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Quality Assessment for an Image Registration Method

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Abstract

One of the challenges in the development of image registration algorithms is their validation. In this article first we overview a recently proposed image registration method [2] and then assess its quality with validation strategies such as visual assessment, masked mean warping index and similarity metric. Secondly we compare the method with some other well-known image registration methods such as curvature, elastic, fluid and divergence - curl based methods mostly developed under the NSF project [7].

Mathematics Subject Classifications: 65D18, 65J05, 97N40

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1 Preliminaries

Image registration can be defined as a process of determining the optimal transform that maps points from a template image, $\mathbf{T}(\mathbf{x})$, to the corresponding points in a reference image, $\mathbf{R}(\mathbf{x})$. Object or motion tracking, detecting tumors, locating diseased areas, remote sensing are popular applications of image registration. Unfortunately, no general theory for image registration has been yet established. Hence, no single standard method of image registration has emerged. A detailed treatment for some well-known image registration techniques can be seen at [1].

The structure of this paper is as follows. First section overview the image registration method developed in [7] and presented in [2]. In the second section we assess the validation of this method. We use validation techniques such as masked mean warping index, similarity metric and simulation. In the third and fourth sections we review some related image registration methods and compare the method in [2] with these methods.

2 Formulation of Image Registration Problem

Assume that both template $\mathbf{T}(\mathbf{x})$ and reference $\mathbf{R}(\mathbf{x})$ images are defined on the same domain Ω. We can formulate the image registration problem as

$$\mathcal{J}[\mathbf{R}(\mathbf{x}), \mathbf{T}(\mathbf{x}); \boldsymbol{\phi}(\mathbf{x})] := \min_{\boldsymbol{\phi}\in\boldsymbol{\Gamma}} \Big\{ C_{sim}[\mathbf{R}(\mathbf{x}), \mathbf{T}(\mathbf{x}); \boldsymbol{\phi}(\mathbf{x})] + \lambda C_{reg}[u(\mathbf{x})] \Big\} \quad (1)$$

where $C_{sim}[\mathbf{R}, \mathbf{T}; \boldsymbol{\phi}(\mathbf{x})]$ is the similarity metric between template $\mathbf{T}(\mathbf{x})$ and reference $\mathbf{R}(\mathbf{x})$ images, $C_{reg}[u(\mathbf{x})]$ is the regularization term due to cracks, folding or other unwanted deformations, λ is the regularization constant, $\boldsymbol{\phi}(\mathbf{x}) = \mathbf{x} + u(\mathbf{x}) = (x_1 + u_1(\mathbf{x}), x_2 + u_2(\mathbf{x}), x_3 + u_3(\mathbf{x}))$ is the deformation field, $u(\mathbf{x})$ is displacement field, Γ is the set of all possible transformations and $\mathbf{x} = (x_1, x_2, x_3) \in \mathbb{R}^3$. Because reference and template images are obtained from different distances, angles, times, sensors and sometimes even from different individuals, a displacement field may occur between these images. We presented [2] a systematic method in order to find the displacement field. Next we overview this method briefly.

3 The Image Registration Method

Given a *reference image* $\mathbf{R}(\mathbf{x})$ and a *template image* $\mathbf{T}(\mathbf{x})$, we express the equation (1) as follows: find a mapping $\boldsymbol{\phi}(\mathbf{x})$ that minimizes the L^2-norm of the difference between $\mathbf{T}(\boldsymbol{\phi}(\mathbf{x}))$ and $\mathbf{R}(\mathbf{x})$ over Ω. In order to achieve this, we define the similarity functional

$$\begin{aligned} \mathcal{J}(\boldsymbol{\phi}, f, \mathbf{g}) = \frac{1}{2} \int_\Omega \left|\mathbf{T}(\boldsymbol{\phi}(\mathbf{x})) - \mathbf{R}(\mathbf{x})\right|^2 d\mathbf{x} \\ + \frac{\omega}{2} \int_\Omega |f(\mathbf{x})|^2 \, d\mathbf{x} + \sum_{i=1}^{3} \frac{w_i}{2} \int_\Omega |g_i(\mathbf{x})|^2 \, d\mathbf{x}, \end{aligned} \quad (2)$$

where

$$\boldsymbol{\phi}(\mathbf{x}) = \mathbf{x} + \mathbf{u}(\mathbf{x}) \quad (3)$$

with $\mathbf{u}$ satisfying

$$\nabla \cdot \mathbf{u} = f - 1 \quad \text{and} \quad \nabla \times \mathbf{u} = \mathbf{g} \quad \text{in } \Omega \qquad \text{and} \qquad \mathbf{u} = \mathbf{0} \quad \text{on } \partial\Omega. \quad (4)$$

In (2), ω and w_i, $i = 1, 2, 3$, are penalty weights and g_i, $i = 1, 2, 3$, denote the components of the vector $\mathbf{g}$. Let us note that the argument $\nabla \cdot \mathbf{u} = f - 1$ improves the conservation of the size. Then, we minimize $\mathcal{J}(\boldsymbol{\phi}, f, \mathbf{g})$ subject to the constraints (3) and (4). We use the Lagrange multiplier method to transform the constrained minimization problem into an unconstrained saddle point problem and the corresponding optimality system in terms of Poisson equations is given by

$$\begin{cases} \Delta \mathbf{u} = \frac{1}{\omega}\nabla q - \frac{1}{w}\nabla \times \mathbf{v} & \text{in } \Omega \\ \mathbf{u} = \mathbf{0} & \text{on } \partial\Omega \end{cases} \tag{5}$$

$$\begin{cases} \Delta q = -\nabla \cdot \Big(\mathbf{T}_{\boldsymbol{\phi}}(\mathbf{x}+\mathbf{u})\big(\mathbf{T}(\mathbf{x}+\mathbf{u}) - \mathbf{R}(\mathbf{x})\big)\Big) & \text{in } \Omega \\ q = 0 & \text{on } \partial\Omega \end{cases} \tag{6}$$

$$\begin{cases} \Delta \mathbf{v} = -\nabla \times \Big(\mathbf{T}_{\boldsymbol{\phi}}(\mathbf{x}+\mathbf{u})\big(\mathbf{T}(\mathbf{x}+\mathbf{u}) - \mathbf{R}(\mathbf{x})\big)\Big) & \text{in } \Omega \\ \mathbf{v} = \mathbf{0} & \text{on } \partial\Omega, \end{cases} \tag{7}$$

where for (5) and (7) we have used the identity $\Delta \mathbf{v} = \nabla(\nabla \cdot \mathbf{v}) - \nabla \times (\nabla \times \mathbf{v})$, for (5) we have set $w_i = w$ for $i = 1, 2, 3$, and for (7), we have adopted the gauge $\nabla \cdot \mathbf{v} = 0$. Although we applied this method to 2D and 3D synthetic and medical images, we did not use any validation technique to calculate its performance and one of the major goals of this paper is to compute the strength of this image registration method. Next we present some validation techniques and use some of them to assess the quality of our method.

4 Quality Assessment Techniques

Method of gold standard, simulation based methods, consistency methods and visual assessment are popular validation techniques in the literature. **Masked Mean Warping Index** is another validation technique which can be described as follows: To properly reflect the quality of the image registration results, we need to exclude the homogeneous background area in the reference image R by imposing a binary mask in the process of computing the warping index. Therefore, the masked mean warping index $\bar{\omega}^*$ is introduced here as

$$\bar{\omega}^* = \frac{1}{\|\Omega^*\|} \sum_{\boldsymbol{\xi} \in \Omega^*} \|\phi(\boldsymbol{\xi}) - \phi^*(\boldsymbol{\xi})\|, \tag{8}$$

and the masked maximum warping index ω^*_{max} is defined as

$$\omega^*_{\max} = \max_{\boldsymbol{\xi} \in \Omega^*} \left(\|\phi(\boldsymbol{\xi}) - \phi^*(\boldsymbol{\xi})\|\right). \tag{9}$$

where Ω^* is the domain excluding the homogeneous background. Figure 1 contains an example for an application of the masked mean warping index method. Rightmost side figure of Figure 1 indicates that our method is quite efficient in the registration of deformable medical images.

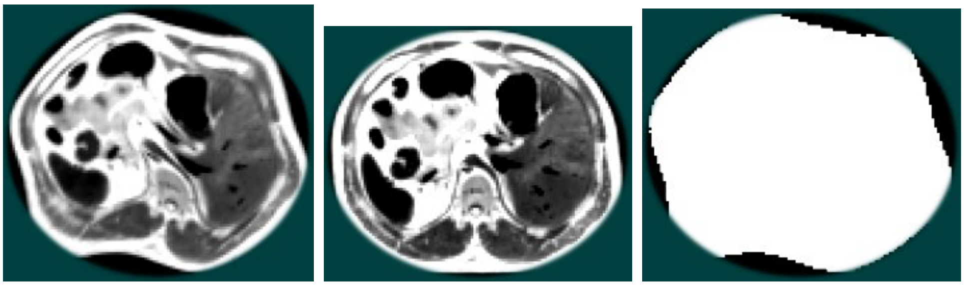

Figure 1: Masked mean warping index applied image

Registration of noisy images is still a quite challenging problem due to the lack of perfect image denoising or restoration techniques. It is highly likely that some important image features such as edge, corner and cusp points in images may be lost at the end of denoising operation. Therefore denoising an image before registration may result inaccurate registration. In [6] we applied our method to the registration of noisy images and experimental results shows [6] that our method is as efficient as the method in [9] in the registration of noisy images.

Similarity metric is another important tool to assess the quality of an image registration method. We used sum of squared differences (SSD) method in the applications of our method. All of the medical and synthetic applications indicate that SSD is small enough at the end of registration operation which shows that our method registers the images in a quite similar way that indicates one of the strengths of our method.

5 Other related image registration methods

Elastic registration, fluid registration, diffusion registration, curvature registration, demons registration are some of the most leading image registration algorithms and detailed information about each of these methods can be seen at [1].

In the NSF project [7] several different geometric image registration algorithms ([2], [10], [11], [12]) were successfully developed. Each of these algorithms are based on the deformation based grid generation method [2] and uses the divergence and curl of image pixels in a different format with different optimization algorithms. An important correlation has been established between these methods and the methods in the previous paragraph. In this section first we review these methods briefly and compare our results with the results of these methods and some other well-known image registration methods. First we overview the non-rigid image registration algorithms developed under the NSF project [7].

Helmholtzs theorem states that, with suitable boundary condition, a vector field is completely determined if both of its divergence and curl are specified everywhere. Based on this, H. Hsiao developed [10] a new parametric non-rigid image registration algorithm. Instead of the displacements of regular control grid points, the curl and divergence at each grid point are employed as the parameters. This leads to a very simple mathematical model which consists of two Poisson equations in 2-D or three Poisson equations in 3-D. By solving these Poisson equations the author obtains the displacement field in a systematic manner.

Re-gridding was first introduced in viscous fluid registration for preventing folding of the transformation and for maintaining the admissible deformation field in large-deformation nonrigid image registration applications. T. Lin investigated [11] the application of re-gridding to some popular non-rigid image registration algorithms, including elastic, fluid, diffusion, curvature, and demons algorithms, and compared the performance and accuracy in each case. He also introduced a grid repairing mechanism based on the adaptive grid generation method to prevent the transformation from folding.

Another method developed under the NSF project was introduced [12] by Chu. He introduced a new approach for non-rigid image registration using mutual information. A fast method for non-rigid registration is developed by adjusting divergence and curl of an intermediate vector field from which the deformation field is computed using finite difference method. The similarity measure mutual information is employed in the gradient-based cost minimization (or mutual information maximization) of the registration. The huge amount of data associated with MRI is handled by fully automated algorithm optimized with a multi-resolution topology.

Example: In the deformable image registration algorithms one of the typical example is to register the C letter looking figure to a disk. Figure 2 illustrates this. In order to compare our method with the registration methods mentioned in the previous paragraph we choose the same example as an experimental example. To register the disk to C letter looking figure at 2 we iterated the disk 240 times and the resulting figure is seen at the rightmost side of figure 2. Figure 2 contains reference, template and iterated template images from left to right, respectively. Based on the information provided in [11] we can write the following: By choosing Lame constants $\mu = 30$ and $\lambda = 50$ in elastic, fluid and

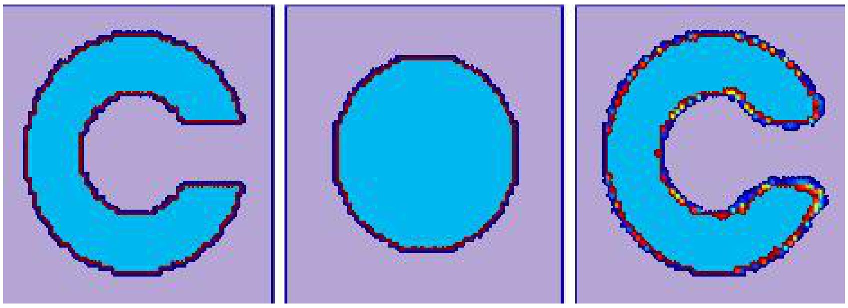

Figure 2: Registration of C shape figure to disk

re-gridding algorithms, elastic registration reduces the SSD from 7000 to 20 at the end of 1550 iterations, re-gridding algorithm reduces SSD from 7000 to 26 at the end of 5000 iterations and fluid registration reduces it from 7000 to 42 at the end of 4000 iterations. Diffusion, curvature and demons registration algorithms does not need to use the Lame constants and SSD start with 7000 in each of these methods and reduces to 25 at the end of 960 and 1037 iterations in diffusion and curvature registration, respectively and Demons algorithm reduces the SSD to 217 at the end 3000 iterations. Our method reduces the SSD from 350 to 4 at the end of 240 iterations which shows that our method is quite efficient as well.

6 Conclusion

One of the challenges in the development of image registration algorithms is their validation. In this article first we reviewed a recently proposed image registration method [6] and then assessed its quality with some well-known validation strategies such as visual assessment and similarity metric. Secondly we compare the method with some other well-known image registration methods such as curvature, elastic, fluid and divergence and curl based methods mostly developed under the NSF project [7].

While there are many existing deformable registration techniques, common approaches are shown to fail when one or more of the images to be registered contains even moderate levels of noise. The deformable image registration technique [6] based on the deformation based grid generation method using divergence and curl of image pixels is effective for registration of noisy images.

We can list the merits of our method as follows: It is based on a solid mathematical foundation. In particular, it accounts for local volume changes through the divergence of the transformation; and it accounts for local rotation through the curl vector of the transformation. The method is based on a linear differential system; its numerical implementation is fast, stable, simple and robust. The method is quite efficient in the registration of images which contain certain level of noise. Finally, the method is general in the sense that it may be used in any optimization problem that involves motion estimation. Thus, it has potential to be numerical kernel for a wide range of applications.

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