33} = 1,$$

$$10x_{11} + 8x_{12} + 15x_{13} + 13x_{21} + 12x_{22} + 13x_{23} + 8x_{31} + 10x_{32} + 9x_{33} + d_1^- - d_1^+ = 38,$$

$$13x_{11} + 15x_{12} + 8x_{13} + 10x_{21} + 20x_{22} + 12x_{23} + 15x_{31} + 10x_{32} + 12x_{33} + d_2^- - d_2^+ = 42,$$

$$x_{ij} = \begin{cases} 1 & \text{if } i\text{th person is assigned } j\text{th job} \\ 0 & \text{if } i\text{th person is not assigned } j\text{th job} \end{cases}$$

$$\beta, d_1^-, d_1^+, d_2^-, d_2^+ \ge 0$$

The problem is solved by using the TORA package. The optimal solution is presented as follows:

$x_{12} = 1,\ x_{21} = 1,\ x_{33} = 1, d_1^- = 8, d_2^- = 5,$

Therefore $Z_1^* = 30$ and $Z_2^* = 37$ with $\beta = 1.28$ and $\lambda = .28$.

Assume decision-maker needs more improvement, then, go to step (5.1)
The new upper and lower bounds are:
$29 \le Z_1 \le 30$ and $28 \le Z_2 \le 37$.
According to algorithm, the new aspiration levels of the two objective functions are 30 and 37, respectively. Thus, the new upper bounds in model S3 will be 30 & 37, respectively and repeat step 2 to step 6. Based on these modifications, the above mathematical model is updated and resolved. Then, we find that the new upper bounds are equivalent to the earlier upper bounds so the algorithm terminates. As the algorithm terminates, the above solution is accepted by the decision maker and validates as best optimal solution. The set of solutions (Z_1, Z_2) can be summarized in the following way :(38, 42), (30, 37), (29, 28) respectively.

6. Conclusions

In this paper, we mainly studied a fuzzy multi-objective assignment problem. As a result, we used an interactive fuzzy goal programming (IFGP) approach to deal with it. The combination of goal programming, fuzzy programming and interactive programming is a powerful method for solving MOAP. In order to obtain the best solution, an algorithm with non-linear membership function (exponential) is designed. The IFGP approach has the following features:

1. The approach provides an optimal compromise solution by updating both upper bounds and aspiration level of each objective function.
2. This is a powerful approach to obtain an appropriate aspiration level of the objective functions.
3. The approach solves all types of MOAP, the vector minimum problem and the vector maximum problem.
4. It is easy and simple to use for the decision- maker and can be easily implemented to solve similar linear multi-objective programming problems.
5. The approach solves a series of classical assignment problem and a linear integer programming problem, which can be solved by any available software.
6. This approach with exponential membership function is a suitable representation in many practical situations. This feature makes this approach more practical than the approach using linear membership function in solving MOAP.

References

1. R.J.Aenaida,N.W.Kwak, A linear goal programming for transshipment problems with flexible supply and demand constraints, Journal of Operational Research Society, 2 (1994),215-224.

2. N.Belacela,M.R.Boulasselb, Multi criteria fuzzy assignment problem: a useful tool to assist medical diagnosis, Artificial Tntelligence in Medicine, 21(2001), 201-207.
3. R.Belmann,L.Zadeh,Decision making in a fuzzy environment, Management Science, 17(1970), 141-164.
4. A.K.Bit,M.P.Biswal,S.S.Alam,Fuzzy programming approach to multi criteria decision making transportation problem, Fuzzy Sets and Systems North-Holland ,50(1992) , 135-141.
5. A.Charnas,W.W.Cooper,Management model and industrial application of linear programming, Wiley,Newyork ,1(1961).
6. J.N.Climaco,C.H.Antunes,M.J.Aives,Interactive decision support for multi-objective transportation problems.European Journal of Operational Research, 65(1993), 58-67.
7. S.P.Gao,S.A.Liu,Two Phase fuzzy algorithms for multi-objective transportation problem, The Journal of Fuzzy mathematics, 12(2004), 147-155
8. S.Geetha,K.P.K.Nair,A variation of the assignment problem, European Journal of Operations Research, 68(1993), 422-426.
9. K.L.Kagade,V.H.Bajaj,Fuzzy approach with linear and some non-linear membership functions for solving multi-objective assignment problem, Advances in computational Research, 1(2009),14-17
10. H.Leberling,On finding compromise solutions in multi-criteria problems using the fuzzy min-operator, Fuzzy sets and Systems, 6(1981),105-118.
11. I.Lee,S.M.Moore, Optimizing tansportation problems with multiple objectives, AIEE Transactions, 5(1973),333-338.
12. J.Liebman,L.Lasdon,L.Schrage,A.Waren, Modeling and Optimization with GINO(The Scientific Press, Palo Alto, CA),1986
13. R.J.Li,S.L.Lee,An exponential membership function for fuzzy multiple objective linear programming,Computers Math.Applic, 22(1991),55-60.
14. C.J.Lin,U.P.Wen, A labeling algorithm for the fuzzy assignment problem, Fuzzy Sets and Systems, 142(2004), 373-391
15. B.Liu,Theory and practice of uncertain programming.Physica-Verlag,Heidelberg,2002.
16. L.I.Lushu,K.K.Lai, A Fuzzy approach to the multi-objective transportation problem, Computers & Operations Research, 27(2000), 43-57
17. S.Pramanik,T.K.Roy,A fuzzy goal programming technique for solving multi-objective transportation problem, Tamsui oxford Journal of management Sciences ,22(2006), 67-89
18. S.Pramanik,T.K.Roy, Multi objective transportation model with fuzzy parameters: priority based fuzzy goal programming approach, Journal of Transportation systems Engineering & Information Technology, 8(2008)

19. J.L.Ringuest,D.B.Rinks,Interactive solutions for the linear multi-objective transportation problem,European Journal of Operations Research 32(1987), 96-106.
20. M.Sakawa,Fuzzy sets and Interactive Multi-Objective Optimization,New York: Plenum Publishing,1993.
21. L.Schrage, Linear, integer and quadratic programming with LINDO(The Scientific Press. Palo Alto, CA),1984
22. L.Shaoyuan,H.U.Chaofang, Satisfying optimization method based on goal programming for fuzzy multiple optimization problem, European Journal of Operational Research, 197(2009), 675-684
23. H.A.Taha,Operations Research, An Introduction ,5th ed.(Macmillan, New York),1992
24. M.Tamiz,D.F.Jones,C.Romero,Goal programming for decision making, An overview of the current state-of-yhr art,European Journal of Operational Research, 111(1998),569-81
25. R.N.Tiwari,S.Dharmar,J.A.Rao, Fuzzy goal programming-An additive model, Fuzzy sets & Systems, 24(1987), 27-34.
26. R.Verma,M.P.Biswal,A.Biswas, Fuzzy programming technique to solve multi-objective transportation problem with some non-linear membership functions,Fuzzy Sets and Systems, 91(1997),37-43.
27. W.F.A.E.Wahed, A multi objective transportation problem under fuzziness, Fuzzy Sets and Systems, 117(2001),27-33.
28. W.F.A.E.Wahed,S.M.Lee, Interactive Fuzzy goal programming for multi objective transportation problems,Omega the International Journal of management Science, 34(2006),158-166.
29.)L.Yang,B.Liu, A multi-objective fuzzy assignment problem: New model and algorithm.IEEE International conference on Fuzzy Systems, (2005),551-556.
30. H.J.Zimmermann, Fuzzy programming and linear programming with several objective functions. Fuzzy Sets and Systems,1(1978),45-66.

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