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Estimation for the Parameters of the Exponentiated Weibull Distribution Based on Progressive Hybrid Censored Samples

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Abstract

Bayes and non-Bayesian estimators for two-parameter exponentiated Weibull distribution have been obtained when sample is progressive hybrid censoring scheme. Maximum likelihood estimators of the parameters and their asymptotic variances have been obtained. Bayes estimators have been developed under squared error loss function using non-informative priors. A numerical illustration for these new results are given.

Key words: The exponentiated Weibull distribution; Progressive hybrid censored; Maximum likelihood estimation; Bayes estimator; Non-informative prior; Square error loss function

1. Introduction

The " exponentiated Weibull family " introduced by Mudholkar and Srivastava (1993) as extension of the Weibull family, contains distribution with bathtub shaped and unimodal failure rates besides a broader class of monotone failure rates. The applications of the exponentiated Weibull (EW) distribution in reliability and survival studies were illustrated by Mudholkar et al. (1995). Its properties have been studied in more detail by Mudholkar and Hutson (1996) and Nassar and Eissa (2003). The cumulative distribution function (c.d.f.), the probability density function (p.d.f.), and the reliability function of the exponentiated Weibull are given respectively by

$$F(x) = \left[1 - e^{-x^{\alpha}}\right]^{\theta}$$

$$f(x) = \alpha\theta x^{\alpha-1} e^{-x^{\alpha}} (1 - e^{-x^{\alpha}})^{\theta-1}, \quad x, \alpha, \theta > 0.$$

and

$$R(t) = 1 - \left[1 - e^{-t^{\alpha}}\right]^{\theta}, \qquad t > 0. \tag{1.1}$$

Properties of EW have been studies in more detail by Mudholkar and Hutson (1996), Jiang and Murthy (1999), Nassar and Eissa (2003) and Gupta et al. (2005). These authors have presented useful applications of the distribution in the modeling of flood data and in reliability. Practically, the failure model of EW is more realistic than that of monotone failure rates and plays an important role to represent such data. The applications of the EW in reliability and survival studies were illustrated by Mudholkar et al. (1995).

Currently, there are little studies for the use of the EW in reliability estimation. Singh et al. (2002), (2005-a,b), and (2006) obtained Bayes estimations of the distribution parameters, reliability function and hazard function with type II censored sample under squared error as well as under LINEX loss function. Nassar and Eissa (2004) obtained the Bayes estimates of the two unknown parameters, the reliability and failure rate function by using Bayes approximation form due to Lindley (1980) under the squared error loss and LINEX loss functions. Elshahat (2006) derived Bayes estimators for the two unknown shape parameters of the EW based on progressive type I interval censored sample. Ashour and Afifiy (2007) considered the analysis of EW family distributed lifetime data observed under type I progressive interval censoring with random removals, maximum likelihood estimators of the parameters and their asymptotic variances are derived. Approximate Bayes estimators for the two unknown shape parameters are derived by Elshahat (2008) based on Lindley (1980) and Tierney and Kadane (1986). Approximate credible intervals for the unknown parameters are obtained with progressive interval censoring. Ashour and Afifiy (2008) derived maximum likelihood estimators for the parameters of EW with type II progressive interval censoring with random removals and their asymptotic variances. Kim et al. (2009) derived the maximum likelihood and Bayes estimators for EW lifetime model using symmetric and asymmetric loss functions.

Succeeding section deals with the computational procedure to obtain the MLE of α and θ and their asymptotic variance-covariance matrix. While section 3 is concerned with Bayes estimators under squared error loss function for the parameters. All estimators are not in nice closed forms, therefore, numerical examples are considered to illustrate the proposed estimators in section 4. Last section includes a brief conclusion.

2. Maximum Likelihood Estimators (MLE)

Consider a life testing experiment in which n units are put on test, successive failure times are recorded, and the test is terminated either at a specified number of failures r (type II) or a specified time T (type I) which ever is reached first. In other words, testing is terminated at time $T^* = min\ (T, x_{(r)})$.This mixture of type I and type II censoring schemes is called the hybrid censoring. The hybrid censoring combines the advantages of both type I and type II censoring.

The main advantage of using a hybrid censored plan in comparison to either type I or type II censoring is that it saves time and money through the reduction achieved in the expected testing time and the expected number of failures observed in the experiment. There is, however, a greater loss in the information provided by the experiment. It is important to note that the results for type I and type II censoring schemes can be obtained as particular cases of the hybrid censoring schemes. The likelihood function for the hybrid censored data may be written as

$$L = \frac{n!}{(n - D^*)!}\left[\prod_{i=1}^{D^*} f(x_{(i)})\right]\left[1 - F(T^*)\right]^{n-D^*} , \qquad (2.1)$$

where D^* denotes the number of units that would fail before the time T^*.

Consider a life testing experiment in which n units are put on a test at time $T_0 = 0$, and successive failure times are recorded. Suppose that censoring occurs in m stages at times $T_j^* > T^*_{j-1}$, $j = 1,2,\ldots,m$. For pre-specified times $T_j > T_{j-1}$ and numbers of failures $D_j > D_{j-1}$, $j = 1,2,\ldots, m$, let $T^*_j = \min (T_j , X_{(Dj)})$, where $X_{(Dj)}$ is the D_j th failure time. Suppose that k_j surviving items are removed (censored) from further observation at the jth stage (T^*_j); the k_j are either fixed or determined independently of the life span X. For example, k_j may be percentages of the remaining live units at T^*_j , $j = 1,2,\ldots,m$. Clearly, K_m represents all remaining live units at time T^*_m, when experimentation is scheduled to terminate. This type of sampling scheme which is a mixture of type I and type II progressive censoring may be called a hybrid progressive censoring.[see Aggarwala et al. (2000)]

Let R be the total number of units that fail up to and including T^*_m . Thus $n = R + \sum_{j=1}^{m} k_j$ the likelihood function for the above data situation is given by

$$L \propto \left[\prod_{i=1}^{R} f(x_{(i)})\right]\left[\prod_{j=1}^{m} (1 - F(T_j^*))^{k_j}\right] \qquad (2.2)$$

Using equations (1.1) and (2.1) the likelihood function is given by

$$L(\underline{x};\alpha,\theta) \propto \left[\alpha^R \theta^R \prod_{i=1}^{R} x^{\alpha-1} e^{-x^\alpha}(1 - e^{-x^\alpha})^{\theta-1}\right]\left[\prod_{j=1}^{m}(1 - (1 - e^{-T_j^{*\alpha}})^{\theta})^{k_j}\right] \qquad (2.3)$$

Taking natural logarithm, we get

$$\ln L(\underline{x};\alpha,\theta) \propto R\ln\alpha + R\ln\theta + (\alpha-1)\sum_{i=1}^{R}\ln x_i - \sum_{i=1}^{R} x_i^{\alpha} + (\theta-1)\sum_{i=1}^{R}\ln(1-e^{-x_i^{\alpha}})$$

$$+\sum_{j=1}^{m} k_j \ln\left[1-(1-e^{-T_j^{*\alpha}})^{\theta}\right]$$

Differentiating $\ln L(\underline{x};\alpha,\theta)$ partially with respect to α and θ and equating the derivatives to zero, we get the following two equations

$$=\frac{R}{\hat{\theta}} + \sum_{i=1}^{R}\ln u_i - \sum_{j=1}^{m} k_j (V^{-\hat{\theta}}-1)^{-1}\ln V = 0$$

and

$$=\frac{R}{\hat{\alpha}} - \sum_{i=1}^{R}(x_i^{\hat{\alpha}}-1)\ln x_i + (\hat{\theta}-1)\sum_{i=1}^{R} q_i u_i^{-1} - \hat{\theta}\sum_{j=1}^{m} z_j \left[V(V^{-\hat{\theta}}-1)\right]^{-1} = 0 \qquad (2.4)$$

where $u_i = (1-e^{-x_i^{\hat{\alpha}}})$, $V = (1-e^{-T_j^{*\hat{\alpha}}})$, $q_i = e^{-x_i^{\hat{\alpha}}} x_i^{\hat{\alpha}} \ln x_i$, and $z_j = e^{-T_j^{*\hat{\alpha}}} T_j^{*\hat{\alpha}} \ln T_j$.

The asymptotic variance covariance matrix of the MLE may be obtained using the following elements

$$-\frac{\partial^2 \ln L}{\partial\theta^2} = \frac{R}{\theta^2} + \sum_{j=1}^{m} k_j V^{\theta} (\ln V)^2 (1-V^{\theta})^{-2}$$

$$-\frac{\partial^2 \ln L}{\partial\alpha^2} = \frac{R}{\alpha^2} + \sum_{i=1}^{R} x_i^{\alpha} (\ln x_i)^2 + (\theta-1)\sum_{i=1}^{R} q_i u_i^{-1} \ln x_i (x_i^{\alpha} + 1 - q_i u_i^{-1})$$

$$+\theta\sum_{j=1}^{m} k_j (1-V^{\theta})^{-1}\left[V^{\theta-1} z_j \ln T_j^{*} - V^{\theta-1} z_j (T_j^{*\alpha} \ln T_j^{*} + (\theta-1) z_j V^{-1})(1-V^{\theta})^{-2}\right]$$

and

$$-\frac{\partial^2 \ln L}{\partial\theta\partial\alpha} = \sum_{j=1}^{m} k_j \left[(1-V^{\theta})^{-1} V^{\theta-1}(q_i + \theta z_j \ln V) + V^{2\theta-1}\theta z_j \ln V\right](1-V^{\theta})^{-2} - \sum_{i=1}^{R} q_i u_i^{-1}$$

Computer facilities and numerical techniques are needed to obtain MLE and their asymptotic variance covariance matrix.

From progressive hybrid censored sample with two parameter exponentiated Weibull distribution, we have the following special cases

1) When $m = 1$, results for single hybrid censoring with likelihood function (2.1) can be obtained as a special case.
2) When T_j^{*} are replaced by T_j and X_{D_j} , respectively, results for multi-stage (progressive) type I and type II censored samples can be obtained as a special cases from current results.
3) If we postpone removing the survivors items until the last stage i. e., $k_1 = k_2 = k_{m-1} = 0$, the corresponding results for the one stage (single) hybrid censored can be obtained.

3. Bayes Estimators

We consider independent non-informative (or vague) priors for the parameters α and θ as

$$g_1(\alpha) \propto \frac{1}{c} \quad , \quad 0 < \alpha < c$$
$$g_2(\theta) \propto \frac{1}{\theta} \quad , \quad \theta > 0 \qquad (3.1)$$

Combining (3.1) with equation (2.3) and using Bayes theorem, the joint posterior distribution is derived as follows:

$$g(\alpha, \theta / \underline{x}) = \frac{1}{\psi_8}\left[\alpha^R \ \theta^{R-1} \prod_{i=1}^{R} x_i^{\alpha-1} \prod_{i=1}^{R} e^{-x_i^{\alpha}} \prod_{i=1}^{R} (1-e^{-x_i^{\alpha}})^{\theta-1}\right] \times \left[\prod_{j=1}^{m}\left[1-(1-e^{-T_j^{\alpha}})^{\theta}\right]^{k_j}\right]$$

where

$$\psi_1 = \int_0^c \int_0^\infty \left[\alpha^R \ \theta^{R-1} \prod_{i=1}^{R} x_i^{\alpha-1} \prod_{i=1}^{R} e^{-x_i^{\alpha}} \prod_{i=1}^{R} (1-e^{-x_i^{\alpha}})^{\theta-1}\right]\left[\prod_{j=1}^{m}\left[1-(1-e^{-T_j^{\alpha}})^{\theta}\right]^{k_j}\right] d\theta\, d\alpha$$

(3.2)

Marginal posterior of unknown parameter is obtained by integrating the joint posterior distribution with respect to the other parameter and hence the marginal posterior of α can be written, after simplification, as

$$g(\alpha / x) = \frac{\alpha^R}{\psi_1} \prod_{i=1}^{R} x_i^{\alpha-1} \prod_{i=1}^{R} e^{-x_i^{\alpha}} \psi_2 ,$$

where

$$\psi_2 = \int_0^\infty \theta^{R-1} \prod_{i=1}^{R} (1-e^{-x_i^{\alpha}})^{\theta-1} \prod_{j=1}^{m}\left[1-(1-e^{-T_j^{\alpha}})^{\theta}\right]^{k_j} d\theta$$

Similarly the marginal posterior of θ can be obtained as

$$g(\theta / x) = \frac{\theta^{R-1}\psi_3}{\psi_1},$$

where

$$\psi_3 = \int_0^c \alpha^R \prod_{i=1}^{R} x_i^{\alpha-1} \prod_{i=1}^{R} e^{-x_i^{\alpha}} \prod_{i=1}^{R} (1-e^{-x_i^{\alpha}})^{\theta-1} \prod_{j=1}^{m}\left[1-(1-e^{-T_j^{\alpha}})^{\theta}\right]^{k_j} d\alpha$$

Under squared error loss function the Bayes estimators for parameters α and θ of EW are will be

$$\tilde{\alpha} = E(\alpha / x) = \int_0^c \alpha\, g(\alpha / x)\, d\alpha,$$

and

$$\tilde{\theta} = E(\theta / x) = \int_0^\infty \theta\, g(\theta / x)\, d\theta,$$

These estimators can be expressed as

$$\tilde{\alpha} = \frac{\psi_4}{\psi_1}$$

and

$$\tilde{\theta} = \frac{\psi_5}{\psi_1} \tag{3.3}$$

where

$$\psi_4 = \frac{1}{\psi_1}\int_0^c\int_0^\infty \alpha^{R+1}\,\theta^{R-1}\prod_{i=1}^{R} x_i^{\alpha-1}\prod_{i=1}^{R} e^{-x_i^\alpha}\prod_{i=1}^{R}(1-e^{-x_i^\alpha})^{\theta-1}\prod_{j=1}^{m}\left[1-(1-e^{-T_j^\alpha})^\theta\right]^{k_j} d\theta\, d\alpha,$$

$$\psi_5 = \frac{1}{\psi_1}\int_0^c\int_0^\infty (\alpha\,\theta)^R \prod_{i=1}^{R} x_i^{\alpha-1}\prod_{i=1}^{R} e^{-x_i^\alpha}\prod_{i=1}^{R}(1-e^{-x_i^\alpha})^{\theta-1}\prod_{j=1}^{m}\left[1-(1-e^{-T_j^\alpha})^\theta\right]^{k_j} d\theta\, d\alpha,$$

Equations (3.3) are hard to obtain. An iterative procedure is applied to solve these equations numerically using "MATHCAD" statistical package.

4. Numerical Illustration

It is clear that, there are no explicit solutions for obtaining new estimators in both non-Bayesian and Bayesian approaches. Therefore artificial data, numerical solution and computer facilities are needed. The main object of this section is to illustrate numerically most of the new theoretical result obtained in sections 2 and 3.

Example (1) Real data-set

To illustrate the usefulness of the proposed estimators obtained in sections 2 and 3 with real situations, we considered the real data-set initially reported by Aarset (1987) to identify the bathtub hazard rate contains lifetime of 50 devices. Mudholkar and Srivastava (1993) used this in context of three-parameter EW distribution to study the suitability of the model with bathtub hazard rat. We obtained the proposed estimators using Aarset (1987) data and summarized it in table (1) by using MATHCAD package.

Bayes estimators are evaluated for the prior hyper-parameter c =4, 10 and 12 and their corresponding values are shown in table (1). Table (1) revealed that the Bayes estimators are not seems very sensitive with variation of "c". It is also worth mentioned that the Bayes estimators are not very far from the estimated values of MLE.

Example (2)

Using MATHCAD package, maximum likelihood estimators and Bayesian (non-informative) estimators for α and θ are obtained. If U has a uniform (0,1) random number, then $x = \left[-\ln(1-U^{1/\theta}\right]^{1/\alpha}$ follows the EW distribution. 100

random samples of sizes 10, 20 and 30 are generated from the two EW distribution with various combinations of α and θ ($\alpha = 2$ and $\theta = 1.5$), ($\alpha = 2$ and $\theta = 2$), ($\alpha = 3$ and $\theta = 1.5$) and ($\alpha = 3$ and $\theta = 2$), it is assumed that number of surviving items k_j are removed from further observation when the number of stages (m) are three since $(j = 1,2)$. By solving the two equations defined in (4.4), the maximum likelihood estimator of α and θ are obtained.

Results are summarized in tables (2) and (4), the square root of mean square error $\sqrt{MSE}$, skewness and kurtosis (to determine Pearson type) and obtaining the probability density function of Pearson type.

From these tables, the following conclusions can be mad

1) For parameter $\alpha = 2$ and $\theta = 2$, the non-Bayesian and Bayesian estimators have a good statistical properties for all samples size.
2) As the sample size increases $\sqrt{MSE}$ of the estimated parameters decreases. This indicates the estimators provide asymptotically normally distributed and consistent estimators for the parameters.

Results in tables (3) and (5) represent the K_p value, Pearson types of estimators and estimate parameters of Pearson types for ML estimators and Bayes estimators of two parameter EW distribution are listed in table (5).

From tables, we conclude that the ML estimators for α and θ have Pearson type IV for all samples size 10, 20 and 30 with the following form

$$f(x) = G\left[k_0 + c_2(x + k_1)^2\right]^{-(2c_2)^{-1}} .\exp\left[\frac{-(c_1 - k_1)}{\sqrt{c_2 k_0}} \tan^{-1}\left(\frac{x + k_1}{\sqrt{\frac{k_0}{c_2}}}\right)\right]$$

where $k_0 = c_0 - \frac{c_1^2 c_2^{-1}}{2}$, $k_1 = \frac{c_1 c_2^{-1}}{2}$

and, c_0, c_1 and c_2 are

$$c_\circ = (4\beta_2 - 3\beta_1)(10\beta_2 - 12\beta - 18)^{-1}\mu_2$$

$$c_1 = a = \sqrt{\beta_1}(\beta_2 + 3)(10\beta_2 - 12\beta_1 - 12)^{-1}\sqrt{\mu_2}$$

$$c_2 = (2\beta_2 - 3\beta_1 - 6)(10\beta_2 - 12\beta_1 - 18)^{-1}$$

Also, we conclude from tables (5), that Bayes estimator for α has Pearson type IV when the parameter $\alpha = 2$, but when $\alpha = 3$ and has Pearson type VI for all samples size 10, 20 and 30, with the following form

$$f(x) = K\left(x - a_1\right)^{m_1}\left(x - a_2\right)^{m_2}$$

with $m_1 = \frac{a + a_1}{c_2(a_2 - a_1)}$ and $m_2 = -\frac{a + a_2}{c_2(a_2 - a_1)}$

where $x > a_2$, a_1 and a_2 are the root of equation $c_\circ + c_1 x + c_2 x^2 = 0$, $m_2 < -1$, K is a constant term and $m_1 + m_2 < 0$. Also, from table (5) we can observed that the Bayes estimator of θ, has Pearson type IV.

Table (1): Bayesian and non-Bayesian estimates for the two shape parameters α and θ of EW distribution for different samples size and various prior parameters "c" under hybrid progressive censored samples with three stages $(m = 3)$ for Aarset (1987) data

n	T_j	D_j	X_{D_j}	ML estimates		Variance		Bayes estimates (Posterior mean)		
				$\hat{\alpha}$	$\hat{\theta}$	$\hat{\alpha}$	$\hat{\theta}$	Prior parameter c	$\tilde{\alpha}$	$\tilde{\theta}$
15	3	4	1	0.285	2.822	0.0028	0.537	5	0.28	2.82
	5	10	6					8	0.27	2.82
	85	12	11					12	0.26	2.80
20	40	4	1	0.208	3.445	0.0013	0.609	5	0.20	3.42
	80	10	6					8	0.23	3.47
	119	12	11					12	0.21	3.45
50	45	4	1	0.173	5.602	0.0012	0.792	5	0.17	5.58
	60	10	6					8	0.15	5.41
	90	12	11					12	0.21	5.87

Table (2): Maximum likelihood estimates, Square root of mean square error (MSE), Skewness β_1 and Kurtosis β_2 for the estimates of various combinations of parameters α and θ and different samples size from EW distribution based on hybrid progressive censored samples with thee stages $(m = 3)$. Number of repetitions $N = 100$

n	Parameters		ML estimates		Square root of MSE		Skewness β_1		Kurtosis β_2	
	α	θ	$\hat{\alpha}$	$\hat{\theta}$	$\hat{\alpha}$	$\hat{\theta}$	$\hat{\alpha}$	$\hat{\theta}$	$\hat{\alpha}$	$\hat{\theta}$
10	2	1.5	1.935	1.668	0.452	0.43	-0.53	0.19	5.24	5.61
	2	2	1.942	2.174	0.451	0.54	-0.51	-0.49	5.39	4.58
	3	1.5	2.961	1.568	0.745	0.42	-0.47	-0.04	4.02	4.09
	3	2	2.954	1.995	0.687	0.50	-0.68	-0.28	4.83	4.20
20	2	1.5	2.183	1.48	0.527	0.37	-0.62	-0.21	4.75	4.66
	2	2	2.138	2.098	0.521	0.51	-0.50	-0.68	4.60	4.62
	3	1.5	3.043	1.588	0.645	0.38	-1.23	0.52	6.77	10.9
	3	2	3.104	2.002	0.609	0.46	-1.29	-0.41	8.25	5.25
30	2	1.5	2.071	1.542	0.446	0.30	-0.65	-1.38	6.33	8.52
	2	2	2.088	2.06	0.399	0.41	-1.21	-1.17	9.26	7.72
	3	1.5	2.98	1.541	0.52	0.30	-1.80	-1.26	11.9	8.02
	3	2	3.138	1.982	0.587	0.37	-1.61	-1.07	9.78	10.16

Table (3): Pearson value K , Pearson types of the sampling distribution for ML estimates $\hat{\alpha}$ and $\hat{\theta}$ of EW distribution and estimate parameters of Pearson types for various combinations of parameters and different samples size. Number of repetitions $N = 100$ and $m = 3$.

n	Param-eters		K value		Pearson type		Estimate parameters of Pearson type for $\hat{\alpha}$				Estimate parameters of Pearson type for $\hat{\theta}$			
	α	θ	$\hat{\alpha}$	$\hat{\theta}$	$\hat{\alpha}$	$\hat{\theta}$	K_0	K_1	C_1	C_2	K_0	K_1	C_1	C_2
10	2	1.5	0.06	0.006	IV	IV	0.12	0.27	0.06	0.117	0.095	0.06	0.02	0.136
	2	2	0.05	0.081	IV	IV	0.12	0.24	0.06	0.122	0.176	0.40	0.08	0.098
	3	1.5	0.13	0.001	IV	IV	0.38	0.89	0.13	0.07	0.125	0.03	0.01	0.095
	3	2	0.17	0.029	IV	IV	0.28	0.79	0.15	0.092	0.173	0.24	0.04	0.094
20	2	1.5	0.14	0.011	IV	IV	0.16	0.51	0.1	0.094	0.088	0.09	0.02	0.114
	2	2	0.06	0.216	IV	IV	0.17	0.33	0.07	0.102	0.15	0.71	0.12	0.081
	3	1.5	0.52	0.02	IV	IV	0.14	1.28	0.24	0.096	0.066	0.09	0.03	0.171
	3	2	0.35	0.035	IV	IV	0.15	0.80	0.20	0.123	0.13	0.20	0.05	0.123
30	2	1.5	0.07	0.424	IV	IV	0.11	0.25	0.07	0.134	0.033	0.45	0.11	0.12
	2	2	0.21	0.279	IV	IV	0.07	0.36	0.10	0.143	0.076	0.49	0.12	0.124
	3	1.5	0.59	0.352	IV	IV	0.07	0.86	0.22	0.13	0.038	0.41	0.1	0.121
	3	2	0.58	0.123	IV	IV	0.09	1.03	0.24	0.119	0.064	0.24	0.07	0.156

Table (4): Bayes estimates, Square root of (MSE), Skewness β_1 and Kurtosis β_2 for the estimates of various combinations of parameters α and θ and different samples size from EW distribution based on progressive hybrid censored samples with thee stages $(m = 3)$. Number of repetitions $N = 100$ and prior parameter $c = 4.5$

n	Paramet-ers		Bayes estimates (Posterior mean)		Square root of MSE		Skewness β_1		Kurtosis β_2	
	α	θ	$\tilde{\alpha}$	$\tilde{\theta}$	$\tilde{\alpha}$	$\tilde{\theta}$	$\tilde{\alpha}$	$\tilde{\theta}$	$\tilde{\alpha}$	$\tilde{\theta}$
10	2	1.5	2.18	1.54	0.45	0.31	-1.24	-0.84	8.52	7.81
	2	2	2.10	1.90	0.43	0.35	-0.99	-2.05	7.44	11.6
	3	1.5	2.93	1.53	0.46	0.33	-2.8	-0.97	18.48	6.25
	3	2	2.94	1.83	0.46	0.36	-2.9	-2.19	18.61	13.9
20	2	1.5	2.34	1.44	0.61	0.36	-0.85	-0.33	5.15	4.63
	2	2	2.23	1.97	0.60	0.4	-0.56	-1.53	4.71	7.38
	3	1.5	3.05	1.58	0.49	0.32	-2.66	-0.92	15.45	8.75
	3	2	3.09	1.93	0.50	0.37	-2.6	-1.57	15.66	9.85
30	2	1.5	2.20	1.52	0.51	0.3	-0.72	-1.32	5.99	7.89
	2	2	2.16	1.99	0.44	0.36	-1.17	-1.81	8.78	10.4
	3	1.5	3.03	1.54	0.45	0.30	-3.11	-1.38	20.6	8.72
	3	2	3.14	1.95	0.52	0.33	-2.61	-2.14	15.8	14.6

Table (5): Pearson value K , Pearson types of the sampling distribution for Bayes estimates $\tilde{\alpha}$ and $\tilde{\theta}$ for EW distribution and estimate parameters of Pearson types for various combinations of parameters and different samples size. Number of repetitions $N = 100$ and prior parameter $c = 4.5$

n	Parame-ers		K value		Pearson type		Estimate parameters of Pearson type for $\widehat{\alpha}$				Estimate parameters of Pearson type for $\widehat{\theta}$			
	α	θ	$\tilde{\alpha}$	$\tilde{\theta}$	$\tilde{\alpha}$	$\tilde{\theta}$	K_0	K_1	C_1	C_2	K_0	K_1	C_1	C_2
10	2	1.5	0.27	0.10	IV	IV	0.077	0.5	0.12	0.13	0.05	0.19	0.05	0.14
	2	2	0.17	1.43	IV	VI	0.091	0.4	0.1	0.13	-0.03	1.07	0.21	0.01
	3	1.5	2.80	0.24	VI	IV	-0.26	2.1	0.39	0.10	0.053	0.39	0.09	0.11
	3	2	3.31	1.10	VI	VI	-0.35	2.4	0.42	0.09	-0.01	0.75	0.18	0.12
20	2	1.5	0.31	0.03	IV	IV	0.138	0.9	0.14	0.08	0.081	0.15	0.03	0.11
	2	2	0.11	1.62	IV	VI	0.171	0.5	0.09	0.01	-0.08	1.82	0.23	0.06
	3	1.5	4.10	0.10	VI	IV	-0.59	3.3	0.47	0.07	0.048	0.19	0.06	0.15
	3	2	3.32	0.50	VI	IV	-0.44	2.8	0.45	0.08	0.039	0.56	0.14	0.12
30	2	1.5	0.11	0.43	IV	IV	0.127	0.4	0.09	0.12	0.034	0.48	0.11	0.11
	2	2	0.20	0.93	IV	IV	0.079	0.4	0.11	0.14	0.007	0.88	0.19	0.11
	3	1.5	4.01	0.40	VI	IV	-0.46	2.6	0.46	0.09	0.033	0.42	0.10	0.12
	3	2	2.80	0.83	VI	IV	-0.34	2.5	0.43	0.09	0.011	0.63	0.16	0.13

References

[1] M.V. Aarset, How to identify a bathtub hazard rate, IEEE Transactions Reliability, R-36(1987), 106-108.

[2] S.K. Ashour, W.M. Afify, Statistical analysis of exponentiated Weibull family under type I progressive interval with random removals, Applied Sciences Research, 3(12) (2007), 1851-1863.

[3] S.K. Ashour, W.M. Afify, Estimation of the parameters of exponentiated Weibull family with type II progressive interval censoring with random removals, Applied Sciences Research, 4(11) (2008), 1428- 1442.

[4] N. Balakrishnan, R. Aggarwala, Progressive censoring theory, Methods and Applications. Birkhauser, Boston, 2000.

[5] M.A.T. Elshahat, Bayesian estimation of the parameters of the exponentiated Weibull distribution with progressively type I interval censored sample, Journal of Commerce Research, Faculty. of Commerce, Zagazig University, Egypt, 28(2) (2006), 5-21.

[6] M.A.T. Elshahat, Approximate Bayes estimators for the exponentiated Weibull parameters with progressive interval censoring, The 20th Annual Conference on Statistics & Modeling In Human & Social Science, Faculty of Economic & Political Science, Cairo University, Egypt, 2008, 123-136.

[7] R. Jiang, O.W.P. Murthy, The exponentiated Weibull family: a graphical approach, IEEE Transaction Reliability, 48 (1999), 68-72.

[8] C. Kim, J. Jung, Y. Chung, Bayesian estimation for the exponentiated Weibull model type II progressive censoring, Statistical Papers, 52(1) (2009), 53-70.

[9] D.V. Lindley, Approximate Bayesian method, Trabjos de Estadistica, 31 (1980), 223-237.

[10] G.S. Mudholkar, D.K. Srivastava, Exponentiated Weibull family for analyzing bathtub failure-real data, IEEE Transaction Reliability, 42 (1993), 299-302.

[11] G.S. Mudholkar, D.K. Srivastava, M. Freimes, The exponentiated Weibull family: a reanalysis of the Bus-Motor-failure data, Technometrics, 37 (1995), 436-445.

[12] G.S. Mudholkar, A.D. Hutson, The exponentiated Weibull family: some properties and a flood data application, Communications Statistics- Theory and Methods, 25 (12) (1996), 3059-3083.

[13] S. Nadarajah, A. K. Gupta, On the moments of the exponentiated Weibull distribution, Communications in Statistics – Theory and Methods, 34 (2005), 253-256.

[14] M.M. Nassar, F.H. Eissa, On the exponentiated Weibull distribution, Communications in Statistics – Theory and Methods, 32 (2003), 1317-1333.

[15] M.M. Nassar, F.H. Eissa, Bayesian estimation for the exponentiated Weibull model, Communications in Statistics – Theory and Methods, 33 (2004), 2343- 2362.

[16] U. Singh, P.K. Gupta, S.K. Upadhaya, Estimation of exponentiated Weibull shape parameters under LINEX loss function, Communications in Statistics – Simulation, 31(4) (2002), 523-537.

[17] U. Singh, P.K. Gupta, S.K. Upadhaya, Estimation of parameters for exponentiated Weibull family under type II censoring scheme, Communications in Statistics–Simulation, 48(2005-a), 509- 523.

[18] U. Singh, P.K. Gupta, S.K. Upadhaya, Estimation of three parameters exponentiated Weibull distribution under type II censoring, Statistical Planning and Inference, 144 (2005-b), 350-372.

[19] U. Singh, P.K. Gupta, S.K. Upadhaya, Some point estimates for the shape parameters of exponentiated Weibull family, Korean Statistical Society, 35 (2006), 63-77.

[20] A. Zellner, Bayesian estimation and prediction using asymmetric loss functions, American Statistical Association, 81(394) (1986), 446-451.

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