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On Uniform Linear Hypergraph Set-indexers of a Graph

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Abstract

For a graph $G = (V, E)$ and a non-empty set X, a *linear hypergraph set-indexer* (LHSI) is a function $f : V(G) \to 2^X$ satisfying the following conditions: (i)f is injective (ii) the ordered pair $H_f(G) = (X, f(V))$, where $f(V) = \{f(v) : v \in V(G)\}$, is a linear hypergraph, (iii) the induced set-valued function $f^{\oplus} : E \to 2^X$, defined by $f^{\oplus}(uv) = f(u) \oplus f(v), \forall\ uv \in E$ is injective, and (iv) $H_{f^{\oplus}}(G) = (X, f^{\oplus}(E))$, where $f^{\oplus}(E) = \{f^{\oplus}(e) : e \in E\}$, is a linear hypergraph. In this paper, we characterize graphs which admit 3-uniform LHSI and establish the relation between the cyclomatic numbers of the given graph, its line graph and the two hypergraphs associated with a 3-uniform LHSI. Also, we determine the upper LHSI number of graph G having $2 \leq \delta(G) \leq \Delta(G) \leq 3$ and girth $g(G) \geq 5$.

Keywords: Set-indexer, linear hypergraph set-indexer

1 Introduction

For all terminology and notation in graphs and hypergraphs , not specifically defined in this paper, we refer the reader to F. Harary [5] and C. Berge [3], respectively. All graphs considered in this paper are simple and without self-loops whereas hypergraphs are simple but may have edges of cardinality one.

Let X be a nonempty finite set, and let $\mathcal{E} = \{E_i : i \in I\}$ be a family of subsets of X. The family $\mathcal{E}$ is called a *hypergraph on* X if (i) $E_i \neq \emptyset$, for every $i \in I$, and (ii) $\bigcup_{i \in I} E_i = X$, then, $H = (X, \mathcal{E})$ is called a *hypergraph* and $|X| = n$ is called its *order*. If all the edges of H are distinct, then H is called simple and, H is *linear* if it satisfies the condition $|E_i \cap E_j| \leq 1$, for all distinct $E_i, E_j \in \mathcal{E}$. Berge [3].

A *set-valuation* of a given graph G is an assignment f of subsets of an arbitrary nonempty X to the vertices of G and the *symmetric difference*

$$f^{\oplus}(uv) = f(u) \oplus f(v) := (f(u) - f(v)) \cup (f(v) - f(u))$$

to each edge $uv \in E(G)$.

For a simple graph $G = (V, E)$ and for an arbitrary set X, if $f : V(G) \to 2^X$, is a set-valuation such that $f(u) \neq \emptyset$, for each $u \in X$ and if $\bigcup_{v \in V(G)} f(v) = X$, then $H_f(G) = (X, f(V))$, $f(V) := \{f(v) : v \in V(G)\}$, is a hypergraph. Hence, given a property $\mathcal{P}$ of the subsets of X a study of $\mathcal{P}$-hypergraphs associated with a given set-valuation of a given graph G could be interesting. Often, specific properties $\mathcal{P}$ are suggested from practical contexts. Acharya. et.al [2] defined LHSI of a graph with $\mathcal{P}$ taken as the property of linearity of hypergraphs, as formulated below.

Definition 1.1. *[2] For a simple graph G, a set-valued function $f : V(G) \to 2^X$ is a linear hypergraph set-indexer (LHSI in short) of G, if f satisfies the following conditions:*

(i) *f is injective*

(ii) *$H_f(G) = (X, f(V))$ is a linear hypergraph, where $f(V) = \{f(v) : v \in V(G)\}$*

(iii) *The induced set-valued function $f^{\oplus} : E \to 2^X$ defined by $f^{\oplus}(uv) = f(u) \oplus f(v)$, $\forall\ uv \in E$, is injective*

(iv) *$H_{f^{\oplus}}(G) = (X, f^{\oplus}(E))$ is a linear hypergraph where, $f^{\oplus}(E) := \{f^{\oplus}(e) : e \in E\}$*

The least (largest) cardinality of the set X with respect to which G admits an LHSI is called the LHSI number (upper LHSI number) of G, and it is denoted by $I_L(G)$ (respectively, $I^{UL}(G)$). ■

An LHSI f of G is said to be *r-uniform* if $|f(u)| = r$ for each $u \in V(G)$

2 Main Results

We need the following known results.

Theorem 2.1. *[2] For a graph G admitting an LHSI $f : V \to 2^X$, if u is any vertex of G with $d(u) \geq 2$, then, $|f(u)| \leq 3$.*

Theorem 2.2. *[2] For a graph G admitting an LHSI $f : V \to 2^X$, if u is any vertex of G with $d(u) \geq 4$, then, $|f(u)| \leq 2$.*

Theorem 2.3. *[2] For a simple graph G admitting an LHSI $f : V(G) \to 2^X$, $|X|$ can be any arbitrary positive integer greater than $I_L(G)$, if and only if G contains a pendant vertex.*

Proposition 2.4. *[2] If G is a (p, q)-graph without pendant vertices and isolated vertices, then $I^{UL}(G) \leq 2p$.*

Theorem 2.5. *[2] For a (p, q)-graph G with $\delta(G) \geq 3$, $I^{UL}(G) \leq \frac{3p}{2}$.*

Theorem 2.6. *If a graph G admits a 3-uniform LHSI, then G contains no cycles of length ≤ 4.*

Proof. Let $f : V(G) \to 2^X$ be a 3-uniform LHSI of G. Suppose, the vertices v_1, v_2, v_3 form a triangle in G and, let $f(v_i) = A_i$. Then, $|A_i \cap A_j| \leq 1$, for all $i \neq j$. Since $H_{f^\oplus}(G)$ is linear, $|(A_1 \oplus A_2) \cap (A_1 \oplus A_3)| \leq 1$, which implies, $|A_1 \cap A_2^c \cap A_3^c) \cup (A_1^c \cap A_2 \cap A_3)| \leq 1$, implies $|A_1 \cap A_2^c \cap A_3^c| \leq 1$. But, $A_1 \subseteq (A_1 \cap A_2^c \cap A_3^c) \cup (A_1 \cap A_2) \cup (A_1 \cap A_3)$. Since $|A_1| = 3$, $|A_1 \cap A_2^c \cap A_3^c| = |(A_1 \cap A_2)| = |(A_1 \cap A_3)| = 1$ and $A_2 \cap A_3 = \emptyset$. Similarly, interchanging the role of A_1 and A_2, we get $|A_2 \cap A_1^c \cap A_3^c| = |(A_1 \cap A_2)| = |(A_2 \cap A_3)| = 1$ and $A_1 \cap A_3 = \emptyset$, a contradiction.
Now, let G contain a cycle of length 4 and let A_1, B_1, A_2, B_2 be the sets assigned to the vertices v_1, v_2, v_3, v_4 in a cyclic order, under the LHSI f. As argued in the previous paragraph, we get, $|A_1 \cap B_1^c \cap B_2^c| = |(A_1 \cap B_1)| = |(A_1 \cap B_2)| = 1$ and $B_1 \cap B_2 = \emptyset$. Similarly, $|B_1 \cap A_1^c \cap A_2^c| = |(A_1 \cap B_1)| = |(B_1 \cap A_2)| = 1$ and $A_1 \cap A_2 = \emptyset$. Hence, there exists an element $x \in A_1 \cap B_2$ and $x \notin A_2 \cup B_1$. Similarly, there exists $y \in B_1 \cap A_2$ and $y \notin B_2 \cup A_1$. Then, $\{x, y\} \subset (A_1 \bigoplus B_1) \cap (A_2 \bigoplus B_2)$, contradicting the linearity of $H_{f^\oplus}(G)$. □

From the arguments given in the above proof, the following proposition is immediate.

Proposition 2.7. *If $f : V(G) \to 2^X$ is an LHSI of a graph G and $u \in V(G)$ with $|f(u)| = 3$ and $d(u) \geq 2$, then $|f(u) \cap f(v_i)| = 1$ and $f(v_i) \cap f(v_j) = \emptyset$, for all $v_i, v_j \in N(u)$, the open neighborhood of u.*

Theorem 2.8. *If G is a (p, q)-graph with $2 \leq \delta(G) \leq \Delta(G) \leq 3$, then $I^{UL}(G) \leq 3p - q$.*

Proof. Let X be a non-empty set and and $f : V(G) \to 2^X$ be an LHSI of a (p, q)-graph G with $2 \le \delta \le \Delta \le 3$. By Theorem 2.1, $|f(u)| \le 3$, for every $u \in V(G)$.
Hence, let V_1, V_2, V_3 be the subsets of $V(G)$ such that $|f(u)| = i$, for every $u \in V_i,\ i = 1, 2, 3$ and, let $|V_i| = p_i$. Then, $p_1 + p_2 + p_3 = p$.
If $|f(u)| = 3$, since $f^{\oplus}E(G) :\to 2^X$ is linear, $|f(u) \cap f(v_i)| = 1$, where v_i is any vertex adjacent to u. Also, $f(v_i) \cap f(v_j) = \emptyset$, where each of v_i and v_j are adjacent to u. If $|f(u)| = 2$, then $|f(u) \cap f(v_i)| = 1$, for all adjacent vertex v_i of u, except possibly one. Now, $X = \bigcup_{u\in V} f(u)$ and $\sum_{u\in V} |f(u)| = p_1 + 2p_2 + 3p_3$.
Therefore, $|X| \le p_1 + 2p_2 + 3p_3 - \frac{1}{2}(\sum_{u\in V_2}(d(u) - 1) + \sum_{u\in V_3} d(u))$. That is, $|X| \le p_1 + 2p_2 + 3p_3 - \frac{1}{2}(\sum_{u\in V_2\cup V_3} d(u) - p_2) = p_1 + \dfrac{5p_2}{2} + 3p_3 - \dfrac{1}{2}(2q - \sum_{u\in V_1} d(u))$
Hence, $|X| \le p_1 + \frac{5p_2}{2} + 3p_3 - q + \frac{3p_1}{2} = \frac{5p_1}{2} + \frac{5p_2}{2} + 3p_3 - q$, which is maximum when $p_3 = p$. Hence, $|X| \le 3p - q$, which implies $I^{UL}(G) \le 3p - q$. □

The following theorem is a characterization of graphs without isolated points, which admit a 3-uniform LHSI.

Theorem 2.9. *A graph G without isolated points admits a 3-uniform LHSI if and only if (1)* $\Delta(G) \le 3$ *and (2) girth* $g(G) \ge 5$.

Proof. Condition (1) of the necessary part follows from Theorem 2.2 and condition (2) follows from Theorem 2.6.
Let $G = (V, E)$ be a (p, q)-graph without containing any K_2 component and satisfying the conditions in the theorem. Let $V = \{v_1, v_2, \dots, v_p\},\ V_0 = \{v_i \in V : d(v_i) = 1 \text{ or } 2\},\ I = \{i : d(v_i) = 1\}$ and $X = E \cup V_0 \cup I$. We denote by E_{v_i}, the set of all edges incident with v_i. Define $f : V(G) \to 2^X$ as follows.

$$f(v_i) = \begin{cases} E_{v_i} & \text{if } d(v_i) = 3 \\ E_{v_i} \cup \{v_i\} & \text{if } d(v_i) = 2 \\ E_{v_i} \cup \{v_i, i\} & \text{if } d(v_i) = 1 \end{cases}$$

Clearly f is injective, $H_f(G)$ is linear and $\cup_{v_i\in V} f(v_i) = X$. The induced edge function $f^{\oplus} : E(G) \to 2^X$ given by, $f^{\oplus}(v_iv_j) = (E_{v_i} \oplus E_{v_j}) \cup (\{v_i, v_j\} \cap V_0) \cup (\{i, j\} \cap I)$ is injective. Since G contains no components of K_2, $\cup_{e\in E} f^{\oplus}(e) = X$. Now, we claim that $H_{f^{\oplus}}(G)$ is linear. On the contrary, suppose $|f^{\oplus}(v_iv_j) \cap f^{\oplus}(v_kv_r)| \ge 2$, where $\{v_i, v_j\} \ne \{v_k, v_r\}$. Let $S = f^{\oplus}(v_iv_j) \cap f^{\oplus}(v_kv_r)$. Then, $S \cap I = \emptyset$ and $|S \cap V_e| \le 1$. Also, $|S \cap V_e| = 1$ if and only if the edges v_iv_j and v_kv_r are incident at common vertex of even degree.
Case 1: The edges v_iv_j and v_kv_r are adjacent. Without loss of generality, let $v_j = v_k = v$. Then, either v or the third edge incident with v belongs to S. Since, $|S| \ge 2$, there exists an edge e_i in S which is not incident with v. Then,

$e_i = v_i v_r$, whence v_i, v_r, v form a triangle in G, a contradiction, since girth $g(G) \geq 5$.
Case 2: The edges $v_i v_j$ and $v_k v_r$ are non-adjacent. Then, $S \cap V_0 = \emptyset$. Let $e_1, e_2 \in S$.
Subcase 1: Let e_1, e_2 be adjacent with v_i as their common vertex. Then v_i, v_k, v_r form a triangle in G, a contradiction.
Subcase 2: The edges e_1 and e_2 are non-adjacent. Each of e_1 and e_2 has one end vertex in $\{v_i, v_j\}$ and the other in $\{v_k, v_r\}$. Therefore, v_i, v_j, v_k, v_r are the vertices of a cycle of length 4 in G, a contradiction, as girth $g(G) \geq 5$.
Thus, f is an LHSI of G and $|X| = 3p - q$.
If $G' = G \cup mK_2$, containing m components of K_2, a 3-uniform LHSI of G can be extended to a 3-uniform LHSI of G' by assigning disjoint sets of cardinality 3 to the vertices of K_2 components. □

Remark 2.10. *For a graph G with $2 \leq \delta(G) \leq \Delta(G) \leq 3$, girth $g(G) \geq 5$ and $V_e = \{v \in V(G) : d(v) = 2$, the function $f : V(G) \to 2^{E(G) \cup V_e}$, defined by*

$$f(v_i) = \begin{cases} E_{v_i} & \text{if } d(v_i) = 3 \\ E_{v_i} \cup \{v_i\} & \text{if } d(v_i) = 2 \end{cases}$$

is an LHSI with the underlying set $E(G) \cup V_e$ of cardinality $3p - q$.

Invoking Theorem 2.8 and Remark 2.10, we get the following result.

Theorem 2.11. *If G is a (p, q)-graph with $2 \leq \delta(G) \leq \Delta(G) \leq 3$ and girth $g(G) \geq 5$, then $I^{UL}(G) = 3p - q$.*

Corollary 2.12. *For a cycle C_n with $n \geq 5$, $I^{UL}(C_n) = 2n$.*

Corollary 2.13. *If G is a 3-regular graph of order p having girth $g(G) \geq 5$, then, $I^{UL}(G) = \frac{3p}{2}$.*

Let $H = (E; X_1, X_2, \ldots, X_n)$ be a hypergraph with n edges. The *representative graph* of H is defined to the simple graph $L(H)$ of order n whose vertices $x_1, x_2, \ldots, x_n$ respectively represent the edges $X_1, X_2, \ldots, X_n$ of H and with vertices x_i and x_j joined by an edge if, and only if, $X_i \cap X_j \neq \emptyset$. For any graph G, the *square of* G, denoted by G^2, has the same vertices as G, with two vertices u and v adjacent if and only if $d(u, v) \leq 2$ in G, where $d(u, v)$ denotes the usual graph distance. We denote square of the line graph of G by $(L(G))^2$.

Theorem 2.14. *For a graph G with $2 \leq \delta(G) \leq \Delta(G) \leq 3$ and girth $g(G) \geq 5$, there exists a 3-uniform LHSI f of G satisfying the following.*

1. *G is isomorphic to the representative graph of $H_f(G)$.*

2. *The line graph $L(G)$ of G is isomorphic to a spanning subgraph of the representative graph of $H_{f^\oplus}(G)$.*

3. $(L(G))^2$ *is isomorphic to the representative graph of* $H_{f^\oplus}(G)$*.*

Proof. Let $V(G) = \{v_1, v_2, \ldots, v_p\}$, $E(G) = \{e_1, e_2, \ldots, e_q\}$ and $V_e = \{v \in V(G) : d(v) = 2$, where G is a graph with $2 \leq \delta(G) \leq \Delta(G) \leq 3$ and girth $g(G) \geq 5$. Define $f : V(G) \to 2^{E(G)\cup V_e}$ as follows. $f(v) = E_v$, the set of edges incident with the vertex v, for all $v \in V - V_e$ and $f(v) = \{v\} \cup E_v$, for all $v \in V_e$. The induced edge function is given by $f^\oplus(uv) = (E_u \oplus E_v) \cup (\{u, v\} \cap V_e)$. Then, f is an LHSI of G as it is shown in the proof of Theorem 2.9.
(1) Two vertices v_i and v_j in G are adjacent if and only if there exists an edge e_k incident with both v_i and v_j, which is true if and only if $e_k \in f(v_i) \cap f(v_j)$, which, in turn, is true if and only if the vertices in the representative graph of $H_f(G)$ corresponding to the sets $f(v_i)$ and $f(v_j)$ are adjacent. Thus, G is isomorphic to the representative graph of $H_f(G)$.
To prove statement (2), we proceed as follows. The number of vertices in $L(G)$ = the number of vertices in the representative graph of $H_{f^\oplus}(G) = q$. Let ${e_i}'$ denote the vertex in $L(G)$ corresponding to the edge e_i in G and let x_i denote the vertex in the representative graph $L(H_{f^\oplus}(G))$, corresponding to the edge $f^\oplus(e_i)$ in $H_{f^\oplus}(G)$. The vertices ${e_i}'$ and ${e_j}'$ are adjacent in $L(G)$ implies, the edges e_i and e_j in G are incident at a common vertex v_k, say. If $d(v_k) = 2$, then $f(v_k) = \{e_i, e_j, v_k\}$ and $v_k \in f^\oplus(e_i) \cap f^\oplus(e_j)$. If $d(v_k) = 3$, then, there is a third edge e_k incident with v_k. Hence, $e_k \in f^\oplus(e_i) \cap f^\oplus(e_j)$. Thus, in each case, x_i and x_j are adjacent in $L(H_{f^\oplus}(G))$ establishing statement 2.
(3) Define $g : V((L(G))^2) \to V(L(H_{f^\oplus}(G)))$ as $g({e_i}') = x_i$, for all $i \in \{1, 2, \ldots, q\}$. We establish statement (3) by proving that g is a graphical isomorphism between the respective graphs.
Let ${e_i}'$ and ${e_j}'$ are incident in $(L(G))^2$. Then, the distance $d({e_i}', {e_j}')$ is either 1 or 2. If $d({e_i}', {e_j}') = 1$, then ${e_i}'$ and ${e_j}'$ are adjacent in $L(G)$, which implies, x_i and x_j are adjacent in $L(H_{f^\oplus}(G))$ as argued in the proof of statement 2.
If $d({e_i}', {e_j}') = 2$, then there exists a vertex ${e_k}'$ in $L(G)$ such that ${e_i}'{e_k}'$ and ${e_k}'{e_j}'$ are edges in $L(G)$. Then e_k is incident with each of e_i and e_j in G, which implies $e_k \in f^\oplus(e_i) \cap f^\oplus(e_j)$ which, in turn implies, x_i and x_j are adjacent in $L(H_{f^\oplus}(G))$.
Conversely, let x_i and x_j are adjacent in $L(H_{f^\oplus}(G))$. Then, $f^\oplus(e_i) \cap f^\oplus(e_j) \neq \phi$ and it contains either a vertex of even degree or an edge.
Case1 : If a vertex $v_k \in f^\oplus(e_i) \cap f^\oplus(e_j)$, then e_i and e_j are incident with v_k, which implies e_i and e_j are adjacent in G, which in turn implies, ${e_i}'$ and ${e_j}'$ are adjacent in $L(G)$. Hence, ${e_i}'$ and ${e_j}'$ are adjacent in $(L(G))^2$.
Case2 : If an edge $e_k \in f^\oplus(e_i) \cap f^\oplus(e_j)$, then e_k is incident with each of e_i and e_j, which implies, either e_i, e_j and e_k are incident at a common vertex in G, or e_k joins the two edges e_i and e_j in G. Hence, $d({e_i}', {e_j}')$ is either 1 or 2 in $L(G)$, which implies, ${e_i}'$ and ${e_j}'$ are adjacent in $(L(G))^2$. □

3 Cyclomatic number of graphs and set-indexed hypergraphs

Let G be a (p,q)-graph with k components, then the cyclomatic number of G is given by $\mu(G) = q - p + k$. Let $H = (X, \xi)$ be a simple hypergraph without isolates. The weighted intersection graph is denoted by $L_w(H)$. The vertex set of $L_w(H)$ is the edge set of H. Two vertices E and E' of $L_w(H)$ are joined by an edge with weight $|E \cap E'|$, if $E \cap E' \neq \emptyset$, and are not jointed otherwise. Let $w(H)$ be the maximal weight of a forest of $L_w(H)$. Acharya and Las Vergnas [1] defined the cyclomatic number of the hypergraph H as $\mu(H) = \Sigma_{E\in\xi}|E| - |X| - w(H)$.
When H is a graph, $\mu(H)$ is the usual cyclomatic number.

Theorem 3.1. *If G is a conn(p, q)-graph with $2 \leq \delta(G) \leq \Delta(G) \leq 3$ and girth $g(G) \geq 5$, there exists a 3-uniform LHSI f of G satisfying the following.*
(1)$\mu(H_f(G) = \mu(G)$
(2)$\mu(H_{f^\oplus}(G)) = \mu(L(G)) + q$, where $L(G)$ represents the line graph of G.

Proof. Let V_e be the set of vertices of G with even degree and $f : V(G) \rightarrow 2^{E(G)\cup V_e}$ be the LHSI of G as mentioned in the proof of Theorem 2.14. Then, G is isomorphic to the representative graph of $H_f(G)$ and $(L(G))^2$ is isomorphic to the representative graph of $H_{f^\oplus}(G)$. Since $H_f(G)$ and $H_{f^\oplus}(G)$ are linear, their weighted intersection graphs will be corresponding representative graphs. Let G contains k components. Then, the number of components of $L(G) =$ the number of components of $(L(G))^2 = k$. Then, $w(H_f(G)) = p - k$ and $w(H_{f^\oplus}(G)) = q - k$.
(1) $\mu(G) = q - p + k$
$\mu(H_f(G) = \sum_{E\in f(V)} |E| - |X| - w(H_f(G)) = 3p - (3p - q) - (p - k) = q - p + k$
Thus, $\mu(G) = \mu(H_f(G))$
(2) Applying fundamental theorem of Graph Theory, $2|V_e| + 3(p - |V_e|) = 2q$, which implies, $|V_e| = 3p - 2q$. Number of vertices of $L(G) = q$.
The number of edges of $L(G) = -q + \frac{\Sigma_i d_i^2}{2}$

$$= -q + \sum_{v\in V_e} \frac{2^2}{2} + \sum_{v\in V-V_e} \frac{3^2}{2}$$

$$= -q + 2|V_e| + \frac{9}{2}(p - |V_e|) = \frac{9p}{2} - q - \frac{5}{2}|V_e|$$

$$= \frac{9p}{2} - q - \frac{5}{2}(3p - 2q) = 4q - 3p$$

$\mu(L(G) = 4q - 3p - q + k = 3q - 3p + k$

$$\mu(H_{f^{\oplus}}(G)) = \sum_{e \in E(G)} |f^{\oplus}(e)| - |X| - w(H_{f^{\oplus}}(G))$$

$$= 4q - (3p - q) - (q - k) = 4q - 3p + k = \mu(L(G)) + q$$

□

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