Int. J. Contemp. Math. Sciences, Vol. 5, 2010, no. 7, 341 - 358

Wave Solutions of the Nonlinear Schrödinger equation

Ramzi S. N. Alsaedi

Mathematics D epartment, Faculty of Science
King Abdul Aziz University, P.O. Box 80203
Jeddah 21589, Saudi Arabia
ramzialsaedi@yahoo.co.uk

Abstract

In this paper we have investigate the exact travelling wave solutions for the nonlinear Schordinger equation by using of the coupled projective Riccati equations. New families of solutions written in the fractional type in hyperbolic function and triangle function are found. These studies reveal that the nonlinear Schordinger possesses of seeking a rich variety of solutions.

1 Introduction

In recent years , nonlinear partial differential equations as well as their exact travelling wave solutions have been studied widely . Many authors (see for example [15-22,40-43,66-71]) presented various powerful methods to deal with this subject . The basic purpose of them is to construct the solitary wave solutions and periodic solutions as a polynomial in hyperbolic and triangle functions.

In this paper we have investigated the exact travelling wave solutions for the following nonlinear cubic Schrodinger equation

$$iq_t + q_{xx} + q_{yy} + q(|q|^2 - S(x, y, t)) = 0, \tag{1}$$

where $i = \sqrt{-1}$. Nonlinear equations of Schrodinger type are of great interest due to their centeral importance to the theory of quantum mechanics. They arise in plasma wave and nonlinear optics and are importance in development of solitons and inverse scattering transform theory . Searching and constructing exact solutions for linear and nonlinear partial differential equations are an ongoing research. A number of effective methods is presented in the articles[5,10,14] ,such as inverse scattering theory, the truncated painleve expansion , invariant method , sin-cosine method ,the extended tanh- function method ,and other methods .

Zayed et al [15], Fan [5,7] have studies the explicit solutions of Hirota Satsuma coupled KdV system using Jacobic elliptic function and q-deformed function. Xie[12] has investigate Hirota Satsuma coupled KdV and has gives exact travelling wave soliton solutions in rational form .

Khuri [12] has recently used a complex tanh -function method for constructing multiple travelling wave solutions for nonlinear Schordinger equations with complex phases and solutions. In this paper we would like to investigate the Nonlinear Schordinger equation (1) using a new transformation based on the coupled projective Riccati equations . As result new exact travelling wave solutions written in rational forms in hyperbolic and triangle functions are found. If we use the transformation

$$q(x, y, t) = u(x, y, t) + iv(x, y, t), \tag{2}$$

then the equation (1) will be transformed to the following system:

$$u_t + v_{xx} + v_{yy} + v(u^2 + v^2) - S(x, y, t)v = 0, \tag{3}$$

$$v_t - u_{xx} - u_{yy} - u(u^2 + v^2) + S(x, y, t)u = 0. \tag{4}$$

2 Method for finding rational solitary wave solutions

Our method is summarized up as follows[12] . For the given partial differential equations

$$F_1(u, v, u_t, v_t, u_x, v_x, u_y, v_y, ...) = 0, \tag{5}$$

$$G_1(u, v, u_t, v_t, u_x, v_x, u_y, v_y, ...) = 0, \tag{6}$$

we seek the following formal travelling wave solutions which are of important physical significance:

$$\begin{aligned} u(x, y, t) &= u(\xi), \\ v(x, y, t) &= v(\xi), \end{aligned} \tag{7}$$

where $\xi = \lambda_1 t + \lambda_2 x + \lambda_3 y$ and λ_1, λ_2 , λ_3 are arbitrary constants. The equations (5) and (6) are reduced to nonlinear ordinary differential equations.

$$F_2(u, v, u', v', u', v', ...) = 0, \tag{8}$$

$$G_2(u, v, u', v', u', v', ...) = 0, \tag{9}$$

where the prime denotes $\frac{d}{d\xi}$. We now seek the solutions of equations (8) and (9) in the form

$$u = \sum_{i=0}^{r} a_i g^i(\xi) + \sum_{j=1}^{r} b_j g^{j-1}(\xi) f(\xi), \tag{10}$$

$$v = \sum_{k=0}^{s} c_k g^k(\xi) + \sum_{n=1}^{s} d_n g^{n-1}(\xi) f(\xi), \tag{11}$$

with

$$\begin{aligned} f(\xi) &= \frac{1}{A\tanh(\xi)+B\operatorname{sech}(\xi)}, \\ g(\xi) &= \frac{\operatorname{sech}(\xi)}{A\tanh(\xi)+B\operatorname{sech}(\xi)}, \end{aligned} \tag{12}$$

which satisfy

$$\begin{aligned} f'(\xi) &= -Ag^2(\xi)+\frac{B}{A}g(\xi)(1-Bg(\xi)), \\ g'(\xi) &= -Ag(\xi)f(\xi), \\ f^2(\xi) &= g^2(\xi)+\frac{1}{A^2}(1-Bg(\xi))^2, \end{aligned} \tag{13}$$

where A ,B , a_i, b_j, c_k, d_n , $i = 1, 2, .., r$,$j = 1, 2, ..., r$,$k = 1, 2, ..s$, $n = 1, 2, .., s$ are constants to be determined later and r, s can be determined by balancing the highest order partial derivative and the nonlinear term in equations (8) and (9).
Mean while ,we can also assume that

$$\begin{aligned} f(\xi) &= \frac{1}{A\tan(\xi)+B\sec(\xi)}, \\ g(\xi) &= \frac{\sec(\xi)}{A\tan(\xi)+B\sec(\xi)}, \end{aligned} \tag{14}$$

which satisfy

$$\begin{aligned} f'(\xi) &= -Ag^2(\xi)-\frac{B}{A}g(\xi)(1-Bg(\xi)), \\ g'(\xi) &= -Ag(\xi)f(\xi), \\ f^2(\xi) &= g^2(\xi)-\frac{1}{A^2}(1-Bg(\xi))^2. \end{aligned} \tag{15}$$

Equations (13) and (15) can obviously be rewritten into the following unified form

$$
\begin{aligned}
f'(\xi) &= -Ag^2(\xi) + \frac{\delta B}{A} g(\xi)(1 - Bg(\xi)), \\
g'(\xi) &= -Ag(\xi)f(\xi), \\
f^2(\xi) &= g^2(\xi) + \frac{\delta}{A^2}(1 - Bg(\xi))^2. \qquad (16)
\end{aligned}
$$

where $\delta = \pm 1$.
The main steps of our method can be written as follows :
Step 1 . Determine the values of r and s by balancing the highest order partial derivative and the nonlinear term in equations (8) and (9).
Step 2. With the aid of Maple or Mathematica , substituting (10),(11) along with the condition (16) into system (8) , (9) and collecting the coefficients of the same power of $g^j(\xi)f^i(\xi)(i = 0, 1; j = 1, 2, .., m)$. Set each of the obtained coefficients to zero to get an over -determined system of nonlinear algebraic equations with respect to the unknown variables a_i, b_j, c_k, d_n , $i = 1, 2, .., r$,$j = 1, 2, ..., r, k = 1, 2, ..s,$ $n = 1, 2, .., s$
Step 3 . We solve the above algebraic equations in step 2 yields the values of a_i, b_j, c_k, d_n , $i = 1, 2, .., r,$ $j = 1, 2, ..., r, k = 1, 2, ..s,$ $n = 1, 2, .., s$.
Step 4. The solutions of the system (8) and (9) are found by substituting these values into (10) and (11) .
Remark. We can use the following forms

$$
f(\xi) = \frac{\coth(\xi)}{A + B\operatorname{csch}(\xi)}, \qquad g(\xi) = \frac{\operatorname{csch}(\xi)}{A + B\operatorname{csch}(\xi)}, \qquad (17)
$$

instead of (12) and the forms

$$
f(\xi) = \frac{\cot(\xi)}{A + B\csc(\xi)}, \qquad g(\xi) = \frac{\csc(\xi)}{A + B\csc(\xi)}, \qquad (18)
$$

instead of (14).

3 Exact solutions of nonlinear Schordinger Equation

According to the above method , using the travelling wave transformations $u(x,y,t) = U(\xi)$, $v(x,y,t) = V(\xi)$, $S(x,y,t) = S(\xi)$, where $\xi = \lambda_1 t + \lambda_2 x + \lambda_3 y$ and λ_1, λ_2 , λ_3 are arbitrary constants. We reduce the equations (3) and (4) to the following nonlinear ordinary differential equations.

$$\lambda_1 \acute{U}(\xi) + \lambda_2^2 \acute{V}(\xi) + \lambda_3^2 \acute{V}(\xi) + V(\xi)(U^2(\xi) + V^2(\xi)) - S(\xi)V(\xi) = 0, \tag{19}$$

$$\lambda_1 \acute{V}(\xi) - \lambda_2^2 \acute{U}(\xi) - \lambda_3^2 \acute{U}(\xi) - U(\xi)(U^2(\xi) + V^2(\xi)) + S(\xi)V(\xi) = 0, \tag{20}$$

To look for the explicit solutions of equations (19) and (20) we choose the following ansatz in terms of step 1 in the above section ,we obtain

$$\begin{aligned} U &= a_0 + a_1 f(\xi) + a_2 g(\xi), \\ V &= b_0 + b_1 f(\xi) + b_2 g(\xi), \\ S &= c_0 + c_1 f(\xi) + c_2 g(\xi) + c_3 g^2(\xi) + c_4 f(\xi) g(\xi), \end{aligned} \tag{21}$$

where $a_i, b_j (i = 0,1,2, j = 0,1,2)$ and $c_k (k = 0,1,..,4)$ are constants to be determined later.
On substituting (21) into (19) and (20) and collecting all terms with the same power in $g^j(\xi) f^i(\xi)(i = 0,1; j = 1,2,..,m)$ .Setting the coefficients of these terms $g^j(\xi) f^i(\xi)(i = 0,1; j = 1,2,..,m)$ to zero ,yields a set of algebraic polynomial equations namely :

$$\begin{aligned}
&2\lambda_1^2 a_2 \delta B^2 + a_2 b_2^2 - c_3 a_2 - 2\lambda_1^2 a_2 A^2 + \tfrac{3a_1^2 a_2 \delta B^2 + a_2 b_1^2 \delta B^2 2a_1 b_1 b_2 \delta B^2 - c_4 a_1 \delta B^2}{A^2} \\
&+2\lambda_3^2 a_2 A^2 + 2a_1 b_1 b_2 + 2\lambda_2^2 a_2 A^2 + 2\lambda_2^2 a_2 \delta B^2 + 2\lambda_3^2 a_2 \delta B^2 + 3a_1^2 a_2 - c_4 a_1 \\
&+a_2 b_1^2 + a_2^3 = 0, \\
&a_1 b_2^2 + 2\lambda_1^2 a_1 \delta B^2 + a_1(1 + \tfrac{\delta B^2}{A^2}) b_1^2 + 2\lambda 3^2 a_1 A^2 + 2\lambda 3^2 a_1 \delta B^2 + 2\lambda_2^2 a_1 \delta B^2 \\
&+3a_1 a_2^2 + 2\lambda_1^2 a_1 A^2 + 2a_2 b_1 b_2 + 2\lambda_2^2 a_1 A^2 - c_4 a_2 - c_3 a_1 + a_1^3(1 + \tfrac{\delta B^2}{A^2}) = 0, \\
&-3\lambda_2^2 a_2 \delta B + \tfrac{3a_1^2 a_0 \delta B^2}{A^2} + 2a_2 b_2 b_0 - 3\lambda_1^2 a_2 \delta B - c_2 a_2 + 3a_2^2 a_0 + a_0 b_1^2 \\
&-\tfrac{4a_1 b_1 b_2 \delta B}{A^2} - \tfrac{c_1 a_1 \delta B^2}{A^2} + \tfrac{2c_4 a_1 \delta B}{A^2} + a_0 b_2^2 + 3a_1^2 a_0 - 3\lambda 3^2 a_2 \delta B - c_3 a_0
\end{aligned}$$

$$
\begin{aligned}
&+\tfrac{\lambda_1 b_1 \delta B^2}{A}+\tfrac{a_0 b_1^2 \delta B^2}{A^2}+2a_1b_1b_0+\tfrac{2a_1b_1b_0\delta B^2}{A^2}-\tfrac{2a_2b_1^2\delta B}{A^2}-\tfrac{6a_1^2a_2\delta B}{A^2}\\
&-c_1a_1+\lambda_1b_1A=0,\\
&\tfrac{2a_1b_1b_0\delta B^2}{A^2}-\tfrac{6a_1^2a_2\delta B}{A^2}-3\lambda_2^2a_2\delta B+\tfrac{3a_1^2a_0\delta B^2}{A^2}+2a_2b_2b_0-3\lambda_1^2a_2\delta B-c_2a_2\\
&+3a_2^2a_0+\tfrac{2c_4a_1\delta B}{A^2}-\tfrac{2a_2b_1^2\delta B}{A^2}+\lambda_1b_1A+a_0b_1^2-\tfrac{c_1a_1\delta B^2}{A^2}+a_0b_2^2+3a_1^2a_0\\
&-3\lambda_3^2a_2\delta B-c_1a_1\ -c_3a_0+\tfrac{\lambda_1b_1\delta B^2}{A}+\tfrac{a_0b_1^2\delta^2B^2}{A}+2a_1b_1b_0\\
&-\tfrac{4a_1b_1b_2\delta B}{A^2}=0,\\
&-\lambda_2^2a_1\delta B-\tfrac{2a_1\delta Bb_1^2}{A^2}-\lambda_1^2a_1\delta B+2a_2b_1b_0-c_2a_1+\lambda_1b_2A+2a_0b_1b_2+\\
&-c_1a_2\ -c_4a_0\ \ +6a_1a_2a_0-\tfrac{2a_1^3\delta B}{A^2}-\lambda 3^2a_1\delta B+2a_1b_2b_0=0,\\
&a_2b_0^2-\tfrac{\lambda_1b_1\delta B}{A}-\tfrac{2a_0b_1^2\delta B}{A^2}+\tfrac{2a_1b_1b_2\delta}{A^2}-\tfrac{6a_1^2a_0\delta B}{A^2}+3a_2a_0^2-\tfrac{c_4a_1\delta}{A^2}+\lambda_2^2a_2\delta\\
&-c_2a_0+\lambda_1^2a_2\delta+\tfrac{3a_1^2a_2\delta}{A^2}+\tfrac{a_2b_1^2\delta}{A^2}-\tfrac{4a_1b_1b_0\delta B}{A^2}-c_0a_2+2a_0b_2b_0+\tfrac{2c_1a_1\delta B}{A^2}\\
&+\lambda_3^2a_2\delta=0,\\
&\tfrac{a_1^3\delta}{A^2}-c_1a_0+3a_1a_0^2-c_0a_1+a_1b_0^2+2a_0b_1b_0+\tfrac{a_1\delta b_1^2}{A^2}=0,\\
&-\tfrac{c_1a_1\delta}{A^2}-c_0a_0+\tfrac{a_0b_1^2\delta}{A^2}+a_0b_0^2+\tfrac{3a_1^2a_0\delta}{A^2}+a_0^3+\tfrac{2a_1b_1b_0\delta}{A^2}=0,\\
&2b_0a_1a_0-c_1b_0+\tfrac{b_1^3\delta}{A^2}+b_1a_0^2-c_0b_1+\tfrac{b_1\delta a_1^2}{A^2}+3b_1b_0^2=0,\\
&\tfrac{-c_1b_1\delta}{A^2}+\tfrac{b_0a_1^2\delta}{A^2}+\tfrac{3b_1^2b_0\delta}{A^2}-c_0b_0+\tfrac{2b_1a_1a_0\delta}{A^2}+b_0a_0^2+b_0^3=0,\\
&3b_1b_2^2+b_1^3(1+\tfrac{\delta B^2}{A^2})+2\lambda_1^2b_1\delta B^2+2\lambda_2^2b_1\delta B^2-c_3b_1+b_1a_2^2-c_4b_2\lambda_3^2\\
&+b_1\delta B^2+2\lambda 3^2b_1A^2+2b_2a_1a_2+b_1(1+\tfrac{\delta B^2}{A^2})a_1^2+2\lambda_1^2b_1A^2\\
&+2\lambda_2^2b_1A^2=0,\\
&b_2^3+2\lambda_2^2b_2A^2-c_4b_1+2\lambda_1^2b_2A^2-\tfrac{c_4b_1\delta B^2}{A^2}+2\lambda_1^2b_2\delta B^2+2\lambda 3^2b_2A^2\\
&+\tfrac{3b_1^2b_2\delta B^2}{A^2}\ -c_3b_2\ +2\lambda_2^2b_2\delta B^2+\tfrac{2b_1a_1a_2\delta B^2}{A^2}+2\lambda 3^2b_2\delta B^2+b_2a_2^2+\tfrac{b_2a_1^2\delta B^2}{A^2}\\
&+2b_1a_1a_2+b_2a_1^2+3b_1^2b_2=0,\\
&-c_4b_0-c_2b_1-\tfrac{2b_1\delta Ba_1^2}{A^2}+2b_1a_2a_0-\tfrac{2b_1^3\delta B}{A^2}-\lambda_1^2b_1\delta B-\lambda_2^2b_1\delta B-c_1b_2\\
&+2b_2a_1a_0+6b_1b_2b_0-\lambda_1a_2A-\lambda 3^2b_1\delta B+2b_0a_1a_2=0,\\
&-\tfrac{\lambda_1a_1\delta B^2}{A}-\tfrac{2b_2a_1^2\delta B}{A^2}-3\lambda_1^2b_2\delta B+2b_2a_2a_0-\lambda_1a_1A-3\lambda_2^2b_2\delta B+\tfrac{2c_4b_1\delta B}{A^2}\\
&-\tfrac{4b_1a_1a_2\delta B}{A^2}+3b_2^2b_0-c_2b_2-c_1b_1-\tfrac{c_1b_1\delta B^2}{A^2}+b_0a_2^2+\tfrac{3b_1^2b_0\delta B^2}{A^2}-c_3b_0\\
&+2b_1a_1a_0-\tfrac{6b_1^2b_2\delta B}{A^2}+3b_1^2b_0+\tfrac{2b_1a_1a_0\delta B^2}{A^2}+b_0a_1^2-3\lambda 3^2b_2\delta B\\
&+\tfrac{b_0a_1^2\delta B^2}{A^2}=0,\\
&\lambda_2^2b_2\delta-\tfrac{6b_1^2b_0\delta B}{A^2}+\tfrac{2b_1a_1a_2\delta}{A^2}-\tfrac{4b_1a_1a_0\delta B}{A^2}+b_2a_0^2+\tfrac{\lambda_1a_1\delta B}{A}+2b_0a_2a_0+\tfrac{3b_1^2b_2\delta}{A^2}\\
&-c_2b_0-\tfrac{c_4b_1\delta}{A^2}+\lambda_1^2b_2\delta+\lambda_3^2b_2\delta-\tfrac{2b_0a_1^2\delta B}{A^2}+\tfrac{b_2a_1^2\delta}{A^2}+\tfrac{2c_1b_1\delta B}{A^2}+3b_2b_0^2\\
&-c_0b_2=0,
\end{aligned}
\tag{22}
$$

With the aid of the Maple ,we can find a set of solutions of the algebraic

equations (22) and we obtain :
(I) When $\delta = 1$ we have the following cases:

Case 1

$$\begin{aligned} a_2 &= \frac{\lambda_1\sqrt{c_0}A}{\sqrt{D^2-\lambda_1^2}}, \qquad b_1 = A\sqrt{c_0}, \\ c_2 &= -\frac{A[2c_0D+D^2+\lambda_1^2]}{\sqrt{D^2-\lambda_1^2}}, \quad B = \frac{AD}{\sqrt{D^2-\lambda_1^2}}, \\ c_3 &= \frac{2A^2D[c_0D+2D^2-\lambda_1^2]}{D^2-\lambda_1^2}, \\ c_1 &= c_4 = a_0 = a_1 = b_2 = b_0 = 0, \end{aligned} \tag{23}$$

where $D = \lambda_1^2+\lambda_2^2+\lambda_3^2$ and $\lambda_1, \lambda_2, \lambda_3, A$ and c_0 are arbitrary constants . In this case the wave soliton solutions take the form

$$u_1 = \frac{\lambda_1\sqrt{c_0}\operatorname{sech}(\lambda_1t+\lambda_2x+\lambda_3y)}{[\sqrt{D^2-\lambda_1^2}\tanh(\lambda_1t+\lambda_2x+\lambda_3y)+D\operatorname{sech}(\lambda_1t+\lambda_2x+\lambda_3y)]},$$

$$v_1 = \frac{\sqrt{c_0D^2-c_0\lambda_1^2}}{[\sqrt{D^2-\lambda_1^2}\tanh(\lambda_1t+\lambda_2x+\lambda_3y)+D\operatorname{sech}(\lambda_1t+\lambda_2x+\lambda_3y)]},$$

$$\begin{aligned} S_1 &= -\frac{[2c_0D+D^2+\lambda_1^2]\operatorname{sech}(\lambda_1t+\lambda_2x+\lambda_3y)}{[\sqrt{D^2-\lambda_1^2}\tanh(\lambda_1t+\lambda_2x+\lambda_3y)+D\operatorname{sech}(\lambda_1t+\lambda_2x+\lambda_3y)]} \\ &\quad +\frac{2D[c_0D+2D^2-\lambda_1^2]\operatorname{sech}^2(\lambda_1t+\lambda_2x+\lambda_3y)}{[\sqrt{D^2-\lambda_1^2}\tanh(\lambda_1t+\lambda_2x+\lambda_3y)+D\operatorname{sech}(\lambda_1t+\lambda_2x+\lambda_3y)]^2} \\ &\quad +c_0, \end{aligned} \tag{24}$$

Case 2

$$a_1 = \frac{-2b_0AD}{\lambda_1}, \qquad b_1 = \frac{2a_0AD}{\lambda_1},$$

$$\begin{aligned} c_0 &= \frac{A(a_0^2+b_0^2)(\lambda_1^2+4D)}{\lambda_1^2}, \quad c_3 = \frac{2A^2D[2(a_0^2+b_0^2)D+\lambda_1^2]}{\lambda_1^2}, \\ c_1 &= c_4 = c_2 = a_2 = b_2 = B = 0, \end{aligned} \tag{25}$$

where $D = \lambda_1^2 + \lambda_2^2 + \lambda_3^2, \lambda_1, \lambda_2, \lambda_3, A, b_0$ and a_0 are arbitrary constants .The wave soliton solutions take the form

$$\begin{aligned} u_2 &= a_0 - \frac{2b_0 D}{\lambda_1}\coth(\lambda_1 t + \lambda_2 x + \lambda_3 y), \\ v_2 &= b_0 + \frac{2a_0 D}{\lambda_1}\text{csch}(\lambda_1 t + \lambda_2 x + \lambda_3 y) \\ S_2 &= \frac{A(a_0^2 + b_0^2)(\lambda_1^2 + 4D)}{\lambda_1^2} \\ &\quad + \frac{2D[2(a_0^2 + b_0^2)D + \lambda_1^2]}{\lambda_1^2}\text{csch}^2(\lambda_1 t + \lambda_2 x + \lambda_3 y) \end{aligned} \tag{26}$$

Case 3

$$\begin{aligned} b_2 &= -\frac{\lambda_1\sqrt{c_0}A}{\sqrt{D^2 - \lambda_1^2}}, \qquad a_1 = A\sqrt{c_0}, \\ c_2 &= \frac{-A[2c_0 D + D^2 + \lambda_1^2]}{\sqrt{D^2 - \lambda_1^2}}, \qquad B = \frac{AD}{\sqrt{D^2 - \lambda_1^2}}, \\ c_3 &= \frac{2A^2 D[c_0 D + 2D^2 - \lambda_1^2]}{D^2 - \lambda_1^2}, \\ c_1 &= c_4 = a_0 = a_2 = b_1 = b_0 = 0, \end{aligned} \tag{27}$$

where $D = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$, $\lambda_1, \lambda_2, \lambda_3, A$ and c_0 are arbitrary constants .The wave soliton solutions take the form

$$u_3 = \frac{\sqrt{c_0(D^2 - \lambda_1^2)}}{\sqrt{D^2 - \lambda_1^2}\tanh(\lambda_1 t + \lambda_2 x + \lambda_3 y) + D\text{sech}(\lambda_1 t + \lambda_2 x + \lambda_3 y)},$$

$$\begin{aligned} v_3 &= -\frac{\lambda_1\sqrt{c_0}\text{sech}(\lambda_1 t + \lambda_2 x + \lambda_3 y)}{\sqrt{D^2 - \lambda_1^2}\tanh(\lambda_1 t + \lambda_2 x + \lambda_3 y) + D\text{sech}(\lambda_1 t + \lambda_2 x + \lambda_3 y)}, \\ S_3 &= -\frac{[2c_0 D + D^2 + \lambda_1^2]\text{sech}(\lambda_1 t + \lambda_2 x + \lambda_3 y)}{\sqrt{D^2 - \lambda_1^2}\tanh(\lambda_1 t + \lambda_2 x + \lambda_3 y) + D\text{sech}(\lambda_1 t + \lambda_2 x + \lambda_3 y)} \\ &\quad + \frac{2D[c_0 D + 2D^2 - \lambda_1^2]\text{sech}^2(\lambda_1 t + \lambda_2 x + \lambda_3 y)}{[\sqrt{D^2 - \lambda_1^2}\tanh(\lambda_1 t + \lambda_2 x + \lambda_3 y) + D\text{sech}(\lambda_1 t + \lambda_2 x + \lambda_3 y)]^2} \\ &\quad + c_0. \end{aligned} \tag{28}$$

.where $D = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$

Case 4

$$\begin{aligned}
a_2 &= \frac{b_1\lambda_1}{\sqrt{D_1}}\,, \qquad\qquad c_0 = \frac{b_1^2+a_1^2}{A^2},\\
c_3 &= \frac{2}{D_1}\{D^2b_1^2+2A^2D^3+a_1^2D-A^2\lambda_1^4-A^2(\lambda_3^2\lambda_1^2+\lambda_1^2\lambda_2^2)\}\\
c_2 &= \frac{-1}{\sqrt{D_1}A}\{A^2D^2+2(b_1^2+a_1^2)D+A^2\lambda_1^2\}\\
B &= \frac{A\lambda_1 D}{\sqrt{D_1}}, \qquad\qquad b_2 = \frac{-a_1\lambda_1}{\sqrt{D_1}}\,,\\
a_0 &= b_0 = c_1 = c_4 = 0,
\end{aligned} \tag{29}$$

where $D_1 = D^2 - \lambda_1^2$ and $A, \lambda_1, \lambda_2, \lambda_3$ b_1, a_1 are arbitrary constants . Thus , the soliton wave solutions take the forms:

$$\begin{aligned}
u_4 &= \frac{a_1\sqrt{D_1}}{A[\sqrt{D_1}\tanh(\lambda_1 t+\lambda_2 x+\lambda_3 y)+\lambda_1 D\mathrm{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)]}+\\
&\quad \frac{b_1\lambda_1\mathrm{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)}{A[\sqrt{D_1}\tanh(\lambda_1 t+\lambda_2 x+\lambda_3 y)+\lambda_1 D\mathrm{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)]},\\
v_4 &= \frac{b_1\sqrt{D_1}}{A[\sqrt{D_1}\tanh(\lambda_1 t+\lambda_2 x+\lambda_3 y)+\lambda_1 D\mathrm{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)]}+\\
&\quad \frac{-a_1\lambda_1\mathrm{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)}{A[\sqrt{D_1}\tanh(\lambda_1 t+\lambda_2 x+\lambda_3 y)+\lambda_1 D\mathrm{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)]},\\
S_4 &= -\frac{\{A^2D^2+2(b_1^2+a_1^2)D+A^2\lambda_1^2\}\mathrm{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)}{A^2[\sqrt{D_1}\tanh(\lambda_1 t+\lambda_2 x+\lambda_3 y)+\lambda_1 D\mathrm{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)]}\\
&\quad +2\{D^2b_1^2+2A^2D^3+a_1^2D-A^2\lambda_1^4-A^2(\lambda_3^2\lambda_1^2+\lambda_1^2\lambda_2^2)\}\times\\
&\quad \frac{\mathrm{sech}^2(\lambda_1 t+\lambda_2 x+\lambda_3 y)}{A^2[\sqrt{D_1}\tanh(\lambda_1 t+\lambda_2 x+\lambda_3 y)+\lambda_1 D\mathrm{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)]^2}\\
&\quad +\frac{b_1^2+a_1^2}{A^2}.
\end{aligned} \tag{30}$$

Case 5

$$\begin{aligned}
a_2 &= \frac{b_1\lambda_1}{\sqrt{D_1}}, \qquad\qquad b_2 = \frac{-ib_1\lambda_1}{\sqrt{D_1}},\\
c_2 &= \frac{-A}{\sqrt{D_1}}\left\{D^2+\lambda_1^2\right\}, \qquad a_1 = ib_1\,,\\
B &= \frac{AD}{\sqrt{D_1}}, \qquad c_3 = \frac{2AD}{D_1}\left\{2D^2-\lambda_1^2\right\}\\
c_1 &= c_4 = b_0 = a_0 = c_0 = 0
\end{aligned} \tag{31}$$

where $b_1, \lambda_1, \lambda_2, \lambda_3$ are arbitrary constants. Consequently the solitary wave solution in this case take the forms:

$$u_5 = \frac{ib_1}{A[\tanh(\lambda_1 t + \lambda_2 x + \lambda_3 y) + \frac{D}{\sqrt{D_1}}\text{sech}(\lambda_1 t + \lambda_2 x + \lambda_3 y)]} + \frac{b_1 \lambda_1 \text{sech}(\lambda_1 t + \lambda_2 x + \lambda_3 y)}{A\sqrt{D_1}[\tanh(\lambda_1 t + \lambda_2 x + \lambda_3 y) + \frac{D}{\sqrt{D_1}}\text{sech}(\lambda_1 t + \lambda_2 x + \lambda_3 y)]}$$

$$v_5 = \frac{b_1}{A[\tanh(\lambda_1 t + \lambda_2 x + \lambda_3 y) + \frac{D}{\sqrt{D_1}}\text{sech}(\lambda_1 t + \lambda_2 x + \lambda_3 y)]} - \frac{ib_1 \lambda_1 \text{sech}(\lambda_1 t + \lambda_2 x + \lambda_3 y)}{A\sqrt{D_1}[\tanh(\lambda_1 t + \lambda_2 x + \lambda_3 y) + \frac{D}{\sqrt{D_1}}\text{sech}(\lambda_1 t + \lambda_2 x + \lambda_3 y)]}$$

$$S_5 = \frac{-\left\{D^2 + \lambda_1^2\right\} \text{sech}(\lambda_1 t + \lambda_2 x + \lambda_3 y)}{\sqrt{D_1}[\tanh(\lambda_1 t + \lambda_2 x + \lambda_3 y) + \frac{D}{\sqrt{D_1}}\text{sech}(\lambda_1 t + \lambda_2 x + \lambda_3 y)]} - \frac{2D\left\{2D^2 - \lambda_1^2\right\} \text{sech}^2(\lambda_1 t + \lambda_2 x + \lambda_3 y)}{D_1[\tanh(\lambda_1 t + \lambda_2 x + \lambda_3 y) + \frac{D}{\sqrt{D_1}}\text{sech}(\lambda_1 t + \lambda_2 x + \lambda_3 y)]^2} \tag{32}$$

Case 6

$$\begin{aligned} b_1 &= A\sqrt{c_0}\,, \qquad c_3 = -2c_0A^2, \\ c_2 &= \frac{-iA\left\{3\lambda_1^2 + 4c_0D_2^2 + 4\lambda_1^2 c_0 + 4c_0\lambda_3^2\right\}}{2(D_2^2 + \lambda_1^2 + \lambda_2^2)}, \\ a_2 &= \frac{2iA\sqrt{c_0}(D_2^2 + \lambda_1^2 + \lambda_2^2)}{\lambda_1}, \qquad \lambda_2 = D_2\,, \\ B &= iA\;\;, \\ a_0 &= a_1 = b_2 = b_0 = c_1 = c_4 = 0, \end{aligned} \tag{33}$$

where D_2 is the root of the equation $2Z^4 + 4(\lambda_1^2 + \lambda_3^2)Z^2 + 4\lambda_1^2\lambda_3^2 + 2\lambda_3^4 - \lambda_1^2 + 2\lambda_1^4 = 0$ and $A, c_0, \lambda_1, \lambda_2, \lambda_3$ are arbitrary constants .The soliton wave solutions take the forms:

$$
\begin{aligned}
u_6 &= \frac{2i\sqrt{c_0}(D_2^2+\lambda_1^2+\lambda_2^2)\text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)}{\lambda_1[\tanh(\lambda_1 t+\lambda_2 x+\lambda_3 y)+i\text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)]},\\
v_6 &= \frac{\sqrt{c_0}}{\tanh(\lambda_1 t+\lambda_2 x+\lambda_3 y)+i\text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)}\\
S_6 &= c_0-\frac{i\left\{3\lambda_1^2+4c_0D_2^2+4\lambda_1^2c_0+4c_0\lambda_3^2\right\}\text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)}{L[\tanh(\lambda_1 t+\lambda_2 x+\lambda_3 y)+i\text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)]}\\
&\quad \frac{-2c_0A\text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)}{\tanh(\lambda_1 t+\lambda_2 x+\lambda_3 y)+i\text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)}. \qquad (34)
\end{aligned}
$$

where $L = 2(D_2^2 + \lambda_1^2 + \lambda_2^2)$

Case 7

$$
\begin{aligned}
a_2 &= ib_2\,, & c_2 &= -3BD,\\
c_3 &= 2D(A^2+B^2), & c_1 &= -A\lambda_1 i,\\
c_0 &= D\,,\\
a_0 &= a_1=b_1=b_0=c_4=0, && \qquad (35)
\end{aligned}
$$

where $A, B, \lambda_1, \lambda_2, \lambda_3$ and b_2 are arbitrary constants .The soliton wave solutions take the forms:

$$
\begin{aligned}
u_7 &= \frac{ib_2\text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)}{A\tanh(\lambda_1 t+\lambda_2 x+\lambda_3 y)+B\text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)]},\\
v_7 &= \frac{b_2\text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)}{A\tanh(\lambda_1 t+\lambda_2 x+\lambda_3 y)+B\text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)},\\
S_7 &= D-\frac{i\lambda_1 A}{A\tanh(\lambda_1 t+\lambda_2 x+\lambda_3 y)+B\text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)]}\\
&\quad -\frac{3BD\text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)}{A\tanh(\lambda_1 t+\lambda_2 x+\lambda_3 y)+B\,\text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)}\\
&\quad +\frac{2D(A^2+B^2)\text{sech}^2(\lambda_1 t+\lambda_2 x+\lambda_3 y)}{[A\tanh(\lambda_1 t+\lambda_2 x+\lambda_3 y)+B\,\text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)]^2}.
\end{aligned}
\qquad (36)
$$

Case 8

$$\begin{aligned}
a_1 &= \frac{ib_2\lambda_1}{\sqrt{D_3}}\ , \qquad\qquad b_0 = \frac{ib_2\lambda_1 D}{\sqrt{D_3}A}, \\
c_3 &= \frac{2D}{D_3}\{Db_2^2 + A^2(2D^2+\lambda_1^2)\} \\
c_2 &= \frac{-i}{\sqrt{D_3}A}\{-2Db_2^2 + A^2[\lambda_1^2 + 2\lambda_2^4 + 2\lambda_1^4 + 2\lambda_3^4 + 4\lambda_1^2\lambda_2^2 + 4\lambda_3^2\lambda_2^2]\} \\
c_0 &= -\frac{b_2^2 M}{D_3A^2}, \\
B &= a_0 = a_2 = b_1 = c_1 = c_4 = 0,
\end{aligned} \tag{37}$$

where $M = (\lambda_1^2+\lambda_2^2+\lambda_3^2)^2+\lambda_1^2,\ D_3 = 2(\lambda_1^2+\lambda_2^2+\lambda_3^2)^2+\lambda_1^2$ and $A, \lambda_1, \lambda_2, \lambda_3$ b_2 are arbitrary constants .The soliton wave solutions take the forms:

$$\begin{aligned}
u_8 &= \frac{ib_2\lambda_1}{A\sqrt{D_3}}\coth(\lambda_1 t + \lambda_2 x + \lambda_3 y), \\
v_8 &= \frac{ib_2\lambda_1 D}{\sqrt{D_3}A} + \frac{b_2}{A}\operatorname{csch}(\lambda_1 t + \lambda_2 x + \lambda_3 y), \\
S_8 &= -\frac{b_2^2 M}{D_3A^2} - \frac{i}{\sqrt{D_3}A^2}\{-2Db_2^2 + A^2[\lambda_1^2 + 2\lambda_2^4 + 2\lambda_1^4 + 2\lambda_3^4 \\
&\quad +4\lambda_1^2\lambda_2^2 + 4\lambda_3^2\lambda_2^2]\}\operatorname{csch}(\lambda_1 t + \lambda_2 x + \lambda_3 y) + \frac{2}{D_3A^2}\{D^2b_2^2 \\
&\quad +A^2[2D^3 + \lambda_1^4 + \lambda_1^2\lambda_3^2 + \lambda_1^2\lambda_2^2]\}\operatorname{csch}^2(\lambda_1 t + \lambda_2 x + \lambda_3 y).
\end{aligned} \tag{38}$$

Case 9

$$\begin{aligned}
b_1 &= -Aa_0\ , \qquad\qquad a_2 = a_0(A+iB), \\
c_3 &= \frac{2(a_0^2 - 3\lambda_1 Ai + \lambda_1 B)}{a_0}, \\
c_2 &= -i\{2a_0^2 + \lambda_1 Ai - 2\lambda_1 B\}\ , \\
\lambda_2 &= \sqrt[4]{(\lambda_1^2+\lambda_3^2)^2+\lambda_1^2}\left\{\cos\left[\frac{1}{2}\tan^{-1}\frac{\lambda_1}{(\lambda_1^2+\lambda_3^2)}\right],\right. \\
&\quad \left. +i\sin\left[\frac{1}{2}\tan^{-1}\frac{\lambda_1}{(\lambda_1^2+\lambda_3^2)}\right]\right\} \\
b_2 &= b_0 = a_1 = c_0 = c_1 = c_4 = 0,
\end{aligned} \tag{39}$$

where B,A, λ_1, λ_3 , a_0 are arbitrary constants. The soliton wave solutions take the forms:

$$\begin{aligned}
u_9 &= \frac{a_0(A+iB)\text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)}{A\tanh(\lambda_1 t+\lambda_2 x+\lambda_3 y)+B\text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)]},\\
v_9 &= \frac{-Aa_0}{A\tanh(\lambda_1 t+\lambda_2 x+\lambda_3 y)+B\text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)},\\
S_9 &= -\frac{i\{2a_0^2+\lambda_1 Ai-2\lambda_1 B\}\ \text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)}{[A\tanh(\lambda_1 t+\lambda_2 x+\lambda_3 y)+B\ \text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)]}\\
&\quad+\frac{2(a_0^2-3\lambda_1 Ai+\lambda_1 B)\text{sech}^2(\lambda_1 t+\lambda_2 x+\lambda_3 y)}{a_0[A\tanh(\lambda_1 t+\lambda_2 x+\lambda_3 y)+B\ \text{sech}(\lambda_1 t+\lambda_2 x+\lambda_3 y)]^2}\ .
\end{aligned} \tag{40}$$

where λ_2 is a constant given by the formula

$$\begin{aligned}
\lambda_2 &= \sqrt[4]{(\lambda_1^2+\lambda_3^2)^2+\lambda_1^2}\left\{\cos\left[\frac{1}{2}\tan^{-1}\frac{\lambda_1}{(\lambda_1^2+\lambda_3^2)}\right],\right.\\
&\quad\left.+i\sin\left[\frac{1}{2}\tan^{-1}\frac{\lambda_1}{(\lambda_1^2+\lambda_3^2)}\right]\right\}.
\end{aligned}$$

(II) Second , when $\delta=-1$,we have the following cases:

Case 10

$$\begin{aligned}
c_0 &= -\frac{b_1^2+a_1^2}{A^2},\\
c_2 &= \frac{B}{A^2}\left\{2(b_1^2+a_1^2)+A^2(\lambda_2^2+\lambda_3^2)\right\},\\
c_3 &= \frac{1}{A^2}\left\{(b_1^2+a_1^2)(A^2-B^2)+2(\lambda_2^2+\lambda_3^2)(A^4-A^2B^2)\right\}\\
c_1 &= c_4=b_0=a_2=b_2=a_0=\lambda_1=0
\end{aligned} \tag{41}$$

where $A, B, a_1, b_1, \lambda_2$ and λ_3 are arbitrary constants. Consequently the soliton solutions take the form:

$$\begin{aligned}
u_{10} &= \frac{a_1}{A\tan(\lambda_2 x+\lambda_3 y)+B\sec(\lambda_2 x+\lambda_3 y)},\\
v_{10} &= \frac{b_1}{A\tan(\lambda_2 x+\lambda_3 y)+B\sec(\lambda_2 x+\lambda_3 y)},\\
S_{10} &= \frac{B\left\{2(b_1^2+a_1^2)+A^2(\lambda_2^2+\lambda_3^2)\right\}\sec(\lambda_2 x+\lambda_3 y)}{A^2[A\tan(\lambda_2 x+\lambda_3 y)+B\sec(\lambda_2 x+\lambda_3 y)]}\\
&\quad+\frac{\left\{(b_1^2+a_1^2)(A^2-B^2)+2(\lambda_2^2+\lambda_3^2)(A^4-A^2B^2)\right\}\sec^2(\lambda_2 x+\lambda_3 y)}{A^2[A\tan(\lambda_2 x+\lambda_3 y)+B\sec(\lambda_2 x+\lambda_3 y)]^2}.
\end{aligned} \tag{42}$$

Case 11

$$\begin{aligned} a_2 &= \pm\frac{2(D_4^2+\lambda_1^2+\lambda_2^2)^2}{A^2}, \qquad \lambda_2 = D_4, \\ c_0 &= \frac{c_3}{2A^2}, \qquad B = \pm A \\ c_2 &= \frac{1}{A}\left\{3D_4^2A^2 + 3(\lambda_1^2+\lambda_3^2)A^2 - c_3)\right\},\ b_1 = \sqrt{\frac{-c_3}{2}} \\ c_1 &= c_4 = a_0 = a_1 = b_2 = b_0 = \lambda_1 = 0 \end{aligned} \tag{43}$$

where D_4 is the root of the equation $2Z^4 + 4(\lambda_1^2+\lambda_3^2)Z^2 + (\lambda_1^2 + 4\lambda_3^2\lambda_1^2 + 2\lambda_3^4 + 2\lambda_1^4) = 0, A,\ c_3, \lambda_1$ and λ_3 are arbitrary constants. Consequently , the soliton solutions take the form:

$$\begin{aligned} u_{11} &= \frac{\pm 2(D_4^2+\lambda_1^2+\lambda_2^2)^2 \sec(\lambda_1 t + D_4 x + \lambda_3 y)}{A^3[\tan(\lambda_1 t + D_4 x + \lambda_3 y) \pm \sec(\lambda_1 t + D_4 x + \lambda_3 y)]}, \\ v_{11} &= \frac{\sqrt{-c_3}}{\sqrt{2}A[\tan(\lambda_1 t + D_4 x + \lambda_3 y) \pm \sec(\lambda_1 t + D_4 x + \lambda_3 y)]}, \\ S_{11} &= \frac{c_3}{2A^2} + \frac{\{3D_4^2A^2 + 3(\lambda_1^2+\lambda_3^2)A^2 - c_3)\}\sec(\lambda_1 t + D_4 x + \lambda_3 y)}{A^2[\tan(\lambda_1 t + D_4 x + \lambda_3 y) \pm \sec(\lambda_1 t + D_4 x + \lambda_3 y)]} \\ &\quad + \frac{c_3 \sec^2(\lambda_1 t + D_4 x + \lambda_3 y)}{A[\tan(\lambda_1 t + D_4 x + \lambda_3 y) \pm \sec(\lambda_1 t + D_4 x + \lambda_3 y)]^2}, \end{aligned} \tag{44}$$

Case 12

$$a_1 = -\frac{\lambda_1 b_0 A}{D}, \qquad b_2 = -\frac{b_0 A\sqrt{D_5}}{D},$$

$$\begin{aligned} c_2 &= \frac{A[-\lambda_1^2 D + 2D^3 - 2b_0^2 D_5]}{D\sqrt{D_5}}, \quad c_0 = \frac{b_0^2\{D^2-\lambda_1^2)\}}{D^2}, \\ c_3 &= \frac{2}{D_5}[2b_0^2A^2D_5 + 2A^2D^3 - A^2(\lambda_1^4 + \lambda_1^2\lambda_3^2 + \lambda_1^2\lambda_2^2)] \\ c_1 &= c_4 = a_0 = a_2 = b_1 = B = 0 \end{aligned} \tag{45}$$

where $D_5 = 2D^2 - \lambda_1^2$ and $A,\ b_0, \lambda_1$,λ_2 and λ_3 are arbitrary constants . Consequently , the soliton solutions take the form:

$$\begin{aligned} u_{12} &= -\frac{\lambda_1 b_0}{D}\cot(\lambda_1 t+\lambda_2 x+\lambda_3 y),\\ v_{12} &= -\frac{b_0 A\sqrt{D_5}}{D}\csc(\lambda_1 t+\lambda_2 x+\lambda_3 y),\\ S_{12} &= \frac{b_0^2\{D^2-\lambda_1^2)\}}{D^2}+\frac{2}{D_5}[2b_0^2D_5+2D^3-\lambda_1^2 D]\csc^2(\lambda_1 t+\lambda_2 x+\lambda_3 y)\\ &\quad \frac{[-\lambda_1^2 D+2D^3-2b_0^2 D_5]}{D\sqrt{D_5}}\csc(\lambda_1 t+\lambda_2 x+\lambda_3 y). \end{aligned} \tag{46}$$

Case 13

$$\begin{aligned} a_1 &= -\,b_0 A, \qquad b_2 = -b_0 B,\\ c_2 &= \frac{3}{2}B\lambda_1, \qquad \lambda_2=\sqrt{\frac{\lambda_1}{2}-(\lambda_1^2+\lambda_3^2)},\\ c_3 &= b_0^2A^2+\lambda_1(A^2-B^2)\\ c_0 &= c_1=c_4=a_0=a_2=b_1=0 \end{aligned} \tag{47}$$

where $\lambda_1 < \frac{1}{4}+\sqrt{\frac{1}{16}-\lambda_3^2}$, $0<\lambda_3<\frac{1}{4}$ and A, B, b_0 are arbitrary constants . Consequently the soliton solutions take the form:

$$\begin{aligned} u_{13} &= -\frac{b_0 A}{A\tan(\xi)+B\sec(\xi)},\\ v_{13} &= -\frac{b_0 B\sec(\xi)}{A\tan(\xi)+B\sec(\xi)},\\ S_{13} &= \frac{3B\lambda_1\sec(\xi)}{2A\tan(\xi)+2B\sec(\xi)}\\ &\quad +\frac{[\,b_0^2A^2+\lambda_1(A^2-B^2)]\sec^2(\xi)}{[A\tan(\xi)+B\sec(\xi)]^2}. \end{aligned} \tag{48}$$

where $\xi=\lambda_1 t+\sqrt{\frac{\lambda_1}{2}-(\lambda_1^2+\lambda_3^2)}x+\lambda_3 y$.

References

[1] Chenglin Bai,”Exact Solutions For Nonlinear Partial Dierential Equation :A New Approach”Phys.Letters A , 288 ,191-195 (2002).

[2] Dong Bo Cao ,"New Exact Solutions For A Class Of Nonlinear Coupled Dierential Equations"Phys.Letters A , 296 , 27-33 (2002).

[3] Engui Fan, "Travelling Wave Solutions in terms Of Special Functions For Nonlinear Coupled Evolution systems" Physics Letter A,300,243-249(2002).

[4] Zuntao Fu, Skikuo Liu and Shida Liu," New Transformations And Exact Approach To Find Exact Solutions To Nonlinear Equations" Phys. Letters A, 299(2002) 507-512 .

[5] Y.C.Hon and E.G.Fan"A Series Of New Exact Solutions For A Complex KdV System" Chaos,Solitons and Fractals, 19 (2004) 515-525.

[6] C.Huai-Tang and Zh.Hong-Qing"New double periodic and multiple soliton solutions of the generalized (2+1)-dimensional Boussinesq equation" Chaos,Solitons and Fractals, 19 (2004) 515-525 .

[7] S.K.Khuri"A Complex Tanh-Function Method Applied To Nonlinear Equation Of Schrodinger Type" Chaos,Solitons and Fractals, 20 (2004) 1037-1040.

[8] S.Liu,Z.Fu and Sh.Liu,""Periodic solutions for a class of coupled nonlinear partial differential equations" J.Phys.Letter A 336 (2005) 175-179.

[9] L.Tian and X.Song,""New Peaked Solitary Wave Solutions Of The Generalized Camassa Holm Equation"Solitons and Fractals,19(2004) 621-637.

[10] Mingling Wang,"Solitary Wave Solutions For Variant Boussinesq Equations" Phys.Letters A , 199 (1995) 169-172.

[11] F.Xie,""New TravellingWave Solutions Of The Generalized Coupled Hirota -Satsuma KdV System" Chaos Solitons and Fractals,20 (2004). 1005-1012.

[12] Zhenya Yan,"Jacobi elliptic function solutions of nonlinear wave equations via the new Sinh-Gordan equation Expansion Method" J.Phys.A:Math.Gen. ,35 (2002) 9923-9930.

[13] E.M.E Zayed and Hassan A.Zedan,""On The Solutions Of The Nonlinear Schrodinger Equation " Chaos,Solitons and Fractals,16 (2003) 133-145 .

[14] E.M.E.Zayed ,Hassan A.Zedan and Khaled A. Gepreel,"On The Solitary Wave Solutions For Nonlinear Hirota -Satsuma Coupled KdV Of Equations"Chaos,Solitons and Fractals, 22 (2004) 285-303.

[15] E.M.E.Zayed ,Hassan A.Zedan and Khaled A. Gepreel,"On The Solitary Wave Solutions For Nonlinear Euler Equations " Applicable Analsysis,11(2004) 1101-1132.

[16] E.M.E.Zayed ,Hassan A.Zedan and Khaled A. Gepreel,"A Modified Extended Method To Find A Series Of Exact Solutions For A System Of Complex Coupled KdV Equations " Applicable Analsysis, in press.

[17] E.M.E.Zayed ,Hassan A.Zedan and Khaled A. Gepreel,"Group Analysis nAnd Modified Extended Tanh- Function To Find The Invariant Solutions And Soliton Solutions For Nonlinear Euler Equations " International Journal of Nonlinear Sciences and Numerical Simulation, 5 (2004) 221-234.

Received: June, 2009