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Some Common Fixed Points for R-Weakly Commuting Maps in Fuzzy Metric Spaces

Rajesh Shrivastava, Vijyakumar M.*[1], Manavi Kohli*[2], Jitendra Singhvi[**]

and Ramakant Bhardwaj[**]

Department of Mathematics, Govt. Sci. and Comm. Beeneezir College, Bhopal India
Department of Mathematics, Truba , Inst. of Eng. & I.T. Bhopal, India

Correspondence Author e-mail:greengirl_ias2001@yahoo.com

Abstract

The present paper we are proving a result introduced R-weakly commuting maps in Metric spaces and proved a common fixed point theorem. This note offers a fuzzy analogue of pant's theorem. Fuzzy metric space which is motivated by pant [4].

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1. Introduction and Preliminaries

Zadeh [7] Introduced the concept of fuzzy set in 1965 and in the next decade were very productive for fuzzy mathematics and the recent literature has observed the fuzzy frication in at most every direction of Mathematics such as arithmetic topology graph theory, probability theory, Logic etc. Fuzzy set theory has applications in applied sciences such as neural network theory, stability theory mathematical programming modeling theory engineering sciences medical sciences (medical genetics nervous system) Image processing control theory, communication etc.

Kramosil and Michalek [3] introduced the concept of fuzzy metric space in 1975, which opened an avenue for further development of analysis in such spaces. R.P. pant introduced the notation of R-weakly commutatively of mappings in metric spaces and proved some common fixed point theorem. In this paper define R-weakly commuting of mapping in fuzzy metric spaces and prove the fuzzy version of pant's theorem.

Before starting the main results first we write some definitions

Definition 2.1:[7] A fuzzy set A in x is a function with domain x and values in [0,1]

Definition 2.2. [5]: A binary operation *: [0.1] x [0.1] $\rightarrow$ [0,1] is continuous t-norm if * is satisfying the following conditions.

(1) * is commutative and associative.
(2) * is continuous.
(3) $a * 1 = a \ \forall\ a \in [0, 1]$
(4) $a * b \leq c * d$ where $a \leq c$ and $b \leq d$ and $a, b, c, d \in [0, 1]$

Example: $a * b = ab$, $a * b = \min \{a, b\}$

Definition: 2.3 [3] A 3-taple (x. M. *) is said to be a fuzzy metric space if x is an arbitrary set * is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions; for all $x, y\ z \in X$, $s, t > 0$

1) $M(x, y, 0) = 0$

2) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$

3) $M(x, y, t) = M(y, x, t) \neq 0$ for $t \neq 0$

4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$

5) $M(x, y, \cdot) : [0, \infty) \rightarrow [0, 1]$ is continuous $\forall\ x, y, x, y, z \in X$ and $s, t > 0$
6) $\lim_{t \to \infty} M(x, y, t) = 1\ \forall\ x, y$ in X.

Definition 2.4: A sequences $\{ x_n \}$ in a fuzzy metric space (X, M, *) is called Cauchy if $\lim_{n\to\infty} M(x_{n+p}, s_n, t) = 1$ for every $t > 0$ and each $p > 0$, (X, M , *) is complete if every Cauchy sequence in X converges in X. A sequence $\{ x_n \}$ in X is convergent to $x \in X$ if $\mathbf{\lim_{n\to\infty} M(x_n, x, t) = 1}$ for each $t > 0$.

Definition 2.5: Two mappings S and T of a fuzzy metric space (×, M, *) into itself are said to be weakly commuting if

$$M(STx, TSx, t) \;\; M(Sx, Tx, t) \text{ for each } x \text{ in } X.$$

Definition 2.6: Two mappings S and T of a metric space (× . d) into it self are said to be R-weakly commuting provided there exists some positive real number R such that

$$d\,(STx, TSx) \leq Rd\,(Sx, Tx) \text{ for each } x \text{ in } X$$

Now we define R-weakly commuting maps in fuzzy metric spaces.

Definition 2.7: Two mappings S and T of a fuzzy metric space (X, M, *) into itself are R-weakly commuting, provided there exists some positive real number R such that

$$M(STx, TSx, t) \geq M(Sx, Tx, t/R) \text{ for all } x \text{ in } X.$$

3.The following theorem was proved by Pant [4]

Theorem 3.1: Let (×,d) be a complete metric space and let f and g be R – weakly commuting self – mapping of × satisfying the condition.

$$d\,(fx, fy) \leq r(d\,(gx, gy)$$

Where $r : R_+ \to R_+$ is a continuous function such that $r(t) < t$ for each $t > 0$: If the range of f contains the range of g and, if either g f is continuous. Then g and f have a unique common fixed point.

Main Result

Theorem 3.2: Let (×, M, * be a complete fuzzy metric space and let S and T be R – weakly commuting self – mappings of × satisfying the condition.

$$M(Sx, Sy, t) \geq r(M\,(Tx, Ty, t) * M(Sx, Tx, t) * M(Sx, Tx, t)) \qquad \text{…. (3.1)}$$

Where $r : [0, 1] \to [0, 1]$ is continuous function such that $r(t) > t$ for each $0 < t < 1$.

The sequence $\{x_n\}$ and $\{y_n\}$ in X are such that $x_n \to x$, $y_n \to y$, $t > 0$ implies $M(x_n, y_n, t) \to M\,(x, y, t)$ …… (3.2)

If the range of T contains the range of S and either S or T is continuous. Then S and T have a unique common fixed point.

Proof: Let x_0 be an arbitrary point in×. Since S (×) ⊂ T(x). Choose a point x_1 in × such that $Sx_0 = Tx_1$ In general choose x_{n+1} such that $Sx_n = Tx_{n+1}$. Then for t > 0.

$$M(Sx_n, Sx_n+1, t) \geq r(M(Tx_n, Tx_{n+1}\ t) * M(Sx_n, Tx_{n+1}\ t) * M(Sx_n, Tx_n, t))$$

$$= r(M(Tx_n, Tx_{n+1}\ t) * M(Tx_n, Tx_{n+1}\ t) * M(Tx_{n+1}, Tx_n, t))$$

$$= r(M(Tx_n, Tx_{n+1}\ t)$$

$$= r(M(sx_{n-1}, sx_n, t), \text{ Since } r(t) > t \text{ for } 0 < t < 1$$

Thus {M (Sx_n, Sx_{n+1}, t) . n ≥ 0} is a increasing sequence of positive real number in [0, 1] and there fore tends to limit L ≤ 1 we claims that L = 1. For if L < 1, on making n → ∞ in (3.3) we get L ≥ r (L) > L, a contradiction. Hence L = 1

Now for any positive Integer P.

$$M(Sx_n, Tx_{n+p}, t) \geq M(Sx_n, Sx_{n+1}, t/p) * \ldots * M(Sx_{n+p-1}, Sx_{n+p}, t/p)$$

$$\geq M(Sx_n, Sx_{n+1}, t/p) * \ldots * M(fx_n, fx_{n+1}\ t/p)$$

Since by the above argument $\lim_{n\to\infty} \mathbf{M(Sx_n, Sx_n + 1, t) = 1}$ for t > 0 it follows that

$$\lim_{n\to\infty} \mathbf{M(Sx_n, Sx_n + p, t) \geq 1 * \ldots\ldots\ldots\ldots * 1 \geq 1}$$

Thus { Sx_n} is cauchy sequence and by the completeness of ×, {Sx_n} converges to z in ×. Also Tx_n → z.

Let us suppose that the mapping S is continuous then SS x_n → Sz and STx_n → Sz., Further we have since S and T are R – weakly commuting.

$$M(STx_n, STx_n, T) \geq M(Sx_n, Tx_n, t/P)$$

On leffing n → ∞. In this inequality we get TSx_n → Sz by (3.2). We now prove that z = Sz. Suppose z ≠ Sz. Then there exists t > 0 such that M (z, Sz, t) < 1 By (3.1) $M(Sx_n, SSx_n, t) \geq r(M(Tx_n, TSx_n, t))$

On making n → ∞ in above Inequality

$$M(z, Sz, t) \geq r(M(z, Sz, t)) > M(z, Sz, t)$$

A contradiction. There fore $z = Sz$, since $(S(x) \subset T(x)$ we can find z_1 in $\times$ such that $z = Sz = Tz_1$

Now

$$M(SSx_n, Sz_1, t) \geq r\,(M\,(TSx_n, Tx_n, t))$$

Taking limit as $n \to \infty$ we get

$$M\,(Sz, Sz_1, t) \geq r(M\,(Sz, Tz_1, t) = 1$$

Since $r(t) = 1$ for $t = 1$

Which implies that $Sz = Sz_1$ i.e. $z = Sz = Sz_1\ Tz_1$ also for any $t > 0$

$$M(Sz, Tz, t) = M\,(STz_1, TSz_1, t) \geq M(Sz_1, z_1, t/R) = 1$$

Which again implies that $Sz = Tz$

Thus z is a common fixed point of S and T now to prove the uniqueness let if possible $z^1 \neq z$ be another common fixed point of S and T. Then there exist $t > 0$ such that $M\,(z, z^1, t) < 1$ and

$$\begin{aligned} M(z, z^1, t) &= M\,(Sz, Sz^1, t) \\ &\geq r(M\,(Tz, tz^1, t) \\ &= r\,(M\,(z, z^1, t)) \\ &> M(z, z^1, t) \text{ since } r(t) > t \text{ for } 0 < t < 1 \end{aligned}$$

which is a contradiction. There fore $z = z^1$ i.e. z is unique common fixed point of S and T.

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