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Separation Axioms in Sequential Topological Spaces in the Light of Reduced and Augmented Bases

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Abstract

In this paper we define regularity of sequential topological spaces in a new way and prove that the new definition is stronger than the definition of regularity given in [3]. We also introduce the concept of normality in sequential topological spaces.

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1 Introduction

In 2002 M.K. Bose and Indrajit Lahiri introduced the concept of sequential topological spaces in [3]. They have developed separation axioms in sequential topological spaces upto regularity. As in [3], any sequence of subsets of a non void set X is called a sequential set in X i.e $A(s) = \{A_n\}_{n=1}^{\infty}$, where each A_n is a subset of X, is a sequential set in X. The subsets A_n, $n \in \mathbb{N}$ are called the components of A(s). If $A_n = \phi$, $\forall\ n \in \mathbb{N}$, then A(s) is called the void sequential set and is denoted by $\phi(s)$; if $A_n = X, \forall\ n \in \mathbb{N}$, then A(s) is called the universal sequential set and is denoted by X(s). For any two sequential sets $A(s) = \{A_n\}_{n=1}^{\infty}$ and $B(s) = \{B_n\}_{n=1}^{\infty}$, if $A_n \subset B_n\ \forall\ n \in \mathbb{N}$, then $A(s)$ is said to be contained in $B(s)$, denoted by $A(s) \subset B(s)$. $A(s)$ is said to be contained weakly in $B(s)$ if $A_n \subset B_n$ for at least one value of n, provided $A_n \neq \phi$. This is expressed symbolically by $A(s) \subset_w B(s)$. Further if $A(s) \subset B(s)$ and $B(s) \subset A(s)$ then we write $A(s) = B(s)$. A set $P \subset \mathbb{N}$ is said to be the base of $A(s)$ if $A_n \neq \phi\ \forall\ n \in P$ and $A_n = \phi\ \forall\ n \in \mathbb{N} - P$. If there exists an $M \subset \mathbb{N}$ such that $A_n \subset B_n\ \forall\ n \in M$ we say that $A(s)$ is $M-$sinking in $B(s)$ and write $A(s) \subset^M B(s)$. Weakly $M-$ sinking sequential sets are defined in the obvious way and in this case we write $A(s) \subset_w^M B(s)$. The union and intersection of the sequential sets $A(s) = \{A_n\}_{n=1}^{\infty}$ and $B(s) = \{B_n\}_{n=1}^{\infty}$ are, respectively, defined as $A(s) \cup B(s) = \{A_n \cup B_n\}_{n=1}^{\infty}$ and $A(s) \cap B(s) = \{A_n \cap B_n\}_{n=1}^{\infty}$.

The sequential set $B(s) - A(s) = \{B_n - A_n\}_{n=1}^{\infty}$ is defined to be the complement of $A(s)$ in $B(s)$. A sequential point p is a sequential set $P(s) = \{P_n\}_{n=1}^{\infty}$ with support $x \in X$ and base $M \subset \mathbb{N}$ if

$$\begin{aligned} P_n &= \{x\} \forall n \in M, \\ &= \phi \forall n \in \mathbb{N} - M \end{aligned}$$

and write $p = (x, M)$.

A sequential point $p = (x, M)$ is said to belong to $A(s)$ if $(x, M) \subset A(s)$ and it is expressed symbolically by $p \in A(s)$; p is said to belong weakly to $A(s)$, notationaly $p \in_w A(s)$, if $x \in A_n$ for at least one $n \in M$.

Let X be a nonvoid set. A collection τ of sequential sets in X is said to form a sequential topology on X if

(i) $\phi(s)$, $X(s) \in \tau$.

(ii) arbitrary union of members of τ is a member of τ.

(iii) finite intersection of members of τ is a member of τ.

A set X equipped with a sequential topology τ is called a sequential toplogical space, denoted by (X, τ). The members of τ are called $\tau-$open sequential sets in X.

Let $\mathbf{D}$ be a topology on X. The collection of sequences of $\mathbf{D}$-open sets forms a sequential topology on X, called the sequential topology generated by $\mathbf{D}$ and is denoted by $\tau < \mathbf{D} >$.

Suppose (X, τ) is a sequential topological space. Then the collection $D_n(\tau)$ of the n^{th} components of members of τ forms a topology on X and is called the n^{th} component topology of τ on X. A sequential set $A(s)$ is said to be closed if its complement is open. The closure of a sequential set $A(s)$, denoted by $\overline{A}(s)$, is defined to be the intersection of all closed sequential sets containing $A(s)$ and the interior of $A(s)$, denoted by $A^{\circ}(s)$, is defined to be the union of all open sequential sets contained in $A(s)$. A sequential set $N(s)$ is called a neighbourhood of a sequential set $A(s)$ if $A(s) \subset N^{\circ}(s)$.

Two sequential points $p = (x, P)$ and $q = (y, Q)$ are said to be identical if $x = y$ and $P = Q$; otherwise they are distinct. Let P and Q be respectively the bases of the sequential sets $A(s) = \{A_n\}_{n=1}^{\infty}$ and $B(s) = \{B_n\}_{n=1}^{\infty}$. $B(s)$ is said to be a reduced sequential set of $A(s)$ if $A_n \subset B_n\ \forall\ n \in Q \subset P$. In this case $A(s)$ is said to be an augmented sequential set of $B(s)$. A sequential topological space (X, τ) is said to be T_o space if for any two distinct sequential points $p = (x, P)$ and $q = (y, Q)$ $\exists$ an open sequential set $U(s)$ in (X, τ) such that $p \in_w^{P-Q} U(s)$ and $q \notin_w U(s)$, whenever q is a reduced sequential point of the sequential point p; otherwise $\exists$ an open sequential set $U(s)$ in (X, τ) such that $p \in_w U(s)$ and $q \notin_w U(s)$. A sequential topological space (X, τ) is said to be T_1 space if for any two distinct sequential points $p = (x, P)$ and $q = (y, Q)$ $\exists$ open sequental sets $U(s)$ and $V(s)$ in (X, τ) such that $p \in_w^{P-Q} U(s)$, $q \in_w V(s)$, $p \notin_w^{P-Q} V(s)$ and $q \notin_w U(s)$, whenever q is a reduced sequential point of the sequential point p; otherwise $\exists$ open sequential sets $U(s)$ and $V(s)$ in (X, τ) such that $p \in_w U(s)$, $q \in_w V(s)$, $p \notin_w V(s)$ and $q \notin_w U(s)$. A sequential topological space (X, τ) is said to be Hausdorff or T_2 space if for any two distinct sequential points $p = (x, P)$ and $q = (y, Q)$ $\exists$ open sequental sets $U(s)$ and $V(s)$ in (X, τ) such that $p \in_w^{P-Q} U(s)$, $q \in_w V(s)$, $p \notin_w^{P-Q} \overline{V(s)}$ and $q \notin_w \overline{U(s)}$, whenever q is a reduced sequential point of the sequential point p; otherwise $\exists$ open sequential sets $U(s)$ and $V(s)$ in (X, τ) such that $p \in_w U(s)$, $q \in_w V(s)$, $p \notin_w \overline{V(s)}$and $q \notin_w \overline{U(s)}$. A sequential topological space (X, τ) is said to be weak Hausdorff or (w) Hausdorff if for any two distinct sequential points $p = (x, P)$ and $q = (y, Q)$ $\exists$ open sequential sets $U(s)$ and $V(s)$ in (X, τ) such that $p \in_w^{P-Q} U(s)$, $q \in_w V(s)$, $U(s) \cap V(s) = \phi(s)$, whenever q is a reduced sequential point of the sequential point p; otherwise $\exists$ open sequential sets $U(s)$ and $V(s)$ in (X, τ) such that $p \in_w U(s)$, $q \in_w V(s)$, $U(s) \cap V(s) = \phi(s)$.

The above definitions of T_0, T_1 and T_2 spaces are different from the corresponding definitions given in [3], but inspite of the differences, all the results related to these spaces given in [3] remain unchanged. For this reason we exclude those portions of separation axioms in this paper.

We present an example which reveals the fact that how the changes in our definition affect the further study and what consequences we are going to encounter. Consider $X = \{a, b\}$ and $\mathbf{D} = \{\phi, \{a\}, \{b\}, X\}$, then $(X, \mathbf{D})$ is a regular topological space. Also consider the sequential topology $\tau < \mathbf{D} >$ generated by $\mathbf{D}$ on X. Then $(X, \tau < \mathbf{D} >)$ is not a regular sequential topological space according to the definition of regularity in [3], since for the closed sequential set $F(s) = \{F_n\}_{n=1}^{\infty}$ where

$$\begin{aligned} F_n &= \{a\} forn = 1, 2 \\ &= \phi forn = 3, 4, 5... \end{aligned}$$

and the sequential point $p = (a, \{1, 2, 3\}) \notin F(s)$, $\nexists$ open sequential sets $U(s)$ and $V(s)$ in $(X, \tau < \mathbf{D} >)$ such that $p \in_w U(s)$, $F(s) \subset_w V(s), p \notin_w \overline{V}(s)$, $F(s) \subset X(s) - \overline{U}(s)$. Thus with definitions in [3] regularity of (X,D) does not imply that of $(X, \tau < \mathbf{D} >)$ (cf.Theorem:15, [3]). Here we define regularity in an appropriate manner and establish the truth of this result in our setting. Also an attempt of defining T_3 -spaces according to definition of regularity in [3] leads to simple contradictions. In this study we develop the concept of T_3 -spaces, normal sequential topological spaces and T_4 -spaces and obtain some important results.

2 Regularity

Definition 1 *A sequential topological space (X, τ) is said to be regular if for any sequential point $p = (x, P)$ and any closed sequential set $F(s)$ with $p \notin F(s)$, $\exists$ open sequential sets $U(s)$ and $V(s)$ in (X, τ) such that*

$$p \in_w^{P-Q} U(s), F(s) \subset_w V(s), p \notin_w^{P-Q} \overline{V}(s), F(s) \subset X(s) - \overline{U}(s)$$

whenever $F(s)$ is a reduced sequential set, with base Q, of the sequential point p; otherwise $\exists$ open sequential sets $U(s)$ and $V(s)$ in (X, τ) such that

$$p \in_w U(s), F(s) \subset_w V(s), p \notin_w \overline{V}(s), F(s) \subset X(s) - \overline{U}(s).$$

Definition 2 *A sequential topological space (X, τ) is said to be weakly regular or (w) regular if for any sequential point $p = (x, P)$ and any closed sequential set $F(s)$ with $p \notin F(s)$, $\exists$ open sequential sets $U(s)$ and $V(s)$ in (X, τ) such that*

$$p \in_w^{P-Q} U(s), F(s) \subset_w V(s), U(s) \cap V(s) = \phi(s).$$

whenever $F(s)$ is a reduced sequential set, with base Q, of the sequential point p; otherwise $\exists$ open sequential sets $U(s)$ and $V(s)$ in (X, τ) such that

$$p \in_w U(s), F(s) \subset_w V(s), U(s) \cap V(s) = \phi(s).$$

Definition 3 *A sequential topological space (X, τ) is said to be a T_3 space if it is regular and T_1.*

Remark 1 *A T_3 space is a T_2. That the converse may not be true is shown by Example 3.*

Remark 2 *Example 4 shows that a regular sequential topological space may not be a T_1.*

Definition 4 *A sequential topological space (X, τ) is said to be a weakly T_3 space or (w) T_3 if it is (w) regular and T_1.*

Theorem 1 *A sequential topological space (X, τ) is regular if and only if for any sequential point $p = (x, P)$ and for any closed sequential set $F(s)$ with $p \notin F(s)$ $\exists$ open sequential sets $G(s)$ and $H(s)$ in (X, τ) such that*

$$p \in^{P-Q} G(s), F(s) \subset_w H(s), G(s) \cap H(s) = \phi(s).$$

and $\exists$ open sequential sets $D(s)$ and $E(s)$ in (X, τ) such that

$$p \in_w^{P-Q} D(s), F(s) \subset E(s), D(s) \cap E(s) = \phi(s).$$

whenever $F(s)$ is a reduced sequential set, with base Q, of the sequential point p; otherwise $\exists$ open sequential sets $G(s)$ and $H(s)$ in (X, τ) such that

$$p \in G(s), F(s) \subset_w H(s), G(s) \cap H(s) = \phi(s).$$

and $\exists$ open sequential sets $D(s)$ and $E(s)$ in (X, τ) such that

$$p \in_w D(s), F(s) \subset E(s), D(s) \cap E(s) = \phi(s).$$

Proof. Suppose (X, τ) is regular. Let $p = (x, P)$ be any sequential point and $F(s)$ be any closed sequential set such that $p \notin F(s)$. Then $\exists$ open sequential sets $U(s)$ and $V(s)$ in (X, τ) such that

$$p \in_w^{P-Q} U(s),\ F(s) \subset_w V(s),\ p \notin_w^{P-Q} \overline{V}(s),\ F(s) \subset X(s) - \overline{U}(s)$$

whenever $F(s)$ is a reduced sequential set, with base Q, of the sequential point p; otherwise $\exists$ open sequential sets $U(s)$ and $V(s)$ in (X, τ) such that

$$p \in_w U(s),\ F(s) \subset_w V(s),\ p \notin_w \overline{V}(s),\ F(s) \subset X(s) - \overline{U}(s).$$

If we take $G(s) = X(s) - \overline{V}(s)$, $H(s) = V(s)$, $D(s) = U(s)$ and $E(s) = X(s) - \overline{U}(s)$, then we are done.

Conversely, suppose the given conditions are true. Let $p = (x, P)$ be any sequential point and $F(s)$ be any closed sequential set such that $p \notin F(s)$. Then $\exists$ open sequential sets $G(s)$ and $H(s)$ in (X, τ) such that

$$p \in^{P-Q} G(s),\ F(s) \subset_w H(s),\ G(s) \cap H(s) = \phi(s)$$

and $\exists$ open sequential sets $D(s)$ and $E(s)$ in (X, τ) such that

$$p \in_w^{P-Q} D(s),\ F(S) \subset E(s),\ D(s) \cap E(s) = \phi(s)$$

whenever $F(s)$ is a reduced sequential set, with base Q, of the sequential point p; otherwise $\exists$ open sequential sets $G(s)$ and $H(s)$ in (X, τ) such that

$$p \in G(s),\ F(s) \subset_w H(s),\ G(s) \cap H(s) = \phi(s)$$

and $\exists$ open sequential sets $D(s)$ and $E(s)$ in (X, τ) such that

$$p \in_w D(s),\ F(s) \subset E(s),\ D(s) \cap E(s) = \phi(s)$$

If we take $U(s) = G(s) \cap D(s)$ and $V(s) = H(s) \cap E(s)$, then we are done. ∎

Corollary 1 *If (X, τ) is regular, then it is (w) regular.*

Theorem 2 *A sequential topological space (X, τ) is regular if and only if for any sequential point $p = (x, P)$ and an open sequential set $G(s)$ with $p \in_w G(s)$, $\exists$ an open sequential set $H(s)$ in (X, τ) such that $p \in^{P-Q} H(s)$, $\overline{H}(s) \subset_w G(s)$ and $\exists$ an open sequential set $B(s)$ in (X, τ) such that $p \in_w^{P-Q} B(s)$, $\overline{B}(s) \subset G(s)$ whenever $X(s) - G(s)$ is a reduced sequential set, with base Q, of the sequential point p; otherwise $\exists$ an open sequential set $H(s)$ in (X, τ) such that $p \in H(s)$, $\overline{H}(s) \subset_w G(s)$ and $\exists$ an open sequential set $B(s)$ in (X, τ) such that $p \in_w B(s)$, $\overline{B}(s) \subset G(s)$.*

Proof. Suppose (X, τ) is a regular sequential topological space. Let $p = (x, P)$ be any sequential point and $G(s)$ be an open sequential set such that $p \in_w G(s)$ i.e $p \notin X(s) - G(s) = F(s)$ (say). Then $\exists$ open sequential sets $U(s)$ and $V(s)$ in (X, τ) such that

$$p \in^{P-Q} U(s),\ F(s) \subset_w V(s),\ U(s) \cap V(s) = \phi(s).$$

and $\exists$ open sequential sets $D(s)$ and $E(s)$ in (X, τ) such that

$p \in_w^{P-Q} D(s)$, $F(s) \subset E(s)$ $D(s) \cap E(s) = \phi(s)$
whenever $X(s) - G(s) = F(s)$ is a reduced sequential set, with base Q, of the sequential point p, otherwise, $\exists$ open sequential sets $U(s)$ and $V(s)$ in (X, τ) such that
$p \in U(s)$, $F(s) \subset_w V(s)$, $U(s) \cap V(s) = \phi(s)$.
and $\exists$ open sequential sets $D(s)$ and $E(s)$ in (X, τ) such that
$p \in_w D(s)$, $F(s) \subset E(s)$, $D(s) \cap E(s) = \phi(s)$.
If we take $H(s) = U(s)$ and $B(s) = D(s)$ we are done.
Conversely, suppose given conditions are true. Let $p = (x, P)$ be any sequential point and $F(s)$ be any closed sequential set such that $p \notin F(s)$ i.e $p \in_w X(s) - F(s) = G(s)$ (say). Then $\exists$ an open sequential set $H(s)$ in (X, τ) such that $p \in^{P-Q} H(s)$, $\overline{H}(s) \subset_w G(s)$ and $\exists$ an open sequential set $B(s)$ in (X, τ) such that $p \in_w^{P-Q} B(s)$, $\overline{B}(s) \subset G(s)$ whenever $F(s)$ is a reduced sequential set, with base Q, of the sequential point p; otherwise $\exists$ an open sequential set $H(s)$ in (X, τ) such that $p \in H(s)$, $\overline{H}(s) \subset_w G(s)$ and $\exists$ an open sequential set $B(s)$ in (X, τ) such that $p \in_w B(s)$, $\overline{B}(s) \subset G(s)$.
If we take $U(s) = H(s)$, $V(s) = X(s) - \overline{H}(s)$, $D(s) = B(s)$ and $E(s) = X(s) - \overline{B}(s)$, then we are done. ■

Theorem 3 *A sequential topological space (X, τ) is (w) regular if and only if for any seuqential point $p = (x, P)$ and any open sequential set $G(s)$ with $p \in_w G(s)$, $\exists$ an open sequential set $H(s)$ in (X, τ) such that $p \in_w^{P-Q} H(s)$, $\overline{H}(s) \subset_w G(s)$ whenever $X(s) - G(s)$ is a reduced sequential set, with base Q, of the sequential point p; otherwise $\exists$ an open sequential set $H(s)$ in (X, τ) such that $p \in_w H(s)$, $\overline{H}(s) \subset_w G(s)$.*

Proof. Suppose (X, τ) is (w) regular. Let $p = (x, P)$ be any sequential point and $G(s)$ be an open sequential set with $p \in_w G(s)$ i.e $p \notin X(s) - G(s) = F(s)$ (say). Then $\exists$ open sequential sets $U(s)$ and $V(s)$ in (X, τ) such that
$p \in_w^{P-Q} U(s)$, $F(s) \subset_w V(s)$, $U(s) \cap V(s) = \phi(s)$.
whenever $F(s)$ is a reduced sequential set, with base Q, of the sequential point p; otherwise, $\exists$ open sequential sets $U(s)$ and $V(s)$ in (X, τ) such that
$p \in_w U(s)$, $F(s) \subset_w V(s)$, $U(s) \cap V(s) = \phi(s)$.
If we take $H(s) = U(s)$, then we are done.
Conversely, suppose the given conditions are true. Let $p = (x, P)$ be any sequential point and $F(s)$ be any closed sequential set such that $p \notin F(s)$ i.e $p \in_w X(s) - F(s) = G(s)$ (say). Then $\exists$ an open sequential set $H(s)$ in (X, τ) such that $p \in_w^{P-Q} H(s)$, $\overline{H}(s) \subset_w G(s)$ whenever $X(s) - G(s)$ is a reduced sequential set, with base Q, of the sequential point p; otherwise, $\exists$ an open sequential set $H(s)$ in (X, τ) such that $p \in_w H(s)$, $\overline{H}(s) \subset_w G(s)$.
If we take $U(s) = H(s)$ and $V(s) = X(s) - \overline{H}(s)$, then we are done. ■

Theorem 4 *If the sequential topological space (X, τ) is regular, then for any* *closed sequential set $A(s)$ in X, $A(s) = \cap\{N(s) : N(s)$ is a closed neighbourhood of $A(s)\}$ (1).*

Proof. Suppose (X, τ) is a regular sequential topological space. Let $A(s)$ be a closed sequential set in X. Let $p = (x, P)$ be a sequential point in X with $p \notin A(s)$. Then $p \in_w X(s) - A(s) = G(s)$ (say). Then $\exists$ an open sequential set $B(s)$ in (X, τ) such that $p \in_w^{P-Q} B(s)$, $\overline{B}(s) \subset G(s)$ whenever $A(s)$ is a reduced sequential set, with base Q, of the sequential point p; otherwise $\exists$ an open sequential set $B(s)$ in (X, τ) such that $p \in_w B(s)$, $\overline{B}(s) \subset G(s)$. This implies $A(s) \subset X(s) - \overline{B}(s) = H(s)$ (say). Again $p \notin X(s) - B(s)$ and hence $p \notin \overline{H}(s)$.
Thus (1) holds. Hence the theorem . ■

Remark 3 *Converse of Theorem 4 may not be true, which is shown by Example 1.*

Example 1 *Let $\mathbf{U}$ be the usual topology on $\mathbb{R}$. Fix $a \in \mathbb{R}$. For every open set $G \in \mathbf{U}$, we consider sequential sets $U^G(s) = \{U_n^G\}_{n=1}^{\infty}$ and $V^G(s) = \{V_n^G\}_{n=1}^{\infty}$ where*

$$U_n^G = V_n^G = G \forall n \neq 2,$$

$$\begin{aligned} U_2^G &= \{a\} if a \in G \\ &= \mathbb{R} - \{a\} if a \notin G \end{aligned}$$

$$\begin{aligned} V_2^G &= \mathbb{R} if a \in G \\ &= \phi if a \notin G \end{aligned}$$

The collection τ_a of sequential sets $U^G(s)$ and $V^G(s)$ $\forall$ $G \in \mathbf{U}$ forms a sequential topology on $\mathbb{R}$. Any closed sequential set $F(s)$ in $(\mathbb{R}, \tau_a)$ is the intersection of all closed nbds of $F(s)$ but $(\mathbb{R}, \tau_a)$ is not regular.

Theorem 5 *A topological space $(X, \mathbf{D})$ is regular if and only if the generated sequential topological space $(X, \tau < \mathbf{D} >)$ is regular.*

Proof. Suppose $(X, \mathbf{D})$ is regular. Let $p = (x, P)$ be any sequential point in X and $F(s)$ be any closed sequential set, with the base Q, such that $p \notin F(s)$.
Case 1. Suppopse $F(s) = \{F_n\}_{n=1}^{\infty}$ is a reduced sequential set of the sequential point p. Let $U \in \mathbf{D}$ such that $F_j \subset U$ for some $j \in Q$. Let $i \in P - Q$. Consider open sequential sets $U(s) = \{U_n\}_{n=1}^{\infty}$ and $V(s) = \{V_n\}_{n=1}^{\infty}$ in $(X, \tau < \mathbf{D} >)$ where

$$U_i = U,\ U_n = \phi\ \forall\ n(\neq i),\ V_j = U,\ V_n = \phi\ \forall\ n(\neq j).$$

Then $p \in_w^{P-Q} U(s)$, $F(s) \subset_w V(s)$, $p \notin_w^{P-Q} \overline{V}(s)$, $F(s) \subset X(s) - \overline{U}(s)$.

Case 2. Suppose $F(s) = \{F_n\}_{n=1}^{\infty}$ such that $x \in F_n \; \forall n \in Q$ and $F_n = \phi \; \forall$ $n \in \mathbb{N} - Q$ with $P \cap Q = \phi$. Let $U \in \mathbf{D}$ such that $F_j \subset U$ for some $j \in Q$. Let $i \in P$. Consider open sequential set $U(s) = \{U_n\}_{n=1}^{\infty}$ and $V(s) = \{V_n\}_{n=1}^{\infty}$ in $(X, \tau < \mathbf{D} >)$ where

$$U_i = U,\, U_n = \phi \;\forall\; n(\neq i),\; V_j = U,\; V_n = \phi \;\forall\; n(\neq j).$$

Then $p \in_w U(s)$, $F(s) \subset_w V(s)$, $p \notin_w \overline{V}(s)$, $F(s) \subset X(s) - \overline{U}(s)$.

Case 3. In any other case different from case1 and case 2, we proceed as follows, $p \notin F(s) \Longrightarrow \exists$ a closed set F in $(X, \mathbf{D})$ such that $x \notin F$. Since $(X, \mathbf{D})$ is regular $\exists\, U, V \in \mathbf{D}$ such that $x \in U$, $F \subset V$, $U \cap V = \phi$. Now consider open sequential sets $U(s) = \{U_n\}_{n=1}^{\infty}$ and $V(s) = \{V_n\}_{n=1}^{\infty}$ in $(X, \tau < \mathbf{D} >)$ where

$$U_n = U \;\forall n,\; V_n = V \;\forall n.$$

Then $p \in_w U(s)$, $F(s) \subset_w V(s)$, $p \notin_w \overline{V}(s), F(s) \subset X(s) - \overline{U}(s)$.

Combining Case 1, Case 2 and Case 3, $(X, \tau < \mathbf{D} >)$ is regular.

Conversely, suppose $(X, \tau < \mathbf{D} >)$ is regular. Let x be any point in X and F be a closed set in $(X, \mathbf{D})$ such that $x \notin F$. Then $p = (x, n)$ is a sequential point for any $n \in \mathbb{N}$ and $F(s) = \{F_n\}_{n=1}^{\infty}$, where $F_n = F \;\forall\; n$, is a closed sequential set in $(X, \tau < \mathbf{D} >)$ such that $p \notin F(s)$. Hence $\exists$ open sequential sets $U(s) = \{U_n\}_{n=1}^{\infty}$ and $V(s) = \{V_n\}_{n=1}^{\infty}$ in $(X, \tau < \mathbf{D} >)$ such that

$$p \in_w U(s),\; F(s) \subset_w V(s),\; p \notin_w \overline{V}(s),\; F(s) \subset X(s) - \overline{U}(s).$$

If we take $U = U_n$ and $V = X - \overline{U_n}$, $\overline{U_n}$ being the n^{th} component of $\overline{U}(s)$, then we are done. ∎

Theorem 6 *For any topological space* $(X, \mathbf{D})$ *, the generated sequential topological space* $(X, \tau < \mathbf{D} >)$*is (w) regular.*

Proof. The proof is omitted. ∎

Theorem 7 *If a space* (X, τ) *is regular its components* $(X, D_n(\tau))$ *are regular.*

Proof. The proof is omitted. ∎

Remark 4 *Converse of Theorem 7 may not be true, which is shown by Example* 2.

Example 2 *Let* X *be non-empty set and* $a \in X$. *Then the sets* ϕ, $\{a\}$, $X - \{a\}$, X *forms a regular topology on* X. *Consider sequential sets* $A^i(s) = \{A_n^i\}_{n=1}^{\infty}$, $B^i(s) = \{B_n^i\}_{n=1}^{\infty}$ *and* $C^i(s) = \{C_n^i\}_{n=1}^{\infty}$ $(i = 1, 2, 3.........)$ *where* $A_n^i = B_n^i = C_n^i = X - \{a\} \;\forall\; n \neq i$ *and* $A_i^i = \{a\}$, $B_i^i = \phi$, $C_i^i = X$. *Then the collection* S *of all sequential sets* $A^i(s)$, $B^i(s)$ *and* $C^i(s)$ $(i = 1, 2, 3......)$ *and* $\phi(s)$ *forms a subbase of a sequential topology say* τ *on* X*;* (X, τ) *is not regular, though the component spaces are regular.*

Example 3 *Let $\mathbb{R}$ be the set of real numbers. We define a topology $\mathbf{D}$ on $\mathbb{R}$ as follows - for any non zero point in $\mathbb{R}$, the $\mathbf{D}$-nbds are as the usual topology in $\mathbb{R}$. The $\mathbf{D}$- nbds of 0 are of the form $N - A$, where N is a nbd of 0 and $A = \{1, 1/2, 1/3......\}$. Then since $\mathbf{D}$ is finer than the usual topology $\mathbf{U}$ and $(\mathbb{R}, \mathbf{U})$ is $T_{2,}$ $(\mathbb{R}, \mathbf{D})$ is a T_2- space. Hence $(\mathbb{R}, \tau < \mathbf{D} >)$ is T_2-space but it is not regular.*

Example 4 *Let $X = \{a, b, c\}$ and $\mathbf{D} = \{\phi, \{a\}, \{b, c\}, X\}$. Then $(X, \mathbf{D})$ is a regular topological space but it is not a T_1 space. Hence the sequential topological space $(X, \tau < \mathbf{D} >)$, where $\tau < \mathbf{D} >$ is the sequential topology generated by $\mathbf{D}$, is a regular sequential topological space but it is not $T_{1.}$*

3 Normality

Definition 5 *Two sequential sets $A(s) = \{A_n\}_{n=1}^{\infty}$ and $B(s) = \{B_n\}_{n=1}^{\infty}$ are said to be weakly disjoint if $A_n \cap B_n = \phi$ for some n where at least one of A_n or B_n is non-empty.*

Definition 6 *A sequential topological space (X, τ) is said to be normal if for any two weakly disjoint closed sequential sets $A(s) = \{A_n\}_{n=1}^{\infty}$ and $B(s) = \{B_n\}_{n=1}^{\infty}$ with bases P and Q respectively i.e $A_m \cap B_m = \phi$ for some $m \in \mathbb{N}$, where either $A_m \neq \phi$ or $B_m \neq \phi$, $\exists$ open sequential sets $U(s) = \{U_n\}_{n=1}^{\infty}$ and $V(s) = \{V_n\}_{n=1}^{\infty}$ in (X, τ) such that $A(s) \subset_w^{P-Q} U(s)$, $B(s) \subset_w V(s)$, $A(s) \subset^{P-Q} X(s) - \overline{V}(s)$, $B(s) \subset X(s) - \overline{U}(s)$, $A_m \subset U_m$, $B_m \subset V_m$ whenever $B(s)$ is a reduced sequential set of $A(s)$; otherwise $\exists$ open sequential sets $U(s) = \{U_n\}_{n=1}^{\infty}$ and $V(s) = \{V_n\}_{n=1}^{\infty}$ in (X, τ) such that $A(s) \subset_w U(s)$, $B(s) \subset_w V(s)$, $A(s) \subset X(s) - \overline{V}(s)$, $B(s) \subset X(s) - \overline{U}(s)$, $A_m \subset U_m$, $B_m \subset V_m$.*

Definition 7 *A sequential topological space (X, τ) is said to be weakly normal or (w) normal if for any two weakly disjoint closed sequential sets $A(s) = \{A_n\}_{n=1}^{\infty}$ and $B(s) = \{B_n\}_{n=1}^{\infty}$ with bases P and Q respectively i.e $A_m \cap B_m = \phi$ for some $m \in \mathbb{N}$, where either $A_m \neq \phi$ or $B_m \neq \phi$, $\exists$ open sequential sets $U(s) = \{U_n\}_{n=1}^{\infty}$ and $V(s) = \{V_n\}_{n=1}^{\infty}$ in (X, τ) such that $A(s) \subset_w^{P-Q} U(s)$, $B(s) \subset_w V(s), U(s) \cap V(s) = \phi(s)$, $A_m \subset U_m$, $B_m \subset V_m$ whenever $B(s)$ is a reduced sequential set of $A(s)$; otherwise $\exists$ open sequential sets $U(s) = \{U_n\}_{n=1}^{\infty}$ and $V(s) = \{V_n\}_{n=1}^{\infty}$ in (X, τ) such that $A(s) \subset_w U(s)$, $B(s) \subset_w V(s)$, $U(s) \cap V(s) = \phi(s)$, $A_m \subset U_m$, $B_m \subset V_m$.*

Definition 8 *A sequential topological space (X, τ) is said to be a T_4 space if it is normal and T_1.*

Remark 5 *A normal sequential topological space may not be T_1, which is shown by Example 5.*

Example 5 *Consider the Sierpinski space* (X, τ)*, where* $X = \{0, 1\}$*,* $\tau = \{\{0\}, X, \phi\}$*. Let* $A(s) = \{A_n\}_{n=1}^{\infty}$*, where* $A_n = \{0\}\ \forall n \in \mathbb{N}$*. Let* $\tau' = \{\phi(s), A(s), X(s)\}$*. Then* (X, τ') *forms a normal sequential topological space but it is not* T_1*.*

Definition 9 *A sequential topological space* (X, τ) *is said to be weakly* T_4 *space or (w)* T_4 *if it is (w) normal and* T_1*.*

Theorem 8 *A sequential topological space* (X, τ) *is normal if and only if for any two weakly disjoint closed sequential sets* $A(s) = \{A_n\}_{n=1}^{\infty}$ *and* $B(s) = \{B_n\}_{n=1}^{\infty}$ *with bases* P *and* Q *respectively i.e* $A_m \cap B_m = \phi$ *for some* $m \in \mathbb{N}$*, where either* $A_m \neq \phi$ *or* $B_m \neq \phi$*,* $\exists$ *open sequential sets* $G(s) = \{G_n\}_{n=1}^{\infty}$ *and* $H(s) = \{H_n\}_{n=1}^{\infty}$ *in* (X, τ) *such that* $A(s) \subset^{P-Q} G(s)$*,* $B(s) \subset_w H(s)$*,* $G(s) \cap H(s) = \phi(s)$*,* $A_m \subset G_m$*,* $B_m \subset H_m$ *and* $\exists$ *open sequential sets* $D(s) = \{D_n\}_{n=1}^{\infty}$ *and* $E(s) = \{E_n\}_{n=1}^{\infty}$ *in* (X, τ) *such that* $A(s) \subset_w^{P-Q} D(s)$*,* $B(s) \subset E(s)$*,* $D(s) \cap E(s) = \phi(s)$*,* $A_m \subset D_m$*,* $B_m \subset E_m$ *whenever* $B(s)$ *is a reduced sequential set of* $A(s)$*; otherwise* $\exists$ *open sequential sets* $G(s) = \{G_n\}_{n=1}^{\infty}$ *and* $H(s) = \{H_n\}_{n=1}^{\infty}$ *in* (X, τ) *such that* $A(s) \subset G(s)$*,* $B(s) \subset_w H(s)$*,* $G(s) \cap H(s) = \phi(s)$*,* $A_m \subset G_m$*,* $B_m \subset H_m$ *and* $\exists$ *open sequential sets* $D(s) = \{D_n\}_{n=1}^{\infty}$ *and* $E(s) = \{E_n\}_{n=1}^{\infty}$ *in* (X, τ) *such that* $A(s) \subset_w D(s)$*,* $B(s) \subset E(s)$*,* $D(s) \cap E(s) = \phi(s)$*,* $A_m \subset D_m$*,* $B_m \subset E_m$*.*

Proof. Suppose (X, τ) is a normal sequential topological space. Let $A(s) = \{A_n\}_{n=1}^{\infty}$ and $B(s) = \{B_n\}_{n=1}^{\infty}$ be two weakly disjoint closed sequential sets with bases P and Q respectively i.e $A_m \cap B_m = \phi$ for some $m \in \mathbb{N}$, where either $A_m \neq \phi$ or $B_m \neq \phi$. Then $\exists$ open sequential sets $U(s) = \{U_n\}_{n=1}^{\infty}$ and $V(s) = \{V_n\}_{n=1}^{\infty}$ in (X, τ) such that $A(s) \subset_w^{P-Q} U(s)$, $B(s) \subset_w V(s)$, $A(s) \subset^{P-Q} X(s) - \overline{V}(s)$, $B(s) \subset X(s) - \overline{U}(s)$, $A_m \subset U_m$, $B_m \subset V_m$ whenever $B(s)$ is a reduced sequential set of $A(s)$; otherwise $\exists$ open sequential sets $U(s) = \{U_n\}_{n=1}^{\infty}$ and $V(s) = \{V_n\}_{n=1}^{\infty}$ in (X, τ) such that $A(s) \subset_w U(s)$, $B(s) \subset_w V(s)$, $A(s) \subset X(s) - \overline{V}(s)$, $B(s) \subset X(s) - \overline{U}(s)$, $A_m \subset U_m$, $B_m \subset V_m$. If we take $G(s) = X(s) - \overline{V}(s)$, $H(s) = V(s)$, $D(s) = U(s)$, $E(s) = X(s) - \overline{U}(s)$, then we are done.

Conversely, suppose the given conditions are true. Let $A(s) = \{A_n\}_{n=1}^{\infty}$ and $B(s) = \{B_n\}_{n=1}^{\infty}$ be two weakly disjoint closed sequential sets with bases P and Q respectively i.e $A_m \cap B_m = \phi$ for some $m \in \mathbb{N}$, where either $A_m \neq \phi$ or $B_m \neq \phi$. Then $\exists$ open sequential sets $G(s) = \{G_n\}_{n=1}^{\infty}$ and $H(s) = \{H_n\}_{n=1}^{\infty}$ in (X, τ) such that $A(s) \subset^{P-Q} G(s)$, $B(s) \subset_w H(s)$, $G(s) \cap H(s) = \phi(s)$, $A_m \subset G_m$, $B_m \subset H_m$ and $\exists$ open sequential sets $D(s) = \{D_n\}_{n=1}^{\infty}$ and $E(s) = \{E_n\}_{n=1}^{\infty}$ in (X, τ) such that $A(s) \subset_w^{P-Q} D(s)$, $B(s) \subset E(s)$, $D(s) \cap E(s) = \phi(s)$, $A_m \subset D_m$, $B_m \subset E_m$ whenever $B(s)$ is a reduced sequential set of $A(s)$; otherwise $\exists$ open sequential sets $G(s) = \{G_n\}_{n=1}^{\infty}$ and $H(s) = \{H_n\}_{n=1}^{\infty}$ in (X, τ) such that $A(s) \subset G(s)$, $B(s) \subset_w H(s)$, $G(s) \cap H(s) = \phi(s)$, $A_m \subset G_m$,

$B_m \subset H_m$ and $\exists$ open sequential sets $D(s) = \{D_n\}_{n=1}^{\infty}$ and $E(s) = \{E_n\}_{n=1}^{\infty}$ in (X, τ) such that $A(s) \subset_w D(s)$, $B(s) \subset E(s)$, $D(s) \cap E(s) = \phi(s)$, $A_m \subset D_m$, $B_m \subset E_m$.
If we take $U(s) = G(s) \cap D(s)$ and $V(s) = H(s) \cap E(s)$, then we are done. ■

Corollary 2 *A normal sequential topological space is (w) normal.*

Remark 6 *Converse of corollary* 2 *is not true. This is shown by Example* 6.

Example 6 *Let* $X = \mathbb{R}_l^2$. *Let* $\mathbf{D}$ *denotes the product topology on* X. *Now consider the sequential topology generated by* $\mathbf{D}$ *on* X *i.e* $\tau < \mathbf{D} >$. *Then* $(X, \tau < \mathbf{D} >)$ *is not normal.*
Now let $A(s) = \{A_n\}_{n=1}^{\infty}$ *and* $B(s) = \{B_n\}_{n=1}^{\infty}$ *be two weakly disjoint closed sequential sets with bases* P *and* Q *respectively i.e* $A_m \cap B_m = \phi$ *for some* $m \in \mathbb{N}$, *where either* $A_m \neq \phi$ *or* $B_m \neq \phi$.
Case 1. *Suppose one of* $A(s)$ *and* $B(s)$, *say* $B(s)$ *is a reduced sequential set of* $A(s)$ *i.e* $A_n \subset B_n$ $\forall n \in Q \subset P$. *Now consider open sequential sets* $U(s) = \{U_n\}_{n=1}^{\infty}$ *and* $V(s) = \{V_n\}_{n=1}^{\infty}$ *in* $(X, \tau < \mathbf{D} >)$ *where*

$$\begin{aligned} U_n &= A_n \forall n \in P - Q, \\ &= \phi, otherwise. \end{aligned}$$

and

$$\begin{aligned} V_n &= B_n \forall n \in Q, \\ &= \phi, otherwise. \end{aligned}$$

Then $A(s) \subset_w^{P-Q} U(s)$, $B(s) \subset_w V(s)$, $U(s) \cap V(s) = \phi(s)$, $A_m \subset U_m$, $B_m \subset V_m$.
Case 2. Suppose none of $A(s)$ and $B(s)$ is reduced from the other. Since in $\mathbb{R}_l^2$ every set is clopen, so are A_n and B_n. Now consider open sequential sets $U(s) = \{U_n\}_{n=1}^{\infty}$ and $V(s) = \{V_n\}_{n=1}^{\infty}$ in $(X, \tau < \mathbf{D} >)$ where $U_m = A_m$, $U_n = \phi$ $\forall n \neq m$ and $V_m = B_m$, $V_n = \phi$ $\forall n \neq m$. Then $A(s) \subset_w U(s)$, $B(s) \subset_w V(s)$, $U(s) \cap V(s) = \phi(s)$, $A_m \subset U_m$, $B_m \subset V_m$. Hence $(X, \tau < \mathbf{D} >)$ is (w) normal but not normal.

Remark 7 *A normal sequential topological space may not be regular, which is shown by Example* 7.

Example 7 *Consider the Sierpinski space* (X, τ) *where* $X = \{0.1\}$, $\tau = \{\{0\}, X, \phi\}$. *Let* $A(s) = \{A_n\}_{n=1}^{\infty}$ *where* $A_n = \{0\}$ $\forall n \in N$. *Let* $\tau' = \{\phi(s), A(s), X(s)\}$. *Then* (X, τ') *forms a normal sequential topological space but it is not regular.*

Theorem 9 *A normal sequential topological space which is T_1 is a regular space i.e a T_4 space is a T_3 space.*

Proof. Let (X,τ) be a normal sequential topological space which is T_1. Let $p=(x,P)$ be a sequential point in X and $F(s)=\{F_n\}_{n=1}^{\infty}$ be a closed sequential set in (X,τ) with $p \notin F(s)$. Since p and $F(s)$ are weakly disjoint closed sequential sets in (X,τ), $\exists$ open sequential sets $U(s)$ and $V(s)$ in (X,τ) such that

$$p \in_w^{P-Q} U(s),\ F(s) \subset_w V(s),\ p \in^{P-Q} X(s)-\overline{V}(s),\ F(s) \subset X(s)-\overline{U}(s)$$
$$\implies p \in_w^{P-Q} U(s),\ F(s) \subset_w V(s),\ p \notin_w^{P-Q} \overline{V}(s),\ F(s) \subset X(s)-\overline{U}(s)$$

whenever $F(s)$ is a reduced sequential set, with base Q, of sequential point p; otherwise $\exists$ open sequential sets $U(s)$ and $V(s)$ in (X,τ) such that

$$p \in_w U(s),\ F(s) \subset_w V(s),\ p \in X(s)-\overline{V}(s),\ F(s) \subset X(s)-\overline{U}(s).$$
$$\implies p \in_w U(s),\ F(s) \subset_w V(s),\ p \notin_w \overline{V}(s),\ F(s) \subset X(s)-\overline{U}(s).$$

Hence (X,τ) is regular. ■

Remark 8 *Example 8 shows that converse of Theorem 9 is not true.*

Example 8 *Let $X=\mathbb{R}_l^2$ and let $\mathbf{D}$ denotes the product topology on X. Now consider the sequential topology $\tau<\mathbf{D}>$ generated by $\mathbf{D}$ on X. Then $(X,\tau<\mathbf{D}>)$ is not normal but it is regular.*

Theorem 10 *If (X,τ) is a regular sequential topological space where X is finite, then it is (w) normal.*

Proof. Let (X,τ) be a regular sequential topological space where X is finite. Let $A(s)=\{A_n\}_{n=1}^{\infty}$ and $B(s)=\{B_n\}_{n=1}^{\infty}$ be two weakly disjoint closed sequential sets in (X,τ) with bases P and Q respectively i.e $A_m \cap B_m=\phi$ for some $m \in \mathbb{N}$, where either $A_m \neq \phi$ *or* $B_m \neq \phi$.

Case 1. Suppose one of $A(s)$ and $B(s)$, say $B(s)$ is a reduced sequential set of $A(s)$ i.e

$$A_n \subset B_n\ \forall n \in Q \subset P.$$

Here $A_m \neq \phi$, $B_m=\phi$ and $m \in P-Q$. Let $x \in A_m$. Then $p=(x,m) \notin B(s)$. So $\exists$ open sequential sets $U^x(s)$ and $V^x(s)$ in (X,τ) such that $p \in_w^{P-Q} U^x(s)$, $B(s) \subset_w V^x(s)$, $p \notin_w^{P-Q} \overline{V^x}(s)$, $B(s) \subset X(s)-\overline{U^x}(s)$. Corresponding to each $x \in A_m$, we can find such $U^x(s)$ and since A_m is finite, so $\exists$ finitely many open sequential sets in (X,τ) say $U_1(s)$, $U_2(s)$, $U_3(s)$,........, $U_k(s)$ such that $p \in_w^{P-Q} U_i(s)$, $B(s) \subset X(s)-\overline{U}_i(s)$, $i=1,2,3........k$.

Let $U(s)=\{U_n\}_{n=1}^{\infty}=\cup_{i=1}^{k} U_i(s)$, $V(s)=\{V\}_{n=1}^{\infty}=\cap_{i=1}^{k}(X(s)-\overline{U}_i(s))=X(s)-\cup_{i=1}^{k}\overline{U}_i(s)$. Then $A(s) \subset_w^{P-Q} U(s)$ and $B(s) \subset V(s)$. Thus $U(s), V(s) \in \tau$ such that $A(s) \subset_w^{P-Q} U(s)$, $B(s) \subset_w V(s)$ and $U(s) \cap V(s)=\phi(s)$, $A_m \subset U_m$, $B_m \subset V_m$.

Case 2. Suppose none of $A(s)$ and $B(s)$ is reduced from the other and let

$A_m \neq \phi$. Let $x \in A_m$. Then $p = (x, m) \notin B(s)$. So $\exists$ open sequential sets $U^x(s)$ and $V^x(s)$ in (X, τ) such that $p \in_w U^x(s)$, $B(s) \subset_w V^x(s)$, $p \notin_w \overline{V^x}(s)$, $B(s) \subset X(s) - \overline{U^x}(s)$. Now proceeding in the same way as in case 1 we can find open sequential sets $U(s)$ and $V(s)$ in (X, τ) such that $A(s) \subset_w U(s)$, $B(s) \subset_w V(s)$ and $U(s) \cap V(s) = \phi(s)$, $A_m \subset U_m$, $B_m \subset V_m$.
Hence (X, τ) is (w) normal. ■

Remark 9 *That a regular sequential topological space (X, τ), where X is infinite, may not be (w) normal is shown by Example* 9.

Example 9 *Let* $\mathbf{U}$ *be the usual topology on* $\mathbb{R}$ *and let* $a \in \mathbb{R}$. *For any open set* $G \in \mathbf{U}$, *let us consider sequential sets* $A^G(s) = \{A_n^G\}_{n=1}^{\infty}$, $B^G(s) = \{B_n^G\}_{n=1}^{\infty}$, $C^G(s) = \{C_n^G\}_{n=1}^{\infty}$, $D^G(s) = \{D_n^G\}_{n=1}^{\infty}$ *where* $A_n^G = B_n^G = C_n^G = D_n^G = G \ \forall n \neq 2$ *and* $A_2^G = \{a\}$, $B_2^G = \mathbb{R} - \{a\}$, $C_2^G = \phi$, $D_2^G = \mathbb{R}$. *Then* τ_a, *the collection of all sequential sets of the form* $A^G(s)$, $B^G(s)$, $C^G(s)$, $D^G(s)$ $\forall G \in \mathbf{U}$, *forms a regular sequential topology on* X. *But* (X, τ_a) *is not (w) normal.*

Remark 10 *A (w) normal sequential topological space may not be regular, which is shown by Example* 7.

Theorem 11 *Let (X, τ) be a normal sequential topological space, then $(X, D_n(\tau))$ is a normal space for each n.*

Proof. Let A and B be two disjoint non empty closed sets in $(X, D_n(\tau))$. Then $\exists$ closed sequential sets $A(s) = \{A_n\}_{n=1}^{\infty}$ and $B(s) = \{B_n\}_{n=1}^{\infty}$ in (X, τ) such that $A_n = A$ and $B_n = B$. So $\exists$ open sequential sets $U(s)$ and $V(s)$ in (X, τ) such that $A(s) \subset_w U(s)$, $B(s) \subset_w V(s)$, $A(s) \subset X(s) - \overline{V}(s)$, $B(s) \subset X(s) - \overline{U}(s)$, $A \subset U_n$, $B \subset V_n$. Taking $U = X - \overline{V_n}$and $V = V_n$, $\overline{V_n}$ being the n^{th} component of $\overline{V}(s)$, we have $A \subset U$, $B \subset V$, $U \cap V = \phi$. Thus $(X, D_n(\tau))$ is normal. ■

Remark 11 *Converse of Theorem* 11 *is not true. This is shown by Example* 10.

Example 10 *Let X be a non empty set and $a \in X$. Let $\tau = \{\phi, \{a\}, X - \{a\}, X\}$. Then (X, τ) is a normal topological space. Now consider sequential sets $A^i(s) = \{A_n^i\}_{n=1}^{\infty}$, $B^i(s) = \{B_n^i\}_{n=1}^{\infty}$, $C^i(s) = \{C_n^i\}_{n=1}^{\infty}$, $i = 1, 2, 3....$ where $A_n^i = B_n^i = C_n^i = X - \{a\} \ \forall n \neq i$ and $A_i^i = \{a\}$, $B_i^i = \phi$, $C_i^i = X$. Then S, the collection of all sequential sets $A^i(s)$, $B^i(s)$, $C^i(s)$ $(i = 1, 2, 3....)$ and $\phi(s)$ forms a subbase of a sequential topology say τ' on X . The sequential topological space (X, τ') is not normal though the component spaces are normal.*

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