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A Proposed Approach for Solving Rough Bi-Level Programming Problems by Genetic Algorithm

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Abstract

We present a new framework to hybridize the rough set theory with the bi-level programming problem, called 'Rough Bi-level Programming Problems (RBLPPs).This paper studies and designs a genetic algorithm (GA) for solving (RBLPPs) by constructing the fitness function of the upper – level programming problems based on the definition of through feasible degree. Finally, a numerical example will be introduced to show the proposed method.

Keywords: Rough set, Rough Bi- level linear programming problems, Rough optimality, Rough feasibility, Rough feasible degree; fitness function; genetic algorithm.

1 Introduction

Since it was pioneered by Pawalk in mid 1980’s and [4] rough set theory has become a hot topic of great interest to researchers in several fields and has been applied to many domains such as pattern recognition, data mining , artificial intelligence, image processing ,machine learning, and medical application. This new approach proved to be useful in many applications such as optimization theory. For mathematical programming problems (MPPs) in the crisp form, the aim is to maximize or minimize an objective function over certain set of feasible solutions. But in many practical situations, the decision maker may not be in a position to specify the objective and/or the feasible set precisely but rather can specify them in a ‘’rough sense’’. In such situations, it is desirable to use some rough programming type of modeling so as to provide more flexibility to the decision maker. Towards this objective, we present a new framework to hybridize the rough set theory with the bi-level programming problem, called ‘Rough Bi-level Programming Problems (RBLPPs).
Since the roughness may appear in a Bi-level Programming Problems in many ways (e.g. the feasible set may be rough and/or the goals may be rough), the definition of rough Bi-level Programming Problems is not unique. This leads us to propose a new classification and characterization of the rough Bi-level Programming Problems (RBLPPs), according to the place of roughness in the problem. We classified the RBLPPs into the following classes:

1. Bi-level programming problems with rough feasible set, crisp objective function of the upper level and crisp objective function of the lower level.
2. Bi-level programming problems with rough feasible set, crisp objective function of the upper level and rough objective function of the lower level.
3. Bi-level programming problems with rough feasible set, rough objective function of the upper level and crisp objective function of the lower level.
4. Bi-level programming problems with crisp feasible set and rough objective function of the upper level and crisp objective function of the lower level.
5. Bi-level programming problems with crisp feasible set and crisp objective function of the upper level and rough objective function of the lower level.
6. Bi-level programming problems with crisp feasible set and rough objective function of the upper level and rough objective function of the lower level.
7. Bi-level programming problems with rough feasible set and rough objective function of the upper level and rough objective function of the lower level.

New definitions concerning rough optimal sets, rough optimal value, rough global optimality and rough feasibility were also proposed and discussed.
This paper studies and designs a genetic algorithm (GA) of (RBLPPs) by constructing the fitness function of the upper –level programming problems based on the definition of the rough feasible degree.

2. Rough set and approximation space

Rough set theory has been proven to be an excellent mathematical tool dealing with vague description of objects [4]. A fundamental assumption in rough set theory is that any object from a universe is perceived trough available information, and such information may not be sufficient to characterize the object exactly. Pawlak has proposed rough set methodology as a new approach in handling classificatory analysis of vague concepts [4]. In this methodology any vague concept is characterized by a pair of precise concepts called the lower and the upper approximations. Rough set theory is based on equivalence relations describing partitions made of classes of indiscernible objects.

Let U be a non-empty finite set of objects, called the universe, and $E \subseteq U \times U$ be an equivalence relation on U. The ordered pair $A = (U, E)$ is called an approximation space generated by E on U. The equivalence relation E generates a partition $U/E = \{Y_1, Y_2, ..., Y_m\}$ where $Y_1, Y_2, ..., Y_m$ are the equivalence classes (also called elementary sets or granules generated by E, represent elementary portion of knowledge we are able to perceive due to E) of the approximation space A. In rough set theory, any subset $M \subseteq U$ is described by the elementary sets of A, and the two sets

$E_*(M) = \bigcup \{Y_i \in U/E \mid Y_i \subseteq M\}$

$E^*(M) = \bigcup \{Y_i \in U/E \mid Y_i \cap M \neq \phi\}$ are called the lower and the upper approximations of M, respectively. Therefore, $E_*(M) \subseteq M \subseteq E^*(M)$. The difference between the upper and the lower approximations is called the boundary of M and is denoted by $BN_E(M) = E^*(M) - E_*(M)$. The set M is called exact in A iff $BN_E(M) = \phi$; otherwise the set M is inexact (rough) in A [5, 6, 8, 9].

As we can see from the definition approximations are expressed in terms of granules of knowledge. The lower approximation of a set is union of all granules which are entirely included in the set; the upper approximation - the union of all granules which have non –empty intersection with the set; the boundary region of the set is the difference between the upper and lower approximation [4]. This definition is clearly depicted in figure 1.

3. Classes of Rough Bi-level Programming Problems (RBLPPs)

The most typical Bi-level Programming Problems (BLPPs) can be stated as:

$$\max_{x_1} F(x_1, x_2)$$

(1)

where x_2 solve

$$\max_{x_2} \; f\left(x_1, x_2\right) \tag{2}$$

subject to

$x = (x_1, x_2) \in M$

where F and f are called the objective functions of the upper and lower level DM, and M is called the feasible set of the problem. In the above formulation, it is assumed that all entries of F , f and M are defined in the crisp sense, and ‘’max’’ is a strict imperative. However, in many practical situations it may not be reasonable to require that the feasible set or the objective functions in bi-level programming problems be specified in precise crisp terms. In such situations, it is desirable to use some type of rough modeling and this leads to the concept of rough bi-level programming problems. When decision is to be made in a rough environment, many possible modifications of the above bi-level programming model exist. Thus, rough bi-level programming models are not uniquely defined as it will very much depend upon the type of roughness and its specification as prescribed by the decision maker. Therefore, the rough bi-level programming problems can be broadly classified as:

1st Class: Bi-level programming problems with rough feasible set, crisp objective function of the upper level and crisp objective function of the lower level.

2nd Class: Bi-level programming problems with rough feasible set, crisp objective function of the upper level and rough objective function of the lower level.

3rd Class: Bi-level programming problems with rough feasible set, rough objective function of the upper level and crisp objective function of the lower level.

4th Class: Bi-level programming problems with crisp feasible set and rough objective function of the upper level and crisp objective function of the lower level.

5th Class: Bi-level programming problems with crisp feasible set and crisp objective function of the upper level and rough objective function of the lower level.

6th Class: Bi-level programming problems with crisp feasible set and rough objective function of the upper level and rough objective function of the lower level.

7th Class: Bi-level programming problems with rough feasible set and rough objective function of the upper level and rough objective function of the lower level.

In RBLPPs, wherever roughness exists, new concepts like rough feasibility and rough global optimality come in the front of our interest. The **rough feasibility** arises only in the 1st , 2nd , 3rd and 7th classes, where solutions have different degrees of feasibility (surely-feasible, possible-feasible, and surely-not feasible) On the other hand, the **rough global optimality** arises in all classes of the RBLPPs where solutions have different degrees of global optimality (surely-global optimal, possible- global optimal, and surely-not global optimal). As a result of these new concepts, the optimal value of the objectives and the optimal set of the problem are defined in rough sense.

Definition 1:
In RBLPPs, the optimal value of the objective function of the upper level DM is a rough real number $\overline{F}$, that is determined roughly by lower and upper bounds denoted by $\overline{F}_*$, $\overline{F}^*$, respectively.

Remark 1:
If $\overline{F}_* = \overline{F}^*$ then the optimal value $\overline{F}$ is exact, otherwise $\overline{F}$ is rough.
Also, the single optimal set of the crisp bi-level programming problem is replaced by four optimal sets covering all possible degrees of feasibility and optimality. See table1,

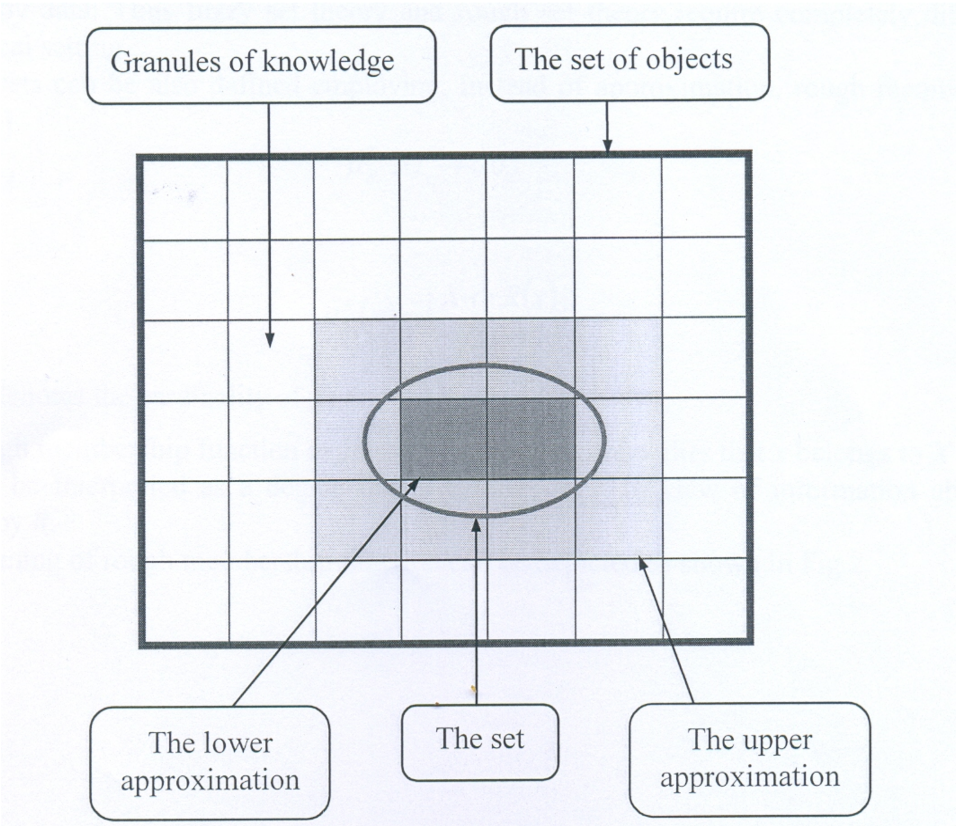

Figure1.

Definition 2:
The set of all surely-feasible, surely- global optimal solutions is denoted by FO_{ss} .

Definition 3:
The set of all surely-feasible, possibly-global optimal solutions is denoted by FO_{sp} .

Definition 4:
The set of all possibly -feasible, surely -global optimal solutions is denoted by FO_{ps} .

Optimality			
Possibly	**Surly**		
FO_{pp}	FO_{ps}	**Possibly**	**Feasibility**
FO_{sp}	FO_{ss}	**Surly**	

Table 1: Possible degrees of feasibility and optimality for problem (RBLPPs).

Definition 5:
The set of all possibly -feasible, possibly -global optimal solutions is denoted by FO_{pp}

4. 1st Class RBLPPs

Suppose that $A=(U,E)$ is an approximation space generated by equivalence relation E on an universe U .A rough bi-level programming problem of the 1stclass takes the following form:

$$\max_{x_1} F(x_1,x_2) \tag{3}$$

where x_2 solve

$$\max_{x_2} f(x_1,x_2) \tag{4}$$

subject to

$$E_*(M) \subseteq M \subseteq E^*(M)$$

where $M \subseteq U$ is a rough set in the approximation space $A=(U,E)$ representing the feasible set of the problem. The sets $E^*(M)=M^*$ and $E_*(M)=M_*$ represent the notion of **rough feasibility** of problem (3) - (4), where M^* is called the set of all possibly –feasible solutions and M_* is called the set of all surely-feasible solutions. On the other hand $U-M^*$ is called the set of all surely-not feasible solutions [3].

Proposition 1:
In problem (3) - (4), the lower and upper bounds of the optimal objective value for the upper level problem $\overline{F}$ are given by

$$\overline{F}_* = \sup\{a,b\}$$

$$\overline{F}^* = \sup\{a,c\}$$

where

$$a = \max_{(x_1,x_2)\in M_*} F(x_1,x_2)$$

$$b = \sup \bigcup_{\substack{Y \in U/E \\ Y \subseteq M_{BN}}} \left\{ \min_{(x_1,x_2) \in Y} F(x_1, x_2) \right\}$$

$$c = \max_{(x_1,x_2) \in M_{BN}} F(x_1, x_2)$$

Definition 6:
A solution $\left(x_1^*, x_2^*\right) \in M^*$ is surely -global optimal solution iff $F(x_1^*, x_2^*) = \overline{F}^*$.
Definition 7:
A solution $\left(x_1^*, x_2^*\right) \in M^*$ is possibly -global optimal solution iff $F(x_1^*, x_2^*) \geq \overline{F}_*$.
Definition 8:
A solution $\left(x_1^*, x_2^*\right) \in M^*$ is surely -not global optimal solution iff $F(x_1^*, x_2^*) < \overline{F}_*$.
Definition 9:
The optimal sets of the 1st class RBLPP for the upper level problem are defined as:

$$FO_{ss} = \left\{ \left(x_1^*, x_2^*\right) \in M_* \middle| \; F(x_1^*, x_2^*) = \overline{F}^* \right\}$$

$$FO_{sp} = \left\{ \left(x_1^*, x_2^*\right) \in M_* \middle| \; F(x_1^*, x_2^*) \geq \overline{F}_* \right\}$$

$$FO_{ps} = \left\{ \left(x_1^*, x_2^*\right) \in M^* \middle| \; F(x_1^*, x_2^*) = \overline{F}^* \right\}$$

$$FO_{pp} = \left\{ \left(x_1^*, x_2^*\right) \in M^* \middle| \; F(x_1^*, x_2^*) \geq \overline{F}_* \right\}.$$

Proposition 2:
The lower and upper bounds of the optimal objective value for the lower level problem $\bar{f}$ are given by

$$\bar{f}_* = \sup\{a', b'\}$$

$$\bar{f}^* = \sup\{a', c'\}$$

where

$$a' = \max_{(x_1^*, x_2) \in M_*} f(x_1^*, x_2)$$

$$b' = \sup \bigcup_{\substack{Y \in U/E \\ Y \subseteq M_{BN}}} \left\{ \min_{(x_1^*, x_2) \in Y} f(x_1^*, x_2) \right\}$$

$$c' = \max_{(x_1^*, x_2) \in M_{BN}} f(x_1^*, x_2)$$

Definition 10:
A solution $\left(x_1^*, x_2'\right) \in M^*$ is surely -global optimal solution iff $f(x_1^*, x_2') = \bar{f}^*$.
Definition 11:
A solution $\left(x_1^*, x_2'\right) \in M^*$ is possibly -global optimal solution iff $f(x_1^*, x_2') \geq \bar{f}_*$.
Definition 12:
A solution $\left(x_1^*, x_2'\right) \in M^*$ is surely not -global optimal solution iff $f(x_1^*, x_2') < \bar{f}_*$.
Definition 13:
The optimal sets of the 1st class RBLPP for the lower level problem are defined as:

$$FO'_{ss} = \left\{ \left(x_1^*, x_2'\right) \in M_* \middle| \; f(x_1^*, x_2') = \bar{f}^* \right\}$$

$$FO'_{sp} = \left\{ \left(x_1^*, x_2'\right) \in M_* \,\middle|\, f(x_1^*, x_2') \geq \bar{f}_* \right\}$$
$$FO'_{ps} = \left\{ \left(x_1^*, x_2'\right) \in M^* \,\middle|\, f(x_1^*, x_2') = \bar{f}^* \right\}$$
$$FO'_{pp} = \left\{ \left(x_1^*, x_2'\right) \in M^* \,\middle|\, f(x_1^*, x_2') \geq \bar{f}_* \right\}.$$

5. Design of GA for RBLPPs

The basic idea for solving RBLPP by GA is: firstly, choose the initial population satisfying the constraints (by using the above definitions and proposition 1,2) , the lower-level decision maker makes the corresponding rough optimal reaction and evaluate the individuals according to the fitness function constructed by the rough feasible degree, until the optimal solution searched by the genetic operation over and over.

6. Design of the Fitness Function

To solve the problem RBLPPs by GA, the definition of the feasible degree is firstly introduced and the fitness function is constructed to solve the problem by GA. Let d denote the large enough penalty interval of the rough feasible region for each $(x_1, x_2) \in X$.

Definition 14:

Let $\theta \in [0,1]$ denotes the rough feasible degree of satisfying the rough feasible region, and describe it by the following function:

$$\theta = \begin{cases} 1, & if \ \| x_2 - Y(x_1) \| = 0 \\ 1 - \left\| \dfrac{x_2 - Y(x_1)}{d} \right\|, & if \ 0 < \| x_2 - Y(x_1) \| \leq d \\ 0, & if \ \| x_2 - Y(x_1) \| > d \end{cases} \tag{5}$$

where $\|.\|$ denotes the norm and $Y(x_1)$ denote the rough optimal solution of lower level problem .

Furthers, the fitness function of the GA can be stated as:

$eval(v_k) = (F(x_1, x_2) - F_{\min}) * \theta$

where $F_{\min}$ is the rough minimal value of $F(x_1, x_2)$ on X .

7. The algorithm

Step 1. Initialization, give the population scale *M*, the maximal iteration generation

MAXGEN, and let the generation $t = 0$ [7].

Step 2. The initial population, *M* individuals are randomly generated in X, making up the nitial population.

Step 3. Evaluation the rough minimal value of upper level problem ($F_{\min}$) and the rough optimal solution of the lower level ($Y(x_1)$) by using proposition 1,2 respectively.

Step 4. Computation of the fitness function. Evaluate the fitness value of the population according to formula (5).

Step 5. Selection, Select the individual by roulette wheel selection operator [2].

Step 6. Crossover, in this step, first, a random number $P_c \in [0,1]$ is generated. This number is the percentage of the population on which the crossover is performed. Then, two individuals are selected randomly from the population as parents. Children are generated using the following procedure:

Random integer c is generated in the interval $[1, l-1]$, where l is the number of components of an individual. The c first components of the children are the same components as respective parents (i.e. the first child from the first parent and the second child from the second parent). The remaining components are selected according to the following rules:

(i) The $(c+i)\,th$ component of the first child is replaced by the $(l-i+1)\,th$ component of the second parent (for $i = 1,2,\ldots,l-c$).

(ii) The $(c+i)\,th$ component of the second child is replaced by the $(l-i+1)\,th$ component of the first parent (for $i = 1,2,\ldots,l-c$).

For example we assume c =5, we obtained the following children

Parents	children
10110 1100	10110 0100
11010 0010	11010 0011

Note that the proposed operator generates individuals with more variety in comparison with the standard operator, because this operator can generate different children from similar parents, where standard operators cannot [1].

Step 7. Mutation, In this step, first, a random number $P_m \in [0,1]$ is generated. This number is the percentage of the population on which the mutation performed. Then one individual is selected randomly from the population. An integer random number u is generated in the interval $[1, l]$, where l is the length of the individual. For generating the new individual, the u th component is changed to 0, if it was initially 1 and to 1 if it was initially 0 [1].

Step 8. Terminations, Jude the condition of the termination. When t is larger than the maximal iteration number, stop the GA and output the rough optimal solution Otherwise, let $t = t + 1$, turn to Step 3.

8. An Example

Let U be a universal set defined as $U=\left\{x=\left(x_1,x_2\right)\in R^2 \,\middle|\, x_1^2+x_2^2\le 9\right\}$ and let K be a polytope generated by the following closed halfplanes

$h_1=x_1+x_2-2\le 0$, $h_2=x_2-x_1-2\le 0$

$h_3=x_2-x_1+2\ge 0$, $h_4=x_1+x_2+2\ge 0$

Suppose that E is an equivalence relation on U such that

$U/E=\{E_1,E_2,E_3\}$

$E_1=\{x\in U : x \text{ is an interior point of polytope } K\}$

$E_2=\{x\in U : x \text{ is a boundary point of polytope } K\}$

$E_3=\{x\in U : x \text{ is an exterior point of polytope } K\}$

Consider the following 1st Class RBLPPs

$$\max_{x_1\in M} F(x_1,x_2)=-(x_1-2.5)^2-x_2^2 \tag{6}$$

where x_2 solve

$$\max_{x_2\in M} f(x_1,x_2)=x_1+x_2 \tag{7}$$

subject to

$M_*=E_1\bigcup E_2$, $M^*=E_1\bigcup E_2\bigcup E_3$

where M is a rough feasible region in the approximation space $A=(U,E)$ and M_* , M^* are the lower and the upper approximation of M ; respectively. Also, the boundary region of M is given by $M_{BN}=E_3$.

The solution

By using the above genetic algorithm we found the following results:

1) Give the population scale $M=100$, the maximal iteration generation MAXGEN =20, and let the generation $t=0$.

2) Computation of the fitness function. Evaluate the fitness value of the population according to formula (5) as:

$eval(v_k)=(-(x_1-2.5)^2-x_2^2-20.44)*\theta$

To evaluate $F_{\min}$, we use proposition 1 and the given definitions

*** Finding the rough minimal value of the upper level problem** $F_{\min}=\overline{F}$

Where $F(x_1,x_2)=(x_1-2.5)^2+x_2^2$

$\overline{F}_*=\sup\{a,b\}$

$a=\max\limits_{(x_1,x_2)\in M_*} F(x_1,x_2)=F(-2.02,0)=20.44$

$\because \inf\limits_{(x_1,x_2)\in E_3} F(x_1,x_2)=F(0.0001,0)=6.25$

$\therefore b=\sup \bigcup\limits_{\substack{Y\in U/E,\\ Y\subseteq M_{BN}}} \left\{\min\limits_{(x_1,x_2)\in Y} F(x_1,x_2)\right\}=\min\limits_{(x_1,x_2)\in E_3} F(x_1,x_2)=F(0.0001,0)=6.25$

$\therefore \overline{F}_*=20.44$

$\because \overline{F}^* = \sup\{a,c\}$

$\because \sup_{(x_1,x_2)\in E_3} F(x_1,x_2) = F(-0.0001,0) = 6.25$

$\therefore c = \max_{(x_1,x_2)\in M_{BN}} F(x_1,x_2) = \max_{(x_1,x_2)\in E_3} F(x_1,x_2) = F(-0.0001,0) = 6.25$

$\therefore \overline{F}^* = 20.44$

$F_{\min} = \overline{F} = 20.44$

**** Finding the rough optimal value of the upper level problem $\overline{F}$:**

$\overline{F}_* = \sup\{a,b\}$

$a = \max_{(x_1,x_2)\in M_*} F(x_1,x_2) = F(2,0) = -0.25$

$\because \inf_{(x_1,x_2)\in E_3} F(x_1,x_2) = F(-3,0) = -30.25$

$\therefore b = \sup \bigcup_{\substack{Y\in U/E,\\ Y\subseteq M_{BN}}} \left\{ \min_{(x_1,x_2)\in Y} F(x_1,x_2) \right\} = \min_{(x_1,x_2)\in E_3} F(x_1,x_2) = F(-3,0) = -30.25$

$\therefore \overline{F}_* = -0.25$

$\because \overline{F}^* = \sup\{a,c\}$

$\because \sup_{(x_1,x_2)\in E_3} F(x_1,x_2) = F(2.5,0) = 0$

$\therefore c = \max_{(x_1,x_2)\in M_{BN}} F(x_1,x_2) = \max_{(x_1,x_2)\in E_3} F(x_1,x_2) = F(2.5,0) = 0$

$\therefore \overline{F}^* = 0$

$\overline{F} \in [-0.25,\ 0]$

Finding the rough optimal sets for the upper level problem:

$FO_{ss} = \{ (x_1^*, x_2^*) \in E_1 \cup E_2 : F(x_1^*, x_2^*) = 0 \} = \phi$

$FO_{sp} = \{ (x_1^*, x_2^*) \in E_1 \cup E_2 : F(x_1^*, x_2^*) = -0.25 \} = \{ (\mathbf{2,0}) \}$

$FO_{ps} = \{ (x_1^*, x_2^*) \in E_1 \cup E_2 \cup E_3 : F(x_1^*, x_2^*) = 0 \} = \{ (2.5,0) \}$

$$FO_{PP} = \{ (x_1^*, x_2^*) \in E_1 \cup E_2 \cup E_3 : F(x_1^*, x_2^*) \geq -0.25 \} \\ = \{ (x_1,x_2) : (x_1 - 2.5)^2 + x_2^2 \leq 0.25 \}$$

But for evaluate θ , we find $Y(x_1)$ by using proposition 2 such that:

***** Finding the rough optimal value of the lower level DM $Y(x_1) = \bar{f} = x_2 + 2$ for fixed $x_1^* = 2$**

$\bar{f}_* = \sup\{a', b'\}$

$a' = \max_{(x_1^*,x_2)\in[-2,2]} f(x_1^*, x_2) = f(2,2) = 4$

$\because \inf_{(x_1^*,x_2)\in E_3} f(x_1^*, x_2) = f(2,-3) = -1$

$\therefore b' = \sup \bigcup_{\substack{Y\in U/E,\\ Y\subseteq M_{BN}}} \left\{ \min_{(x_1^*,x_2)\in Y} f(x_1^*, x_2) \right\} = \min_{(x_1^*,x_2)\in E_3} f(x_1^*, x_2) = f(2,-3) = -1$

$\bar{f}_* = 4$

$\bar{f}^* = \sup\{a', c'\}$

$\because \sup_{(x_1^*, x_2) \in E_3} f(x_1^*, x_2) = f(2,3) = 5$

$\therefore c' = \max_{(x_1^*, x_2) \in M_{BN}} f(x_1^*, x_2) = \max_{(x_1^*, x_2) \in E_3} f(x_1^*, x_2) = f(2,3) = 5$

$\therefore \bar{f}^* = 5$

$\bar{f} \in [4, 5]$.

Finding the rough optimal sets for the lower level

$FO'_{ss} = \{ (x_1^*, x_2') \in E_1 \cup E_2 : f(x_1^*, x_2') = \bar{f}^* = 5 \} = \phi$

$FO'_{sp} = \{ (x_1^*, x_2') \in E_1 \cup E_2 : f(x_1^*, x_2') = \bar{f}_* = 4 \} = \{2\}$

$FO'_{ps} = \{ (x_1^*, x_2') \in E_1 \cup E_2 \cup E_3 : f(x_1^*, x_2') = \bar{f}^* = 5 \} = \{3\}$

$FO'_{PP} = \{ (x_1^*, x_2') \in E_1 \cup E_2 \cup E_3 : f(x_1^*, x_2') \geq 4 \} = [2,3]$

then $Y(x_1) = 4$ and $d = 8$

3) The operators of the genetic algorithm [2, 7] are applied such that $P_c = 0.17$, $P_m = 0.3$.

Obtain the optimal solution $(x_1, x_2, \theta) = (2.5, -3, 0.5152)$.

Elapsed time is 4.45 minutes.

9. Conclusions

This paper proposes a new formulation, classification and definition of the rough bi-level programming problems. Only the 1st class of the RBLPPs is defined and its optimal sets are characterized in this work. Also, it designs GA for solving RBLPPs of which the rough optimal solution of the lower-level problem is dependent on the upper-level problem.

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