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$\sigma-$Connectivity in $L-$Topological Spaces

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Abstract

In this paper, the concept of $\sigma-$connectivity in $L-$topological spaces is introduced. Some properties for $\sigma-$connectivity in $L-$topological spaces are characterized systematically. The famous K.Fan's Theorem holds for $\sigma-$connectivity in $L-$topological spaces.

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1 Introduction

Connectivity is one of the most important notions in topology. Pu and Liu [10] introduced the definition of connectedness in fuzzy topological spaces. Since then, many other authors have presented different kinds of connectivity in fuzzy setting([8], [11], [7], [12]). In [3, 4, 5], Chen proposed the concept of $\sigma-$closed $L-$set in fuzzy lattices and further developed different important topological concepts such as $\sigma-$convergence and $\sigma-$continuous order homomorphisms by means of fuzzy $\sigma-$open $L-$set.

In this paper, along the line of [4], we introduce the concept of $\sigma-$ connectedness in $L-$ topological spaces based on $\sigma-$ open $L-$sets. $\sigma-$connectedness preserves many nice properties of connectedness in general topological spaces. Meanwhile, the famous K.Fan's Theorem can be generalized to $L-$topological spaces for $\sigma-$ connectedness.

2 Preliminaries

Throughout this paper, $(L, \bigvee, \bigwedge, ')$ will denote a complete $DeMorgan$ algebra. For a nonempty set X, L^X denotes the set of all $L-$fuzzy sets ($L-$sets for short) on X. The smallest element and the largest element in L^X are denoted by $\underline{0}$ and $\underline{1}$ respectively.

A non-null element a in L is called $\bigvee-$irreducible element if $a \leq b \bigvee c$ implies $a \leq b$ or $a \leq c$. The set of all $\bigvee-$irreducible elements in L is denoted by $M^*(L)$.

An $L-$topological space is a pair (L^X, δ), where δ is a subfamily of L^X which contains $\underline{0}$,$\underline{1}$ and is closed for any suprema and finite infima. δ is called an $L-$topology on X. Every member of δ is called an open $L-$set and its quasicomplementation is called a closed $L-$set. The interior and closure of $P \in L^X$ will be denoted by $int(P)$ and $cl(P)$ respectively.

Definition 2.1 [3] *Let (L^X, δ) be an $L-$topological space, $e \in M^*(L^X)$ and $P, Q \in \delta$. Then P, Q are called an ordered pair of closed remote-neighborhood of e, in symbol $< P, Q >\in \eta(e) \times \eta(e)$, if $e \not\leq P$ and $Q \leq int(P)$.*

Definition 2.2 [4] *Let (L^X, δ) be an $L-$topological space, $A \in L^X$ and $e \in M^*(L^X)$.*

(1) *e is said to be a $\sigma-$adherence point of A, if $A \not\leq Q$ for each $< P, Q >\in \eta(e) \times \eta(e)$. The union of all $\sigma-$adherence points of A is called the $\sigma-$closure of A which denoted by $cl_\sigma(A)$.*

(2) *A is called a $\sigma-$closed set if $cl_\sigma(A) \leq A$.*

(3) *A is called a $\sigma-$open set if A' is a $\sigma-$closed set.*

Definition 2.3 [5] *Let $(L^{X_i}, \delta_i)(i = 1, 2)$ be $L-$topological spaces and $f : L^{X_1} \to L^{X_2}$ an order-homomorphism. Then f is called $\sigma-$ continuous if $f^{-1}(A)$ is $\sigma-$open for each $\sigma-$open set A in (L^{X_2}, δ_2).*

3 $\sigma-$connectivity in $L-$topological spaces

In this section, we will introduce the concept of $\sigma-$connectivity in $L-$topological spaces and discuss its basic properties.

Definition 3.1 *Let (L^X, δ) be an $L-$topological space and $A, B \in L^X$. Then A and B are called $\sigma-$separated if $cl_\sigma(A) \bigwedge B = A \bigwedge cl_\sigma(B) = \underline{0}$.*

Definition 3.2 *Let (L^X, δ) be an $L-$topological space and $A \in L^X$. A is called $\sigma-$connected subset if there do not exist two non-null $\sigma-$separated subsets B, C in L^X such that $A = B \bigvee C$. (L^X, δ) is said to be a $\sigma-$connected space if $\underline{1}$ is $\sigma-$connected.*

Theorem 3.3 *Let (L^X, δ) be an $L-$topological space . Then the following statements are equivalent:*

(1) *(L^X, δ) is not a $\sigma-$connected space.*

(2) *There exist two non-null $\sigma-$closed subsets A, B such that $A \bigvee B = \underline{1}$ and $A \bigwedge B = \underline{0}$.*

(3) *There exist two non-null $\sigma-$open subsets A, B such that $A \bigvee B = \underline{1}$ and $A \bigwedge B = \underline{0}$.*

Proof (1) $\Rightarrow$ (2) Suppose (L^X, δ) is not a $\sigma-$connected space. Then there exist two non-null $\sigma-$separated subsets A, B such that $A \bigvee B = \underline{1}$. It is obvious that $A \bigwedge B = \underline{0}$. The fact that A is $\sigma-$closed follows from

$$cl_\sigma(A) = cl_\sigma(A) \bigwedge (A \bigvee B) = (cl_\sigma(A) \bigwedge A) \bigvee (cl_\sigma(A) \bigwedge B) = A.$$

Similarly, one can see that B is a $\sigma-$closed subset.

(2) $\Rightarrow$ (1) It follows from Definition 3.1 and Definition 3.2.

(2) $\Rightarrow$ (3) Assume that there exist two non-null $\sigma-$closed subsets C, D such that $C \bigvee D = \underline{1}$ and $C \bigwedge D = \underline{0}$. Then, C', D' are two non-null $\sigma-$open sets such that $C' \bigvee D' = \underline{1}$ and $C' \bigwedge D' = \underline{0}$. Let $A = C', B = D'$, thereby, A and B satisfy (3). Similarly, one can prove (3) $\Rightarrow$ (2).

Corollary 3.4 *Let (L^X, δ) be an $L-$topological space, then (L^X, δ) is not a $\sigma-$connected space if and only if there exist non-null subset A in L^X such that A is both $\sigma-$closed and $\sigma-$open .*

Theorem 3.5 *Let (L^X, δ) be an $L-$topological space and $A \in L^X$. Then the following statements are equivalent:*

(1) *A is $\sigma-$connected.*

(2) *There do not exist two $\sigma-$closed subsets C, D such that*

$$C \bigwedge A \neq \underline{0}, D \bigwedge A \neq \underline{0}, A \leq C \bigvee D \text{ and } C \bigwedge D \bigwedge A = \underline{0}.$$

(3) *There do not exist two $\sigma-$closed subsets C, D such that*

$$A \not\leq C, A \not\leq D, A \leq C \bigvee D \text{ and } C \bigwedge D \bigwedge A = \underline{0}.$$

Proof (1) $\Rightarrow$ (2) Suppose A is $\sigma-$connected and there exist two $\sigma-$closed subsets C, D such that

$$C \bigwedge A \neq \underline{0}, D \bigwedge A \neq \underline{0}, A \leq C \bigvee D \text{ and } C \bigwedge D \bigwedge A = \underline{0}.$$

Then $(C \bigwedge A) \bigvee (D \bigwedge A) = (C \bigvee D) \bigwedge A = A$. One can see that $cl_\sigma(C \bigwedge A) \bigwedge (D \bigwedge A) = \underline{0}$ by

$$cl_\sigma(C \bigwedge A) \bigwedge (D \bigwedge A) \leq cl_\sigma(C) \bigwedge (D \bigwedge A) = C \bigwedge D \bigwedge A = \underline{0}.$$

Similarly, $cl_\sigma(D \bigwedge A) \bigwedge (C \bigwedge A) = \underline{0}$. Hence, A is not $\sigma-$connected which is a contradiction.

(2) ⇒ (3) Suppose there exist two $\sigma-$closed subsets C, D such that

$$A \not\leq C, A \not\leq D, A \leq C \bigvee D \text{ and } C \bigwedge D \bigwedge A = \underline{0}.$$

One can easily prove that $C \wedge A \neq \underline{0}, D \wedge A \neq \underline{0}$ which is a contradiction.

(3) ⇒ (1) Suppose A is not $\sigma-$connected. Then there exist two non-null $\sigma-$separated subsets E, F in L^X such that $A = E \vee F$. Take $C = cl_\sigma(E)$ and $D = cl_\sigma(F)$. Then $A = E \vee F \leq cl_\sigma(E) \vee cl_\sigma(F) = C \vee D$ and $C \wedge D \wedge A = \underline{0}$ by that

$$\begin{aligned} cl_\sigma(E) \bigwedge cl_\sigma(F) \bigwedge A &= cl_\sigma(E) \bigwedge cl_\sigma(F) \bigwedge (E \bigvee F) \\ &= (cl_\sigma(E) \bigwedge cl_\sigma(F) \bigwedge E) \bigvee (cl_\sigma(E) \bigwedge cl_\sigma(F) \bigwedge F) \\ &= (cl_\sigma(F) \bigwedge E) \bigvee (cl_\sigma(E) \bigwedge F) = \underline{0}. \end{aligned}$$

Moreover one can get that $A \not\leq C$ and $A \not\leq D$. In fact, if $A \leq C$, then $D \wedge A = D \wedge (A \wedge C) = \underline{0}$, i.e. $cl_\sigma(F) \wedge A = \underline{0}$. Hence, $F = F \wedge A \leq cl_\sigma(F) \wedge A = \underline{0}$. This is a contradiction. Similarly, one can have $A \not\leq D$. By the contradiction to (3), we have that A is $\sigma-$connected.

Theorem 3.6 *Let (L^X, δ) be an $L-$topological space and $A \in L^X$. Then the following statements are equivalent:*

(1) *A is $\sigma-$connected.*

(2) *For any two non-null points $a, b \leq A$, there exists a $\sigma-$connected subset E in (L^X, δ) such that $a, b \leq E \leq A$.*

(3) *For any two non-null points $a, b \in M^*(A)$, there exists a $\sigma-$connected subset E in (L^X, δ) such that $a, b \leq E \leq A$.*

Proof (1) ⇒ (2) and (2) ⇒ (3) are obvious, we only prove (3) ⇒ (1).

Suppose A is not $\sigma-$connected in (L^X, δ), then there exist two $\sigma-$closed subsets $C, D \in L^X$ such that

$$A \not\leq C, A \not\leq D, A \leq C \bigvee D \text{ and } C \bigwedge D \bigwedge A = \underline{0}.$$

By $A = \bigvee M^*(A)$, there exist $a, b \in M^*(A)$ such that $a \not\leq C, b \not\leq D$. Let E be a $\sigma-$connected set in (L^X, δ) such that $a, b \leq E \leq A$. Then we have that

$$E \not\leq C, E \not\leq D, E \leq C \bigvee D \text{ and } C \bigwedge D \bigwedge E = \underline{0}.$$

By Theorem 3.5, E is not $\sigma-$connected which is a contradiction.

Theorem 3.7 *Let A be $\sigma-$connected in an $L-$topological space (L^X, δ). If $A \leq B \leq cl_\sigma(A)$, then B is a $\sigma-$connected subset in (L^X, δ).*

Proof Suppose B is not a $\sigma-$connected subset in (L^X, δ). Then there exist two non-null $\sigma-$separated subsets C, D in L^X such that $B = C \bigvee D$. Let $E = A \bigwedge C$ and $F = A \bigwedge D$. Then

$$E \bigvee F = (A \bigwedge C) \bigvee (A \bigwedge D) = A \bigwedge (C \bigvee D) = A \bigwedge B = A$$

and

$$cl_\sigma(E) \bigwedge F = E \bigwedge cl_\sigma(F) = \underline{0}.$$

Hence, $E = \underline{0}$ or $F = \underline{0}$ for A is $\sigma-$connected. If $E = \underline{0}$, then $A = F = A \bigwedge D \leq D$, $cl_\sigma(A) \leq cl_\sigma(D)$. On the other hand, $C \leq B \leq cl_\sigma(A)$, $C = C \bigwedge cl_\sigma(A) \leq C \bigwedge cl_\sigma(D) = \underline{0}$. That is, $C = \underline{0}$ which leads to a contradiction. Similarly, one can get $D = \underline{0}$ if $F = \underline{0}$. So, B is $\sigma-$connected in (L^X, δ).

Corollary 3.8 *If A is $\sigma-$connected in (L^X, δ), then so is $cl_\sigma(A)$.*

Theorem 3.9 *Let (L^X, δ) be an $L-$topological space and $A \in L^X$ is $\sigma-$connected. If there exist two $\sigma-$separated subsets B and C in L^X such that $A \leq B \bigvee C$, then $A \leq B$ or $A \leq C$.*

Proof Let B and C be two $\sigma-$separated subsets in (L^X, δ) such that $A \leq B \bigvee C$. Then $A \bigwedge B$ and $A \bigwedge C$ are two $\sigma-$separated subsets by the followings

$$cl_\sigma(A \bigwedge C) \bigwedge (A \bigwedge B) \leq cl_\sigma(C) \bigwedge B = \underline{0}$$

and

$$cl_\sigma(A \bigwedge B) \bigwedge (A \bigwedge C) \leq cl_\sigma(B) \bigwedge C = \underline{0}.$$

Since A is $\sigma-$connected and $A = A \bigwedge (B \bigvee C) = (A \bigwedge B) \bigvee (A \bigwedge C)$, we get $A \bigwedge B = \underline{0}$ or $A \bigwedge C = \underline{0}$. If $A \bigwedge B = \underline{0}$, then $A = A \bigwedge C \leq C$. Similarly, if $A \bigwedge C = \underline{0}$, then $A \leq B$.

Theorem 3.10 *Let $\{A_t\}_{t \in T} \subset L^X$ be a family of $\sigma-$connected subsets in (L^X, δ). Suppose there exists $s \in T$ such that A_t and A_s are not $\sigma-$separated for each $t \in T \backslash \{s\}$, then $A = \bigvee_{t \in T} A_t$ is $\sigma-$connected in (L^X, δ).*

Proof Let B and C be two $\sigma-$separated subsets in (L^X, δ) such that $A = B \bigvee C$. We will show that $B = \underline{0}$ or $C = \underline{0}$ by Definition 3.2.

Take $B_t = A_t \bigwedge B, C_t = A_t \bigwedge C$ for each $t \in T$. Then $cl_\sigma(B_t) \bigwedge C_t = cl_\sigma(C_t) \bigwedge B_t = \underline{0}$. That is, B_t and C_t are $\sigma-$separated for each $t \in T$. One can see that $B_t = \underline{0}$ or $C_t = \underline{0}$ by $A_t = A_t \bigwedge A = (A_t \bigwedge B) \bigvee (A_t \bigwedge C) = B_t \bigvee C_t$ and and A_t is $\sigma-$connected. It follows that $A_t = C_t \leq C$ or $A_t = B_t \leq B$. Especially, $A_s = C_s \leq C$ or $A_s = B_s \leq B$. Without loss of generality, we may assume that $A_s = C_s \leq C$. Then $A_t \leq C$ for each $t \in T\backslash\{s\}$. In fact, if $A_t \nleq C$ for some $t \in T\backslash\{s\}$, then $A_t \leq B$ and

$$cl_\sigma(A_t) \bigwedge A_s = cl_\sigma(A_t) \bigwedge C_s \leq cl_\sigma(B) \bigwedge C = \underline{0},$$

$$A_t \bigwedge cl_\sigma(A_s) = A_t \bigwedge cl_\sigma(C_s) \leq B \bigwedge cl_\sigma(C) = \underline{0}.$$

That is, A_t and A_s are $\sigma-$separated which is a contradiction. Therefore, $A_t \leq C$ for each $t \in T$. It follows that $A \leq C$ and $B = B \bigwedge A \leq B \bigwedge C \leq B \bigwedge {C_\sigma}^- = \underline{0}$, i.e.,$B = \underline{0}$. Hence, $A = \bigvee\limits_{t \in T} A_t$ is a $\sigma-$connected subset in (L^X, δ).

Corollary 3.11 *Let* $\{A_t\}_{t \in T} \subset L^X$ *be a family of* $\sigma-$*connected subsets in* (L^X, δ). *If* $\bigwedge\limits_{t \in T} A_t \neq \underline{0}$, *then* $A = \bigvee\limits_{t \in T} A_t$ *is* $\sigma-$*connected in* (L^X, δ).

Definition 3.12 *Let* (L^X, δ) *be an* $L-$*topological space and* $A \in L^X$. *A is called a* $\sigma-$*connected component in* (L^X, δ) *if A is a maximal* $\sigma-$*connected* $L-$*set, i.e.,* $A = B$ *for each* $\sigma-$*connected* $L-$*set B in* (L^X, δ) *such that* $A \leq B$.

Theorem 3.13 *Let* (L^X, δ) *be an* $L-$*topological space, then*

(1) *The union of all the* $\sigma-$*connected components of* (L^X, δ) *equals* $\underline{1}$.

(2) *The intersection of different* $\sigma-$*connected components of* (L^X, δ) *is empty.*

(3) *Each* $\sigma-$*connected component of* (L^X, δ) *is a* $\sigma-$*closed* $L-$*set.*

Proof (1) Firstly, for each $e \in M^*(L^X)$, e is a $\sigma-$connected subset. In fact, if e is not a $\sigma-$connected subset, there exist two non-null $\sigma-$separated subsets A and B such that $e = A \bigvee B$. Since e is a molecule, $e = A$ or $e = B$. Then, $A = \underline{0}$ or $B = \underline{0}$ which is a contradiction.

For each $e \in M^*(L^X)$, define $\mathcal{A} = \{A(e) \in L^X | A(e)$ is $\sigma-$connected such that $e \leq A(e)\}$. Obviously, $\mathcal{A} \neq \emptyset$. Let $A = \bigvee \mathcal{A}$, then A is $\sigma-$connected by Corollary 3.11. Clearly, A is a $\sigma-$connected component (L^X, δ). And $A = \underline{1}$ follows from that $\bigvee M^*(L^X) = \underline{1}$.

(2) Suppose A and B are different $\sigma-$connected components and $A \bigwedge B \neq \emptyset$. Then $A \bigvee B$ is $\sigma-$connected by Corollary 3.11 which is a contradiction.

(3) Suppose A is a $\sigma-$connected component in (L^X, δ), then $cl_\sigma(A)$ is $\sigma-$connected and $A \leq cl_\sigma(A)$. Therefore, $A = cl_\sigma(A)$ and A is $\sigma-$closed by Definition 2.2, .

Theorem 3.14 *Let* $(L^{X_i}, \delta_i)(i = 1, 2)$ *be two* $L-$*topological spaces and* $f : L^{X_1} \to L^{X_2}$ *be a* $\sigma-$*continuous order homomorphism. If A is* $\sigma-$*connected in* (L^{X_1}, δ_1), *then* $f(A)$ *is* $\sigma-$*connected in* (L^{X_2}, δ_2).

Proof Suppose $f(A)$ is not $\sigma-$connected. Then there are two $\sigma-$closed $L-$sets $C, D \in L^{X_2}$ such that

$$f(A) \nleq C, f(A) \nleq D, f(A) \leq C \bigvee D \text{ and } C \bigwedge D \bigwedge f(A) = \underline{0}.$$

So,

$$A \nleq f^{-1}(C), A \nleq f^{-1}(D), A \leq f^{-1}(C \bigvee D) = f^{-1}(C) \bigvee f^{-1}(D)$$

and

$$\begin{aligned} f^{-1}(C) \bigwedge f^{-1}(D) \bigwedge A &\leq f^{-1}(C) \bigwedge f^{-1}(D) \bigwedge f^{-1}(f(A)) \\ &= f^{-1}(C \bigwedge D \bigwedge f(A)) \\ &= \underline{0}. \end{aligned}$$

Since f is $\sigma-$continuous, $f^{-1}(C)$ and $f^{-1}(D)$ are two $\sigma-$ closed $L-$sets in (L^{X_1}, δ_1). This shows that A is not $\sigma-$connected which is a contradiction. Therefore $f(A)$ is $\sigma-$connected in (L^{X_2}, δ_2).

Corollary 3.15 *Let $(L^{X_i}, \delta_i)(i = 1, 2)$ be two $L-$topological spaces and $f : L^{X_1} \rightarrow L^{X_2}$ be a $\sigma-$continuous onto order homomorphism. If (L^{X_1}, δ_1) is a $\sigma-$connected $L-$topological space, then so is (L^{X_2}, δ_2).*

Definition 3.16 *Let (L^X, δ) be an $L-$topological space and $e \in M^*(L^X)$. Then $P \in L^X$ is called $\sigma-$closed remote neighborhood of e if P is a $\sigma-$closed subset such that $e \nleq P$. The family of all $\sigma-$closed remote neighborhoods of e is denoted by $\eta^-_\sigma(e)$. $Q \in L^X$ is called $\sigma-$ remote neighborhood of e if there exists $P \in \eta^-_\sigma(e)$ such that $Q \leq P$. The family of all $\sigma-$ remote neighborhoods of e is denoted by $\eta_\sigma(e)$.*

Theorem 3.17 *(K.Fan Theorem) Let (L^X, δ) be an $L-$topological space and $A \in L^X$. Then A is $\sigma-$connected if and only if for each pair a, b in $M^*(A)$ and each $\sigma-$remote neighborhood mapping $P : M^*(A) \rightarrow \bigcup\{\eta_\sigma(e) | e \in M^*(A)\}$ where $P(e) \in \eta_\sigma(e)$ for each $e \in M^*(A)$, there exists a finite number of points $e_1 = a, e_2, \cdots, e_n = b$ in $M^*(A)$ such that $A \nleq P(e_i) \vee P(e_{i+1}), i = 1, 2, \cdots, n-1$.*

Proof Sufficiency. Suppose that A is not $\sigma-$connected. Then there exist two non-null $\sigma-$separated $L-$subsets $B, C \in L^X$ such that $A = B \vee C$. Define the mapping $P : M^*(A) \rightarrow \bigcup\{\eta_\sigma(e)|e \in M^*(A)\}$ as the following:

$$P(e) = \begin{cases} cl_\sigma(C), & \text{if } e \leq B, \\ cl_\sigma(B), & \text{if } e \leq C. \end{cases}$$

We have $e \not\leq P(e)$ since $cl_\sigma(B) \bigwedge C = B \bigwedge cl_\sigma(C) = \underline{0}$. By $P(e)$ is a $\sigma-$closed $L-$subset, $P(e) \in \eta_\sigma(e)$ for each $e \in M^*(A)$. Take the points $a, b \in M^*(A)$ such that $a \leq B, b \leq C$. Since for arbitrary finite points $e_1 = a, e_2, \cdots, e_n = b$ there is only one of $e_i \leq B$ and $e_i \leq C$ holds, we have $P(e_i) = cl_\sigma(B)$ or $P(e_i) = cl_\sigma(C)$. But $P(e_1) = cl_\sigma(C)$ and $P(e_n) = cl_\sigma(B)$, hence there exists some $j(1 \leq j \leq n-1)$ such that $P(e_j) = cl_\sigma(C)$ and $P(e_{j+1}) = cl_\sigma(B)$. This shows that

$$A = B \bigvee C \leq P(e_j) \bigvee P(e_{j+1})$$

which is a contradiction.

Necessity. Suppose that condition of theorem is not true, i.e, there are two points $a, b \in M^*(A)$ and a $\sigma-$remote neighborhood mapping $P : M^*(A) \to \bigcup\{\eta_\sigma(e)|e \in M^*(A)\}$ such that

$$A \not\leq P(e_i) \bigvee P(e_{i+1}), i = 1, 2, \cdots, n-1$$

is not true for arbitrary finite points $e_1, \cdots, e_n \in M^*(A)$. For the sake of convenience, we follow the agreement that two points r and k are $\sigma-$linked if there exist finite points $e_1 = r, e_2, \cdots, e_n = k$ in $M^*(A)$ such that $A \not\leq P(e_i) \bigvee P(e_{i+1}), i = 1, 2, \cdots, n-1$. Otherwise, r and k are not $\sigma-$ linked. Let

$$\Phi = \{e \in M^*(A)|a \text{ and } e \text{ are } \sigma-\text{linked}\},$$

$$\Psi = \{e \in M^*(A)|a \text{ and } e \text{ are not } \sigma-\text{linked}\},$$

$$B = \bigvee \Phi, C = \bigvee \Psi.$$

Obviously, a and a are $\sigma-$linked for that $a \not\leq P(a)$ implies $A \not\leq P(a)$. So, $a \in \Phi, a \leq B$. By the hypothesis, a and b are not $\sigma-$linked, then $b \in \Psi$ and $b \leq C$. Hence, $B \neq \underline{0}, C \neq \underline{0}$. Since for each $e \in M^*(A)$, $e \in \Phi$ or $e \in \Psi$, we have $A = B \bigvee C$. We will prove $cl_\sigma(B) \bigwedge C = B \bigwedge cl_\sigma(C) = \underline{0}$. Hence, A is not $\sigma-$connected which is a contradiction.

In fact, suppose $cl_\sigma(B) \bigwedge C \neq \underline{0}$ and take point $d \leq cl_\sigma(B) \bigwedge C$. By $d \leq cl_\sigma(B)$, we have $d \not\leq P(d)$ and $B \not\leq P(d)$. So there is $e \in \Phi$ such that $e \not\leq P(d)$. Hence $e \not\leq P(d) \bigvee P(e)$ and $e \leq B \leq A$. Thus, $A \not\leq P(d) \bigvee P(e)$. For e and a are $\sigma-$linked, then a and d are $\sigma-$linked. On the other hand, by $d \leq C$, we have $C \not\leq P(d)$. There exists $\lambda \in \Psi$ such that $\lambda \not\leq P(d)$. Hence $\lambda \not\leq P(d) \bigvee P(\lambda)$ and $\lambda \leq C \leq A$. Therefore, $A \not\leq P(d) \bigvee P(\lambda)$. By d and a are $\sigma-$linked, we have a and λ are $\sigma-$linked. This contradicts that $\lambda \in \Psi$! Thus, $cl_\sigma(B) \bigwedge C = \underline{0}$. Similarly, one can verify that $B \bigwedge cl_\sigma(C) = \underline{0}$.

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