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Generalized Near Rough Probability in Topological Spaces

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Abstract

Rough set theory has been introduced by Pawlak [13]. It is considered as a base for all researches in the rough set area introduced after this date. Most of these researches concentrated on developing results and techniques based on Pawlak's results [13]. In this paper we shall introduce generalized rough probability from topological view. The basic concepts of some generalized near open, generalized near rough, and generalized near exact sets are introduced and sufficiently illustrated. Moreover, proved results, examples and counter examples are provided. The topological structure which suggested in this paper opens up the way for applying rich amount of topological facts and methods in the process of granular computing.

Keywords: Topological space; Generalized near rough set, Generalized near rough probability

1. Introduction

One of the most powerful notions in system analysis is the concept of topological structures [6] and their generalizations. Rough set theory, introduced by Pawlak in 1982 [13], is a mathematical tool that supports also the uncertainty reasoning but qualitatively. In this paper, we shall integrate some ideas in terms of concepts in topology. Topology is a branch of mathematics, whose concepts exist not only in almost all branches of mathematics, but also in many real life applications. We believe that topological structure will be an important base for modification of knowledge extraction and processing.

2. Preliminaries

A topological space [6] is a pair (X,τ) consisting of a set X and family τ of subsets of X satisfying the following conditions:
(T1) $\phi \in \tau$ and $X \in \tau$.
(T2) τ is closed under arbitrary union.
(T3) τ is closed under finite intersection.

Throughout this paper (X,τ) denotes a topological space, the elements of X are called points of the space, the subsets of X belonging to τ are called open sets in the space, the complement of the subsets of X belonging to τ are called closed sets in the space, and the family of all open sets of (X,τ) is denoted by τ and the family of all closed sets of (X,τ) is denoted by $C(X)$.

For a subset A of a space (X,τ), $Cl(A)$ denote the closure of A and is given by $Cl(A)=\cap\{F \subseteq X : A \subseteq F \text{ and } F \in C(X)\}$. Evidently, $Cl(A)$ is the smallest closed subset of X which contains A. Note that A is closed iff $A = Cl(A)$. $Int(A)$ denote the interior of A and is given by $Int(A)=\cup\{G \subseteq X : G \subseteq A \text{ and } G \in \tau\}$ Evidently, $Int(A)$ is the largest open subset of X which contained in A. Note that A is open iff $A = Int(A)$.

We shall recall some concepts about some near open sets which are essential for our present study.

Definition 2.1. A subset A of a space (X,τ) is called:

i) Semi-open [8] (briefly $s-$open) if $A \subseteq Cl(Int(A))$.
ii) Pre-open [10] (briefly $p-$open) if $A \subseteq Int(Cl(A))$.
iii) α-open [11] if $A \subseteq Int(Cl(Int(A)))$.
iv) β-open [1] **(** = semi-pre-open [2]) if $A \subseteq Cl(Int(Cl(A)))$.

The complement of a $s-$open (resp. $p-$open, α-open and β-open) set is called $s-$closed (resp. $p-$closed , α-closed and β-closed) set .

The family of all $s-$open (resp. $p-$open, α-open and β-open) sets of (X,τ) is denoted by $SO(X)$ (resp. $PO(X)$, $\alpha O(X)$ and $\beta O(X)$).

The family of all $s-$closed (resp. $p-$closed, α- closed and β-closed) sets of (X,τ) is denoted by $SC(X)$ (resp. $PC(X)$, $\alpha C(X)$ and $\beta C(X)$).

The semi-closure (resp. α-closure, pre-closure, semi-pre-closure) of a subset A of (X,τ), denoted by ${}_sCl(A)$ (resp. ${}_\alpha Cl(A)$, ${}_pCl(A)$, ${}_{sp}Cl(A)$) and defined to be the intersection of all semi-closed (resp. α-closed, p-closed, sp-closed) sets containing A. The semi-interior (resp. α-interior, pre-interior, semi-pre-interior) of a subset A of (X,τ), denoted by ${}_sInt(A)$ (resp. ${}_\alpha Int(A)$, ${}_pInt(A)$, ${}_{sp}Int(A)$) and defined to be the union of all semi-open (resp. α-open,

p -open, sp -open) sets contained in A .

Definition 2.2. A subset A of a space (X,τ) is said to be:
(i) generalized closed[7] (briefly, g -closed) if $Cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in (X,τ),
(ii) generalized semi-closed[3] (briefly, gs -closed) if ${}_sCl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in (X,τ),
(iii) generalized semi-preclosed[4] (briefly, gsp -closed) if ${}_{sp}Cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in (X,τ),
(iv) α -generalized closed[9] (briefly, αg -closed) if ${}_\alpha Cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in (X,τ),
(v) generalized preclosed[12] (briefly, gp -closed) if ${}_pCl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in (X,τ).

The complement of a g -closed (resp. gs -closed, gsp -closed, gp -closed and αg -closed) set is called g -open (resp. gs -open, gsp -open, gp -open and αg -open). The family of all g -open (resp. gs -open, gp -open, αg -open and gsp -open) sets of (X,τ) is denoted by $gO(X)$ (resp. $gSO(X)$, $gPO(X)$, $\alpha gO(X)$ and $gspO(X)$). The family of all g -closed (resp. gs -closed, gp -closed, αg -closed and gsp -closed) sets of (X,τ) is denoted by $gC(X)$ (resp. $gSC(X)$, $gPC(X)$, $\alpha gC(X)$, and $gspC(X)$).

The generalized interior (briefly g -interior) of A is denoted by ${}_gInt(A)$ and is defined by ${}_gInt(A)=\cup\{G\subseteq X:G\subseteq A, G$ is a g -open$\}$, and the generalized near interior (briefly gj -interior) of A is denoted by ${}_{gj}Int(A)$ for all $j\in\{s,p,\alpha,\beta\}$ and is defined by ${}_{gj}Int(A)=\cup\{G\subseteq X:G\subseteq A, G$ is a gj -open$\}$.

The generalized closure (briefly g -closure) of A is denoted by ${}_gCl(A)$ and is defined by ${}_gCl(A)=\cap\{F\subseteq X:A\subseteq F, F$ is a g -closed set$\}$, and the generalized near closure (briefly gj -closure) of A is denoted by ${}_{gj}Cl(A)$ for all $j\in\{s,p,\alpha,\beta\}$ and is defined by $gjCl(A)=\cap\{F\subseteq X:A\subseteq F, F$ is a gj closed set$\}$.

The generalized boundary (briefly g -boundary) region of A is denoted by ${}_gBN(A)$ and is defined by ${}_gBN(A)={}_gCl(A)-{}_gInt(A)$ and the generalized near boundary (briefly gj -boundary) region of A is denoted by ${}_{gj}BN(A)$ for all $j\in\{s,p,\alpha,\beta\}$ and is defined by ${}_{gj}BN(A)={}_{gj}Cl(A)-{}_{gj}Int(A)$.

3. Pawlak's approach

Consider the approximation space $K=(U,R)$, where U is a set called the universe and R is an equivalence relation. The order triple $S=(U,R,p)$ is called the stochastic approximation space [14], where p is a probability measure; any subset of U will be called an event. The probability measure p has the following properties:

$p(\phi)=0$, $p(U)=1$ and if $A=\bigcup_{i=1}^{n} X_i$ is an observable set in K, then $p(A)=\sum_{i=1}^{n} p(X_i)$.

It is clear that A is a union of disjoint sets, since R is an equivalence relation. Pawlak introduced the definitions of the lower and upper probabilities of an event A in the stochastic approximation space $S=(U,R,p)$. These definitions are:

- The lower probability of A is denoted by $\underline{p}(A)$ and is given by $\underline{p}(A)=p(\underline{R}A)$.
- The upper probability of A is denoted by $\overline{p}(A)$ and is given by $\overline{p}(A)=p(\overline{R}A)$.

Clearly, $0\le \underline{p}(A),\ \overline{p}(A)\le 1$.

4. Generalized near rough probability in topological spaces

Definition 4.1. [5]. Let $K=(X,R)$ be an approximation space with general relation R and τ_k is the topology associated to K. Then the triple (X,R,τ_k) is called a topologized approximation space.

Definition 4.2 [5]. Let $K=(U,R)$ be an approximation space with general relation R and τ_K is the topology associated to K. Then the order 4-tuples $\mathrm{S}=(U,R,p,\tau_K)$ is called the topologized stochastic approximation space.

4.1. Generalized near rough probability

We obtain some rules to find $gj-$lower and $gj-$upper probabilities in a topologized stochastic approximation spaces for all $j\in\{s,p,\alpha,\beta\}$.

Definition 4.1.1. Let A be an event in the topologized stochastic approximation space $\mathrm{S}=(U,R,p,\tau_K)$. Then the $gj-$lower (resp. $gj-$upper) probability of A for all $j\in\{s,p,\alpha,\beta\}$ is given by

$$ {}_{gj}\underline{p}(A) = p({}_{gj}Int(A)) \text{ (resp. } {}_{gj}\overline{p}(A) = p({}_{gj}Cl(A))\text{)}. $$

Definition 4.1.2. Let A be an event in the topologized stochastic approximation space $\mathrm{S} = (U, R, p, \tau_K)$. Then the gj –rough probability of A for all $j \in \{s, p, \alpha, \beta\}$ is denoted by ${}_{gj}p^*(A)$ and is given by

$$ {}_{gj}p^*(A) = \left\langle {}_{gj}\underline{p}(A), {}_{gj}\overline{p}(A) \right\rangle . $$

Proposition 4.1.1. Let A be an event in the topologized stochastic approximation space $\mathrm{S} = (U, R, p, \tau_K)$, then the implications between the gj –lower probability of A are given by the following diagram for all $j \in \{p, s, \alpha, \beta\}$.

$$
\begin{tikzcd}
& {}_{gs}\underline{p}(A) \arrow[dr, "\leq"] & \\
{}_{g\alpha}\underline{p}(A) \arrow[ur, "\leq"] \arrow[dr, "\leq"'] & & {}_{g\beta}\underline{p}(A) \\
& {}_{gp}\underline{p}(A) \arrow[ur, "\leq"'] &
\end{tikzcd}
$$

Figure 4.1.1.
The relation between the gj –lower probability

Proof. The proof is obvious.

Proposition 4.1.2. Let A be an event in the topologized stochastic approximation space $\mathrm{S} = (U, R, p, \tau_K)$, then the implications between the gj – upper probability of A are given by the following diagram for all $j \in \{p, s, \alpha, \beta\}$.

$$
\begin{tikzcd}
& {}_{gs}\overline{p}(A) \arrow[dr, "\leq"] & \\
{}_{g\beta}\overline{p}(A) \arrow[ur, "\leq"] \arrow[dr, "\leq"'] & & {}_{g\alpha}\overline{p}(A) \\
& {}_{gp}\overline{p}(A) \arrow[ur, "\leq"'] &
\end{tikzcd}
$$

Figure 4.1.2.
The relation between the gj –upper probability

Proof. The proof is obvious.

4.2. Generalized near rough distribution function

We shall introduce the concept of generalized near rough (briefly gj –rough) distribution function of a random variable X for all $j \in \{s, p, \alpha, \beta\}$.

In the following definition we define the gj – lower and the gj – upper distribution functions of a random variable X for all $j \in \{s, p, \alpha, \beta\}$.

Definition 4.2.1. Let X be a random variable in the topologized stochastic approximation space $S = (U, R, p, \tau_K)$. Then the gj – lower distribution (resp. gj – upper distribution) function of X for all $j \in \{s, p, \alpha, \beta\}$ is given by

$$_{gj}\underline{F}(x) = {}_{gj}\underline{p}(X \leq x) \text{ (resp. } _{gj}\overline{F}(x) = {}_{gj}\overline{p}(X \leq x)\text{)}.$$

Definition 4.2.2. Let X be a random variable in the topologized stochastic approximation space $S = (U, R, p, \tau_K)$. Then the gj – rough distribution function of X for all $j \in \{s, p, \alpha, \beta\}$ is denoted by $_{gj}F^*(x)$ and is given by

$$_{gj}F^*(x) = \left\langle {}_{gj}\underline{F}(x),\ {}_{gj}\overline{F}(x) \right\rangle.$$

Example 4.2.1. Consider the experiment of choosing one from four cards have numbered from one to four. The collection of the five elements forms the outcome space. Hence,

$$U = \{1, 2, 3, 4\},$$

Let R be a binary relation defined on U such that

$$R = \{(1,1), (1,2), (1,3), (2,3), (3,3), (3,4), (4,2)\}.$$

Thus $U / R = \{\{1,2,3\}, \{3\}, \{3,4\}, \{2\}\}$. Let $K = (U, R)$ be an approximation space and τ_K is the topology associated to K. Thus,

$$\tau_K = \{U, \phi, \{2\}, \{3\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{2,3,4\}\} \text{ . Then}$$

$$SO(U) = \{U, \phi, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,3,4\}, \{2,3,4\}\},$$

$$SC(U) = \{\phi, U, \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,2,4\}, \{1,3,4\}\},$$

$$_{g}SO(U) = \{U, \phi, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,3,4\}, \{2,3,4\}\},$$

$$_{g}SC(U) = \{U, \phi, \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,2,4\}, \{1,3,4\}\}.$$

Define the random variable X to be the number on the chosen card. We can construct Table 5.3.1 which contains the gj – lower and the gj – upper probabilities of a random variable $X = x$ for $j = s$.

Table 4.2.1.

X	1	2	3	4
$_{gs}\underline{p}(X = x)$	0	$\frac{1}{4}$	$\frac{1}{4}$	0
$_{gs}\overline{p}(X = x)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

Then the gs – lower distribution function of X is

$$_{gs}\underline{F}(x)=\begin{cases}0 & -\infty < x < 2,\\ \frac{1}{4} & 2 \le x < 3,\\ \frac{2}{4} & 3 \le x < \infty.\end{cases}$$

And the $gs-$upper distribution function of X is

$$_{gs}\overline{F}(x)=\begin{cases}0 & -\infty < x < 1,\\ \frac{1}{4} & 1 \le x < 2,\\ \frac{2}{4} & 2 \le x < 3,\\ 1 & 3 \le x < 4,\\ \frac{5}{4} & 4 \le x < \infty.\end{cases}$$

4.3. Generalized near rough expectation

We shall introduce the generalized near rough (briefly $gj-$rough) expectation of a random variable X for all $j \in \{s, p, \alpha, \beta\}$. We shall define the $gj-$lower and the $gj-$upper expectations of a random variable X for all $j \in \{s, p, \alpha, \beta\}$ in the following definition.

Definition 4.3.1. Let X be a random variable in the topologized stochastic approximation space $\mathrm{S}=(U, R, p, \tau_K)$. Then the $gj-$lower (resp. $gj-$upper) expectation of X for all $j \in \{s, p, \alpha, \beta\}$ is given by

$$_{gj}\underline{\mu} = {}_{gj}\underline{E}(X) = \sum_{k=1}^{n} x_k \; _{gj}\underline{p}(X = x_k)$$

$$(\text{resp. } _{gj}\overline{\mu} = {}_{gj}\overline{E}(X) = \sum_{k=1}^{n} x_k \; _{gj}\overline{p}(X = x_k)).$$

Definition 4.3.2. Let X be a random variable in the topologized stochastic approximation space $\mathrm{S}=(U, R, p, \tau_K)$. Then the $gj-$rough expectation of X for all $j \in \{s, p, \alpha, \beta\}$ is denoted by $_{gj}E^*(X)$ and is given by

$$_{gj}E^*(X) = \left\langle {}_{gj}\underline{E}(X),\ {}_{gj}\overline{E}(X) \right\rangle;$$

The $gj-$rough expectation of X also denoted by $_{gj}\mu^* = \left\langle {}_{gj}\underline{\mu},\ {}_{gj}\overline{\mu} \right\rangle$ for all $j \in \{s, p, \alpha, \beta\}$.

4.4. Generalized near rough variance and generalized near rough standard deviation

We shall introduce the near rough (briefly $gj-$rough) variance and the near rough (briefly $gj-$rough) standard deviation of a random variable X for all $j \in \{s, p, \alpha, \beta\}$.

Definition 4.4.1. Let X be a random variable in the topologized stochastic approximation space $S = (U, R, p, \tau_K)$. Then the $gj-$lower (resp. $gj-$upper) variance of X for all $j \in \{s, p, \alpha, \beta\}$ is given by

$$_{gj}\underline{V}(X) = {_{gj}\underline{E}}(X - {_{gj}\underline{\mu}})^2 \quad \text{(resp. } {_{gj}\overline{V}}(X) = {_{gj}\overline{E}}(X - {_{gj}\overline{\mu}})^2\text{).}$$

Definition 4.4.2. Let X be a random variable in the topologized stochastic approximation space $S = (U, R, p, \tau_K)$. Then the $gj-$rough variance of X for all $j \in \{s, p, \alpha, \beta\}$ is denoted by $_{gj}V^*(X)$ and is given by

$$_{gj}V^*(X) = \left\langle {_{gj}\underline{V}}(X),\ {_{gj}\overline{V}}(X) \right\rangle.$$

Definition 4.4.3. Let X be a random variable in the topologized stochastic approximation space $S = (U, R, p, \tau_K)$. Then the $gj-$lower (resp. $gj-$upper) standard deviation of X for all $j \in \{s, p, \alpha, \beta\}$ is given by

$$_{gj}\underline{\sigma}(X) = \sqrt{_{gj}\underline{V}(X)} \quad \text{(resp. } {_{gj}\overline{\sigma}}(X) = \sqrt{_{gj}\overline{V}(X)}\text{).}$$

Definition 4.4.4. Let X be a random variable in the topologized stochastic approximation space $S = (U, R, p, \tau_K)$. Then the $gj-$rough standard deviation of X for all $j \in \{s, p, \alpha, \beta\}$ is denoted by $_{gj}\sigma^*(X)$ and is given by

$$_{gj}\sigma^*(X) = \left\langle {_{gj}\underline{\sigma}}(X),\ {_{gj}\overline{\sigma}}(X) \right\rangle.$$

Example 4.4.1. Consider the same experiment as in Example 4.2.1. From Table 4.2.1 it is easy to see the following:

- Neither of the $gj-$lower and the $gj-$upper probabilities summed to one for $j = s$.
- The value 2 of X has $gs-$exact probability since $_{gs}\underline{p}(X) = {_{gs}\overline{p}}(X) = \frac{1}{4}$ at $X = 2$.
- The $gs-$lower and the $gs-$upper expectations of X are:

$$_{gs}\underline{\mu} = {_{gs}\underline{E}}(X) = 1.25, \quad {_{gs}\overline{\mu}} = {_{gs}\overline{E}}(X) = 3.25$$

- The $gs-$rough mean (or $gs-$rough expectation) of X equals:

$$_{gs}\mu^* = \langle 1.25,\ 3.25 \rangle$$

- The $gs-$lower and the $gs-$upper variances of X are:

$$_{gs}\underline{V}(X)=\frac{29}{32}=0.906\,,\quad _{gs}\overline{V}(X)=1.828$$

- The $gs-$rough variance of X equals:

$$_{gs}V^{*}(X)=\langle 0.906,\,1.828\rangle$$

- Finally, the $gs-$rough standard deviation of X is given by:

$$_{gs}\sigma^{*}(X)=\langle 0.952,\,1.352\rangle .$$

4.5. Generalized near rough moments

We shall define the near rough (briefly $gj-$rough) moment of a random variable X for all $j\in\{s,p,\alpha,\beta\}$.

Definition 4.5.1. Let X be a random variable in the topologized stochastic approximation space $\mathrm{S}=(U,R,p,\tau_K)$. Then the $gj-$lower (resp. $gj-$upper) rth moment of X about the $gj-$lower mean $_{gj}\underline{\mu}$ (resp. the $gj-$upper mean $_{gj}\overline{\mu}$) for all $j\in\{s,p,\alpha,\beta\}$, also called the generalized lower (resp. generalized upper) rth $gj-$central moment, is defined as

$$_{gj}\underline{\mu_r}={}_{gj}\underline{E}(X-{}_{gj}\underline{\mu})^r=\sum_{k=1}^{n}\left(x_k-{}_{gj}\underline{\mu}\right)^r{}_{gj}\underline{p}(X=x_k)$$

$$(\text{resp. } _{gj}\overline{\mu_r}={}_{gj}\overline{E}(X-{}_{gj}\overline{\mu})^r=\sum_{k=1}^{n}\left(x_k-{}_{gj}\overline{\mu}\right)^r{}_{gj}\overline{p}(X=x_k)),$$

Where $r=0,1,2,\ldots$.

The r^{th} $gj-$lower (resp. $gj-$upper) moment of X about origin is defined as

$$_{gj}\underline{\mu_r'}={}_{gj}\underline{E}(X^r)\quad(\text{resp. } _{gj}\overline{\mu_r'}={}_{gj}\overline{E}(X^r)),$$

where $r=0,1,2,\ldots$

Definition 4.5.2. Let X be a random variable in the topologized stochastic approximation space $\mathrm{S}=(U,R,p,\tau_K)$. Then the $gj-$rough r^{th} moment of X for all $j\in\{s,p,\alpha,\beta\}$ is denoted by $_{gj}\mu_r^{*}$ and is defined by

$$_{gj}\mu_r^{*}(X)=\left\langle {}_{gj}\underline{\mu_r},\ {}_{gj}\overline{\mu_r}\right\rangle .$$

In the following definition we shall define the $gj-$lower and the $gj-$upper moment generating function of a random variable X for all $j\in\{s,p,\alpha,\beta\}$.

Definition 4.5.3. Let X be a random variable in the topologized stochastic approximation space $\mathrm{S}=(U,R,p,\tau_K)$. Then the $gj-$lower (resp. $gj-$upper) moment generating function of X for all $j\in\{s,p,\alpha,\beta\}$ is defined by

$$_{gj}\underline{M_X}(t) = {}_{gj}\underline{E}\left(e^{tX}\right) = \sum_{k=1}^{n} e^{tx_k}\ {}_{gj}\underline{p}(X = x_k)$$

$$\text{(resp. } {}_{gj}\overline{M_X}(t) = {}_{gj}\overline{E}\left(e^{tX}\right) = \sum_{k=1}^{n} e^{tx_k}\ {}_{gj}\overline{p}(X = x_k)).$$

Definition 4.5.4. Let X be a random variable in the topologized stochastic approximation space $S = (U, R, p, \tau_K)$. Then the gj −rough moment generating function of X for all $j \in \{s, p, \alpha, \beta\}$ is defined by

$$_{gj}M_X^*(t) = \left\langle {}_{gj}\underline{M_X}(t),\ {}_{gj}\overline{M_X}(t)\right\rangle.$$

Example 4.5.1. Consider the same experiment as in Example 4.2.1. From Table 4.2.1 it is easy to see the following:

- The gs −lower r^{th} moment of X about the gs −lower mean $_{gj}\underline{\mu}$ is

$$_{gs}\underline{\mu_r} = {}_{gs}\underline{E}(X - {}_{gs}\underline{\mu})^r = \sum_{k=1}^{4}\left(x_k - {}_{gs}\underline{\mu}\right)^r\ {}_{gs}\underline{p}(X = x_k)$$

$$= \frac{1}{4}\left[(\frac{3}{4})^r + (\frac{7}{4})^r\right] = \left[\frac{3^r + 7^r}{4^{r+1}}\right], \text{ where } r = 0,1,2,...$$

- The gs −upper r^{th} moment of X about the gs −upper mean $_{gs}\overline{\mu}$ is

$$_{gs}\overline{\mu_r} = {}_{gs}\overline{E}(X - {}_{gs}\overline{\mu})^r = \sum_{k=1}^{n}\left(x_k - {}_{gs}\overline{\mu}\right)^r\ {}_{gs}\overline{p}(X = x_k)$$

$$= \left[\frac{(-9)^r + (-5)^r + 2(-1)^r + (3)^r}{(4)^{r+1}}\right], \text{ where } r = 0,1,2,...$$

- The r^{th} gs −lower moment of X about origin is

$$_{gs}\underline{\mu'_r} = {}_{gs}\underline{E}(X^r) = \frac{1}{4}\left[(2^r + 3^r)\right], \text{ where } r = 0,1,2,...$$

- The r^{th} gs −upper moment of X about origin is

$$_{gs}\overline{\mu'_r} = {}_{gs}\overline{E}(X^r) = \left[\frac{1}{4}(1 + 2^r + 2(3)^r + 4^r)\right], \text{ where } r = 0,1,2,...$$

- The gs −lower moment generating function of X is
- $_{gs}\underline{M_X}(t) = {}_{gs}\underline{E}\left(e^{tX}\right) = \sum_{k=1}^{n} e^{tx_k}\ {}_{gs}\underline{p}(X = x_k) = \left[\frac{1}{4}(e^{2t} + e^{3t})\right]$
- The gs −upper moment generating function of X is

$$_{gs}\overline{M_X}(t) = {}_{gs}\overline{E}\left(e^{tX}\right) = \sum_{k=1}^{n} e^{tx_k}\ {}_{gs}\overline{p}(X = x_k)$$

$$=\left[\frac{1}{4}(e^{t}+e^{2t}+2e^{3t}+e^{4t})\right]$$

5. Conclusions

In this paper, we used topological concepts to introduce definitions to generalized rough probability, generalized rough distribution function, generalized rough expectation, …etc. The topological applications which introduced help for measuring generalized rough probability, generalized rough expectation, …etc.

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