Int. J. Contemp. Math. Sciences, Vol. 6, 2011, no. 22, 1079 - 1089

Fixed Point Theorems in Fuzzy Metric Spaces

K.S. Dersanambika

Department of Mathematics
Fatima Mata National College, Kollam, India
dersanapdf@yahoo.com

Aswathy M.R.

Department of Mathematics
Mar Thoma College, Tiruvalla, India
ashwathysree@gmail.com

Abstract

Common fixed point theorem in fuzzy metric space by employing reciprocal continuity were obtained by Urmila Mishra et.al. [9] In this paper we extend the above result to fuzzy 2-metric space and fuzzy 3-metric space.

Mathematics Subject Classification: 47H10, 54A40

Keywords: Fuzzy 2-metric space , Fuzzy 3-metric space, Common fixed point

1 Introduction

Fixed point theorems in fuzzy metric spaces satisfying some contractive condition is a central area of research now a days. The concept of fuzzy sets was introduced by Zadeh[10] in 1965.After this fuzzy set theory was further developed and a series of research were done by several Mathematicians. Kramosil and Michlek [5] introduced the concept of fuzzy metric space in 1975 and fixed point theorems for fuzzy metric space was first obtained by Helpern [4] in 1981. Later in 1994, A.George and P.Veeramani [3] modified the notion of fuzzy metric space with the help of t-norm. Some fixed point theorem in metric space are generalized to fuzzy metric space by several authors.

There are various ways to define a fuzzy metric space, here we adopt the notion that, the distance between objects is fuzzy, the objects themselves may be fuzzy or not.

Gahler [1],[2] investigated the properties of 2-metric space in his papers, and many authors investigated contraction mappings in 2-metric spaces. Succeeding this, the notion of 3-metric space were also introduced. We know that 2-metric space is a real valued function of a point triples on a set X, which abstract properties were suggested by the area function in the Eucledian space, whereas the 3-metric space was suggested by the volume function.

The idea of fuzzy 2-metric space and fuzzy 3-metric space were used by Sushil Sharma [8] and obtained some fruitful results.

Motivated by Sushil Sharma [8], we prove some common fixed point theorem in fuzzy 2-metric space and fuzzy 3-metric space by employing the notion of reciprocal continuity, of which we can widen the scope of many interesting fixed point theorems in fuzzy metric space.

2 Preliminary Notes

Definition 2.1. *A tiangular norm* $*$ *(shortly* $t-$ *norm) is a binary operation on the unit interval* $[0,1]$ *such that for all* $a,b,c,d \in [0,1]$ *the following conditions are satisfied:*

1. $a * 1 = a$;
2. $a * b = b * a$;
3. $a * b \leq c * d$ *whenever* $a \leq c$ *and* $b \leq d$
4. $a * (b * c) = (a * b) * c$.

Definition 2.2. *The 3-tuple* $(X, M, *)$ *is called a fuzzy metric space, if* X *is an arbitrary set,* $*$ *is a continuous* t*-norm and* M *is a fuzzy set in* $X^2 \times [0, \infty]$ *satisfying the following conditions: for all* $x, y, z \in X$ *and* $s, t > 0$

C'-1 $M(x, y, 0) = 0$

C'-2 $M(x, y, t) = 1$, *for all* $t > 0$, *if and only if* $x = y$

C'-3 $M(x, y, t) = M(y, x, t)$

C'-4 $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$

C'-5 $M(x, y, .) : [0, \infty) \to [0, 1]$ *is left continuous,*

C'-6 $\lim_{t\to\infty} M(x, y, t) = 1$

Example 2.3. *Let* (X, d) *be a metric space. Define* $a * b = ab$ *(or* $a * b = min\{a, b\}$*) and for all* $x, y \in X$ *and* $t > 0$, $M(x, y, t) = \frac{t}{t+d(x,y)}$. *Then* $(X, M, *)$ *is a fuzzy metric space and this metric* d *is the standard fuzzy metric.*

Definition 2.4. *A sequence* $\{x_n\}$ *in a fuzzy metric space* $(X, M, *)$ *is said to converge to* x *in* X *if and only if* $M(x_n, x, t) = 1$ *for each* $t > 0$.

Definition 2.5. *Let* $(X, M, *)$ *be a fuzzy metric space A sequence* $\{x_n\}$ *in* X *is called Cauchy sequence if and only if* $M(x_{n+p}, x_n, t) = 1$ *for each* $p > 0, t > 0$.

Definition 2.6. *A fuzzy metric space* $(X, M, *)$ *is said to be complete if and only if every Cauchy sequence in X is convergent in X.*

Definition 2.7. *A pair* (f, g) *of self maps of a fuzzy metric space* $(X, M, *)$ *is said to be reciprocal continuous if* $\lim_{n\to\infty} fgx_n = fx$ *and* $\lim_{n\to\infty} gfx_n = gx$ *whenever there exist a sequence* $\{x_n\}$ *such that* $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ *for some* $x \in X$.

Definition 2.8. *Two self maps* A *and* B *of a fuzzy metric space* $(X, M, *)$ *are said to be weak compatible if they commute at their coincidence points, that is* $Ax = Bx$ *implies* $ABx = BAx$.

Definition 2.9. *A pair* (A, S) *of self maps of a fuzzy metric space* $(X, M, *)$ *is said to be semi-compatible if* $\lim_{n\to\infty} ASx_n = Sx$ *whenever there exists a sequence* $\{x_n\}$ *in* X *such that* $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ *for some* $x \in X$.

Definition 2.10. *A binary operation* $* : [0, 1] \times [0, 1] \times [0, 1] \to [0, 1]$ *is called a continuous t-norm if* $([0, 1]), *)$ *is an abelian topological monoid with unit 1 such that* $a_1 * b_1 * c_1 \leq a_2 * b_2 * c_2$ *whenever* $a_1 \leq a_2$, $b_1 \leq b_2$,$c_1 \leq c_2$ *for all* a_1, a_2, b_1, b_2 *and* c_1, c_2 *are in* $[0, 1]$.

Definition 2.11. *The 3-tuple* $(X, M, *)$ *is called a fuzzy 2-metric space if* X *is an arbitrary set,* $*$ *is a continuous t-norm and* M *is a fuzzy set in* $X^3 \times [0, \infty]$ *satisfying the following conditions: for all* $x, y, z, u \in X$ *and* $t_1, t_2, t_3 > 0$.

C"-1 $M(x, y, z, 0) = 0$,

C"-2 $M(x, y, z, t) = 1$, $t > 0$ *and when at least two of the three points are equal,*

C"-3 $M(x, y, z, t) = M(x, z, y, t) = M(y, z, x, t)$
(Symmetry about three variables)

C"-4 $M(x, y, z, t_1 + t_2 + t_3) \geq M(x, y, u, t_1) * M(x, u, z, t_2) * M(u, y, z, t_3)$
(This corresponds to tetrahedron inequality in 2-metric space)
The function value $M(x, y, z, t)$ *may be interpreted as the probability that the area of triangle is less than* t.

C"-5 $M(x, y, z, .) : [0, \infty) \to [0, 1]$ *is left continuous.*

Definition 2.12. *A sequence $\{x_n\}$ in a fuzzy 2-metric space $(X, M, *)$ is said to converge to x in X if and only if $\lim_{n\to\infty} M(x_n, x, a, t) = 1$ for all $a \in X$ and $t > 0$.*

Definition 2.13. *Let $(X, M, *)$ be a fuzzy 2-metric space. A sequence $\{x_n\}$ in X is called Cauchy sequence, if and only if $\lim_{n\to\infty} M(x_{n+p}, x_n, a, t) = 1$ for all $a \in X$ and $p > 0, t > 0$.*

Definition 2.14. *A fuzzy 2-metric space $(X, M, *)$ is said to be complete if and only if every Cauchy sequence in X is convergent in X.*

Definition 2.15. *A binary operation $* : [0, 1] \times [0, 1] \times [0, 1] \times [0, 1] \to [0, 1]$ is called a continuous t-norm if $([0, 1]), *)$ is an abelian topological monoid with unit 1 such that $a_1 * b_1 * c_1 * d_1 \leq a_2 * b_2 * c_2 * d_2$ whenever $a_1 \leq a_2$, $b_1 \leq b_2$, $c_1 \leq c_2$ and $d_1 \leq d_2$ for all $a_1, a_2, b_1, b_2, c_1, c_2$ and d_1, d_2 are in $[0, 1]$.*

Definition 2.16. *The 3-tuple $(X, M, *)$ is called a fuzzy 3-metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set in $X^4 \times [0, \infty]$ satisfying the following conditions: for all $x, y, z, w, u \in X$ and $t_1, t_2, t_3, t_4 > 0$.*

C"'-1 $M(x, y, z, w, 0) = 0$,

C"'-2 $M(x, y, z, w, t) = 1$, *for all* $t > 0$,
(Only when the three simplex $< x, y, z, w >$ *degenerate)*

C"'-3 $M(x, y, z, w, t) = M(x, w, z, y, t) = M(y, z, w, x, t) = M(z, w, x, y, t) = \ldots$

C"'-4 $M(x, y, z, w, t_1+t_2+t_3+t_4) \geq M(x, y, z, u, t_1)*M(x, y, u, w, t_2)*M(x, u, z, w, t_3)* M(u, y, z, w, t_4)$

C"'-5 $M(x, y, z, w, .) : [0, \infty) \to [0, 1]$ *is left continuous.*

Definition 2.17. *A sequence $\{x_n\}$ in a fuzzy 3-metric space $(X, M, *)$ is said to converge to x in X if and only if $\lim_{n\to\infty} M(x_n, x, a, b, t) = 1$ for all $a, b \in X$ and $t > 0$.*

Definition 2.18. *Let $(X, M, *)$ be a fuzzy 3-metric space. A sequence $\{x_n\}$ in X is called Cauchy sequence, if and only if $\lim_{n\to\infty} M(x_{n+p}, x_n, a, b, t) = 1$ for all $a, b \in X$, $p > 0$,and $t > 0$.*

Definition 2.19. *A fuzzy 3-metric space $(X, M, *)$ is said to be complete if and only if every Cauchy sequence in X is convergent in X.*

3 Main Results

Urmila Mishra et.al[9] proved a common fixed point theorem in complete fuzzy metric space by employing the notion of reciprocal continuity. This result can be extended here to fuzzy 2-metric and fuzzy 3-metric spaces.

Theorem 3.1. *Let A, B, S, T be self maps on a complete fuzzy 2-metric space $(X, M, *)$ where $*$ is a continuous t-norm, satisfying*

T-1 $AX \subseteq TX$, $BX \subseteq SX$.

T-2 (B, T) *is weak compatible and reciprocal continuous,*

T-3 for each $x, y \in X$ *and* $t > 0$, $M(Ax, By, z, t) \geq \Phi(M(Sx, Ty, z, t))$, *where* $\Phi : [0, 1] \longrightarrow [0, 1]$ *is a continuous function such that* $\Phi(1) = 1$, $\Phi(0) = 0$ *and* $\Phi(a) > a$ *for each* $0 < a < 1$.

If (A, S) is semicompatible and reciprocal continuous, then A, B, S, T have a unique common fixed point.

Proof : Let $x_0 \in X$ be an arbitrary point. Then there exists $x_1, x_2 \in X$ such that $Ax_0 = Tx_1$ and $Bx_1 = Sx_2$. Thus we can construct sequences $\{y_n\}$ and $\{x_n\}$ in X such that $y_{2n+1} = Ax_{2n} = Tx_{2n+1}$, $y_{2n+2} = Bx_{2n+1} = Sx_{2n+2}$ for $n = 0, 1, ...$

$$\begin{aligned} M(y_{2n+1}, y_{2n+2}, z, t) &= M(Ax_{2n}, Bx_{2n+1}, z, t) \\ &\geq \Phi(M(Sx_{2n}, Tx_{2n+1}, z, t)) \\ &> \Phi(M(y_{2n}, y_{2n+1}, z, t)) \end{aligned}$$

similarly $M(y_{2n+2}, y_{2n+3}, z, t) > \Phi(M(y_{2n+1}, y_{2n+2}, z, t))$.
More generally, $M(y_{n+1}, y_n, z, t) > \Phi(M(y_n, y_{n-1}, z, t))$
Therefore $\{M(y_{n+1}, y_n, z, t)\}$ is an increasing sequence of positive real numbers in $[0, 1]$ and tends to limit $l \leq 1$. We claim that $l = 1$. If $l < 1$ then $M(y_{n+1}, y_n, z, t) > \Phi(M(y_n, y_{n-1}, z, t))$. On letting $n \to \infty$ we get,

$$\lim_{n\to\infty} M(y_{n+1}, y_n, z, t) \geq \Phi\left(\lim_{n\to\infty} M(y_n, y_{n-1}, z, t)\right)$$
that is $l \geq \Phi(l) > l$

a contradiction. Now for any positive integer p,

$$\begin{aligned} M\left(y_n, y_{n+p}, z, t\right) \geq\ & M\left(y_n, y_{n+1}, y_{n+p}, \frac{t}{2(p-1)+1}\right) \\ & * M\left(y_{n+1}, y_{n+2}, y_{n+p}, \frac{t}{2(p-1)+1}\right) \\ & * \ldots * M\left(y_{n+p-2}, y_{n+p-1}, y_{n+p}, \frac{t}{2(p-1)+1}\right) \\ & * M\left(y_n, y_{n+1}, z, \frac{t}{2(p-1)+1}\right) \\ & * M\left(y_{n+1}, y_{n+2}, z, \frac{t}{2(p-1)+1}\right) \\ & * \ldots * M\left(y_{n+p-1}, y_{n+p}, z, \frac{t}{2(p-1)+1}\right) \\ & * M\left(y_{n+p-1}, y_{n+p}, z, \frac{t}{2(p-1)+1}\right) \end{aligned}$$

Taking limits

$$\begin{aligned} \lim_{n\to\infty} M\left(y_n, y_{n+p}, z, t\right) \geq\ & \lim_{n\to\infty} M\left(y_n, y_{n+1}, y_{n+p}, \frac{t}{2(p-1)+1}\right) \\ & * \lim_{n\to\infty} M\left(y_{n+1}, y_{n+2}, y_{n+p}, \frac{t}{2(p-1)+1}\right) \\ & * \ldots * \lim_{n\to\infty} M\left(y_{n+p-2}, y_{n+p-1}, y_{n+p}, \frac{t}{2(p-1)+1}\right) \\ & * \lim_{n\to\infty} M\left(y_n, y_{n+1}, z, \frac{t}{2(p-1)+1}\right) \\ & * \lim_{n\to\infty} M\left(y_{n+1}, y_{n+2}, z, \frac{t}{2(p-1)+1}\right) \\ & * \ldots * \lim_{n\to\infty} M\left(y_{n+p-1}, y_{n+p}, z, \frac{t}{2(p-1)+1}\right) \\ & * \lim_{n\to\infty} M\left(y_{n+p-1}, y_{n+p}, z, \frac{t}{2(p-1)+1}\right) \end{aligned}$$

that is

$$\lim_{n\to\infty} M\left(y_n, y_{n+p}, z, t\right) \geq 1 * 1 * \ldots * 1 = 1$$

Which means $\{y_n\}$ is a Cauchy sequence in X. Since X is complete $y_n \to w$ in X. That is $\{Ax_{2n}\}, \{Tx_{2n+1}\}, \{Bx_{2n+1}\}, \{Sx_{2n+2}\}$ also converges to w in X.

That is

$$\lim_{n\to\infty} Sx_{2n} \to w$$

and

$$\lim_{n\to\infty} Ax_{2n} \to w$$

Since (A, S) is semi-compatible,

$$\lim_{n\to\infty} ASx_{2n} = Sw$$

Also (A, S) is reciprocal continuous also, therefore,

$$\lim_{n\to\infty} ASx_{2n} = Aw$$

Combining these two we get $Aw = Sw$. Now to prove that $Aw = w$, for let us assume that $Aw \neq w$. Then by the contractive condition,

$$M\left(Aw, Bx_{2n+1}, z, t\right) \geq \Phi\left(M\left(Sw, Tx_{2n+1}, z, t\right)\right)$$

Letting $n \to \infty$,

$$M\left(Aw, w, z, t\right) \geq \Phi\left(M\left(Sw, w, z, t\right)\right) > M\left(Aw, w, z, t\right)$$

a contradiction. Therefore $Aw = w = Sw$.
Since (B, T) is weak compatible and reciprocal continuous, as above we get $Bw = w = Tw$.
Therefore A, B, S and T has a common fixed point. To prove the uniqueness. Let w_1 and w_2 be two common fixed points of A, B, S and T.Assume $w_1 \neq w_2$. Then by the contractive condition,

$$\begin{aligned} M\left(w_1, w_2, z, t\right) &= M\left(Aw_1, Bw_2, z, t\right) \\ &\geq \Phi\left(M\left(Sw_1, Tw_2, z, t\right)\right) \\ &= \Phi\left(M\left(w_1, w_2, z, t\right)\right) \\ &> M\left(w_1, w_2, z, t\right) \end{aligned}$$

a contradiction. Therefore $w_1 = w_2$.

Theorem 3.2. *Let A, B, S, T be self maps on a complete fuzzy 3-metric space $(X, M, *)$ where $*$ is a continuous t-norm, satisfying*

T'-1 $AX \subseteq TX$, $BX \subseteq SX$.

T'-2 (B,T) *is weak compatible and reciprocal continuous,*

T'-3 *for each* $x,y\in X$ *and* $t>0$, $M(Ax,By,a,b,t)\geq\Phi(M(Sx,Ty,a,b,t))$, *where* $\Phi:[0,1]\longrightarrow[0,1]$ *is a continuous function such that* $\Phi(1)=1$, $\Phi(0)=0$ *and* $\Phi(l)>l$ *for each* $0<l<1$.

If (A,S) *is semicompatible and reciprocal continuous, then* A,B,S,T *have a unique common fixed point.*

Proof : Let $x_0\in X$ be an arbitrary point. Then there exists $x_1,x_2\in X$ such that $Ax_0=Tx_1$ and $Bx_1=Sx_2$. Thus we can construct sequences $\{y_n\}$ and $\{x_n\}$ in X such that $y_{2n+1}=Ax_{2n}=Tx_{2n+1},\ y_{2n+2}=Bx_{2n+1}=Sx_{2n+2}$ for $n=0,1,...$

$$\begin{aligned}M\left(y_{2n+1},y_{2n+2},a,b,t\right)&=M\left(Ax_{2n},Bx_{2n+1},a,b,t\right)\\&\geq\Phi\left(M\left(Sx_{2n},Tx_{2n+1},a,b,t\right)\right)\\&>\Phi\left(M\left(y_{2n},y_{2n+1},a,b,t\right)\right)\end{aligned}$$

similarly $M\left(y_{2n+2},y_{2n+3},a,b,t\right)\geq\Phi\left(M\left(y_{2n+1},y_{2n+2},a,b,t\right)\right)$
More generally, $M\left(y_{n+1},y_n,a,b,t\right)\geq\Phi\left(M\left(y_n,y_{n-1},a,b,t\right)\right)$
Therefore $\{M\left(y_{n+1},y_n,a,b,t\right)\}$ is an increasing sequence of positive real numbers in $[0,1]$ and tends to limit $l\leq 1$. We claim that $l=1$. If $l<1$ then $M\left(y_{n+1},y_n,a,b,t\right)\geq\Phi\left(M\left(y_n,y_{n-1},a,b,t\right)\right)$. On letting $n\to\infty$ we get,

$$\lim_{n\to\infty}M\left(y_{n+1},y_n,a,b,t\right)\geq\Phi\left(\lim_{n\to\infty}M\left(y_n,y_{n-1},a,b,t\right)\right)$$
that is $l\geq\Phi\left(l\right)>l$

a contradiction. Now for any positive integer p,

$$\begin{aligned}M\left(y_n,y_{n+p},a,b,t\right)\geq{}&M\left(y_n,y_{n+p},a,y_{n+1},\tfrac{t}{4}\right)*M\left(y_n,y_{n+p},y_{n+1},b,\tfrac{t}{4}\right)\\&*M\left(y_n,y_{n+1},a,b,\tfrac{t}{4}\right)*M\left(y_{n+1},y_{n+p},a,b,\tfrac{t}{4}\right)\end{aligned}$$

Continuing this process we obtain

$$\begin{aligned}M\left(y_n,y_{n+p},a,b,t\right)\geq{}&M\left(y_n,y_{n+p},a,y_{n+1},\tfrac{t}{3(p-1)+1}\right)*M\left(y_{n+1},y_{n+p},a,y_{n+2},\tfrac{t}{3(p-1)+1}\right)\\&*...*M\left(y_{n+p-2},y_{n+p},a,y_{n+p-1},\tfrac{t}{3(p-1)+1}\right)*M\left(y_n,y_{n+p},y_{n+1},b,\tfrac{t}{3(p-1)+1}\right)\\&*M\left(y_{n+1},y_{n+p},y_{n+2},b,\tfrac{t}{3(p-1)+1}\right)*...*M\left(y_{n+p-2},y_{n+p},y_{n+p-1},b,\tfrac{t}{3(p-1)+1}\right)\\&*M\left(y_n,y_{n+1},a,b,\tfrac{t}{3(p-1)+1}\right)*M\left(y_{n+1},y_{n+2},a,b,\tfrac{t}{3(p-1)+1}\right)\\&*...*M\left(y_{n+p-1},y_{n+p},a,b,\tfrac{t}{3(p-1)+1}\right)\end{aligned}$$

Taking limit as $n \to \infty$,

$$\begin{aligned}\lim_{n\to\infty} M\left(y_n, y_{n+p}, a, b, t\right) \geq & \lim_{n\to\infty} M\left(y_n, y_{n+p}, a, y_{n+1}, \tfrac{t}{3(p-1)+1}\right) \\ & * \lim_{n\to\infty} M\left(y_{n+1}, y_{n+p}, a, y_{n+2}, \tfrac{t}{3(p-1)+1}\right) \\ & *\ldots * \lim_{n\to\infty} M\left(y_{n+p-2}, y_{n+p}, a, y_{n+p-1}, \tfrac{t}{3(p-1)+1}\right) \\ & * \lim_{n\to\infty} M\left(y_n, y_{n+p}, y_{n+1}, b, \tfrac{t}{3(p-1)+1}\right) \\ & * \lim_{n\to\infty} M\left(y_{n+1}, y_{n+p}, y_{n+2}, b, \tfrac{t}{3(p-1)+1}\right) * \ldots \\ & * \lim_{n\to\infty} M\left(y_{n+p-2}, y_{n+p}, y_{n+p-1}, b, \tfrac{t}{3(p-1)+1}\right) \\ & * \lim_{n\to\infty} M\left(y_n, y_{n+1}, a, b, \tfrac{t}{3(p-1)+1}\right) \\ & * \lim_{n\to\infty} M\left(y_{n+1}, y_{n+2}, a, b, \tfrac{t}{3(p-1)+1}\right) \\ & *\ldots * \lim_{n\to\infty} M\left(y_{n+p-1}, y_{n+p}, a, b, \tfrac{t}{3(p-1)+1}\right)\end{aligned}$$

that is

$$\lim_{n\to\infty} M\left(y_n, y_{n+p}, a, b, t\right) \geq 1 * 1 * \ldots * 1 = 1$$

Which means $\{y_n\}$ is a Cauchy sequence in X. Since X is complete $y_n \to w$ in X. That is $\{Ax_{2n}\}, \{Tx_{2n+1}\}, \{Bx_{2n+1}\}, \{Sx_{2n+2}\}$ also converges to w in X. That is

$$\lim_{n\to\infty} Sx_{2n} \to w$$

and

$$\lim_{n\to\infty} Ax_{2n} \to w$$

Since (A, S) is semi-compatible,

$$\lim_{n\to\infty} ASx_{2n} = Sw$$

Also (A, S) is reciprocal continuous also, therefore,

$$\lim_{n\to\infty} ASx_{2n} = Aw$$

Combining these two we get $Aw = Sw$. Now to prove that $Aw = w$, for let us assume that $Aw \neq w$. Then by the contractive condition,

$$M\left(Aw, Bx_{2n+1}, a, b, t\right) \geq \Phi\left(M\left(Sw, Tx_{2n+1}, a, b, t\right)\right)$$

Letting $n \to \infty$,

$$M(Aw, w, z, u, t) \geq \Phi(M(Sw, w, a, b, t)) > M(Aw, w, a, b, t)$$

a contradiction. Therefore $Aw = w = Sw$.
Since (B, T) is weak compatible and reciprocal continuous, as above we get $Bw = w = Tw$.
Therefore A, B, S and T has a common fixed point. To prove the uniqueness. Let w_1 and w_2 be two common fixed points of A, B, S and T.Assume $w_1 \neq w_2$. Then by the contractive condition,

$$\begin{aligned} M(w_1, w_2, a, b, t) &= M(Aw_1, Bw_2, a, b, t) \\ &\geq \Phi(M(Sw_1, Tw_2, a, b, t)) \\ &= \Phi(M(w_1, w_2, a, b, t)) \\ &> M(w_1, w_2, a, b, t) \end{aligned}$$

a contradiction. Therefore $w_1 = w_2$.

References

[1] Gähler, S.: Linear 2-normierte Räume, *Math. Nachr.*, **28**(1964), 143.

[2] Gähler, S.: Ü ber 2-Banach Raume, *Math. Nachr.* **42**(1969), 335347.

[3] George, A., Veeramani, P.: On some results in fuzzy metric spaces, *Fuzzy Sets and Systems*, **64**(1994), 395399.

[4] Heipern S. : Fuzzy mappings and fixed point theorems, *J.Math.Anal.Appl.*, **83**(1981), 566-569.

[5] O.Kramosil & J.Michalek : Fuzzy metric and statistical metric spaces, *Kybernelika*, **11** (1975), 326-334.

[6] B.Singh & S.Jain : Semi-compatible and fixed point theorems in fuzzy metric spaces, *Chungcheong Math.Soc*, **18**(2005), 1-22.

[7] Sharma, P.L., Sharma, B.K., Iseki, K.: Contractive type mapping on 2-metric space, *Math. Japonica*, **21**(1976), 6770.

[8] Sushil Sharma : On fuzzy metric space, *Southeast Asian Bulletin of Mathematics*, **26**, 2002, 133-145.

[9] Urmila Mishra, Abhay Sharad Ranavide & Dhananjay Gopal : Some fixed point theorems in fuzzy metric space, *Tamkang Journal of Mathematics*, **39**(4)(2008), 309-316.

[10] L.A.Zadeh : Fuzzy sets, *Inform and Control*, **8**(1965), 338-353.

Received: October, 2010