Int. J. Contemp. Math. Sciences, Vol. 5, 2010, no. 30, 1455 - 1469

Intuitionistic Fuzzy Generalized Semi-Pre Continuous Mappings

R. Santhi and D. Jayanthi

Department of Mathematics
NGM College, Pollachi, Tamil Nadu, India
madhujiithcharu@gmail.com

Abstract

In this paper we introduce intuitionistic fuzzy generalized semi-pre continuous mappings and intuitionistic fuzzy generalized semi-pre irresolute mappings. We investigate some of their properties. Also we provide some characterization of intuitionistic fuzzy generalized semi-pre continuous mappings and intuitionistic fuzzy generalized semi-pre irresolute mappings.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy generalized semi-pre $T_{1/2}$ space, intuitionistic fuzzy generalized semi-pre continuous mappings and intuitionistic fuzzy generalized semi-pre irresolute mappings

1. Introduction

After the introduction of fuzzy sets by Zadeh [14], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [2] is one among them. Using the notion of intuitionistic fuzzy sets, Coker [4] introduced the notion of intuitionistic fuzzy topological spaces. Intuitionistic fuzzy semi-pre continuous mappings in intuitionistic fuzzy topological spaces are introduced by Young Bae Jun and Seok- Zun Song [13]. In this paper we introduce the notion of intuitionistic fuzzy generalized semi-pre continuous mappings and intuitionistic fuzzy generalized semi-pre irresolute mappings and studied some of their properties. We provide some characterizations of intuitionistic fuzzy generalized semi-pre continuous mappings and intuitionistic fuzzy generalized semi-pre irresolute mappings.

2. Preliminaries

Definition 2.1: [2] An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$$

where the functions $\mu_A(x): X \to [0,1]$ and $\nu_A(x): X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non -membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS (X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [2] Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x) \rangle / x \in X\}$.
Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in X\}$
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X\}$
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X\}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$.
The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1 \rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0 \rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: [11] The IFS $c(\alpha, \beta) = \langle x, c_\alpha, c_{1-\beta} \rangle$ where $\alpha \in (0, 1]$, $\beta \in [0, 1)$ and $\alpha + \beta \leq 1$ is called an intuitionistic fuzzy point (IFP for short) in X.

Definition 2.4: [11] Two IFSs are said to be q-coincident ($A\ _q\ B$ in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.5: [4] An *intuitionistic fuzzy topology* (IFT for short) on X is a family τ of IFSs in X satisfying the following axioms.

(i) $0_\sim, 1_\sim \in \tau$
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.6:[4] Let (X, τ) be an IFTS and A = $\langle$ x, μ_A, ν_A $\rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by
int (A) = $\cup$ { G / G is an IFOS in X and G $\subseteq$ A }
cl (A) = $\cap$ { K / K is an IFCS in X and A $\subseteq$ K }

Note that for any IFS A in (X, τ), we have cl(A^c) = $[\text{int}(A)]^c$ and int(A^c) = $[\text{cl}(A)]^c$ [13].

Definition 2.7:[6] An IFS A = $\langle$ x, μ_A, ν_A $\rangle$ in an IFTS (X, τ) is said to be an
(i) *intuitionistic fuzzy semi closed set* (IFSCS in short) if int(cl(A)) $\subseteq$ A
(ii) *intuitionistic fuzzy pre closed set* (IFPCS in short) if cl(int(A)) $\subseteq$ A
(iii) *intuitionistic fuzzy α closed set* (IFαCS in short) if cl(int(cl(A)) $\subseteq$ A.

The respective complements of the above IFCSs are called their respective IFOSs.
The family of all IFSCSs, IFPCSs, and IFαCSs (respectively IFSOSs, IFPOSs and IFαOSs) of an IFTS (X,τ) are respectively denoted by IFSC(X), IFPC(X) and IFαC(X) (respectively IFSO(X), IFPO(X) and IFαO(X)).

Definition 2.8:[13] An IFS A = $\langle$ x, μ_A, ν_A $\rangle$ in an IFTS (X, τ) is said to be an
(i) *intuitionistic fuzzy semi-pre closed set* (IFSPCS for short) if there exists an IFPCS B such
that int(B) $\subseteq$ A $\subseteq$ B.
(ii) *intuitionistic fuzzy semi-pre open set* (IFSPOS for short) if there exists an intuitionistic
fuzzy pre open set (IFPOS for short) B such that B $\subseteq$ A $\subseteq$ cl(B).

The family of all IFSPCSs (respectively IFSPOSs) of an IFTS (X,τ) is denoted by IFSPC(X) (respectively IFSPO(X)).

Every IFSCS (respectively IFSOS) and every IFPCS (respectively IFPOS) is an IFSPCS (respectively IFSPOS). But the separate converses need not be true in general [13].

Note that an IFS A is an IFSPCS if and only if int(cl(int(A))) $\subseteq$ A and an IFSPOS if and only if A $\subseteq$ cl(int(cl(A))) [9].

Definition 2.9:[9] Let A be an IFS in an IFTS (X, τ). Then
spint (A) = $\cup$ { G / G is an IFSPOS in X and G $\subseteq$ A }.
spcl (A) = $\cap$ { K / K is an IFSPCS in X and A $\subseteq$ K }.

Note that for any IFS A in (X, τ), we have spcl(A^c) = $[\text{spint}(A)]^c$ and spint(A^c) = $[\text{spcl}(A)]^c$ [9].

Definition 2.10:[10] An IFS A is an

(i) *intuitionistic fuzzy regular closed set* (IFRCS for short) if A = cl(int(A)).

(ii) *intuitionistic fuzzy generalized closed set* (IFGCS for short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFOS.

(iii) *intuitionistic fuzzy regular generalized closed set* (IFRGCS for short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFROS.

The family of all IFRCSs and IFGCSs (respectively IFROSs and IFGOSs) of an IFTS (X,τ) are respectively denoted by IFRC(X) and IFGC(X) (respectively IFRO(X) and IFGO(X)).

Definition 2.11:[9] An IFS A in an IFTS (X,τ) is said to be an *intuitionistic fuzzy generalized semi-pre closed set* (IFGSPCS for short) if spcl(A) ⊆ U whenever A ⊆ U and U is an IFOS in (X, τ).

Every IFCS, IFGCS, IFSCS, IFPCS, IFRCS, IFαCS and IFSPCS is an IFGSPCS but the separate converses may not be true in general [9].
The family of all IFGSPCSs of an IFTS (X,τ) is denoted by IFGSPC(X).

Definition 2.12:[9] The complement A^c of an IFGSPCS A in an IFTS (X,τ) is called an *intuitionistic fuzzy generalized semi-pre open set* (IFGSPOS for short) in X.

Every IFOS, IFGOS, IFSOS, IFPOS, IFROS, IFαOS and IFSPOS is an IFGSPOS but the separate converses may not be true in general [9].
The family of all IFGSPOSs of an IFTS (X,τ) is denoted by IFGSPO(X).

Definition 2.13:[6] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an *intuitionistic fuzzy continuous* (IF continuous for short) mapping if $f^{-1}(B) \in$ IFO(X) for every $B \in \sigma$

Definition 2.14:[7] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an

(i) *intuitionistic fuzzy semi continuous* (IFS continuous for short) mapping if $f^{-1}(B) \in$ IFSO(X) for every $B \in \sigma$

(ii) *intuitionistic fuzzy α- continuous* (IFα- continuous for short) mapping if $f^{-1}(B) \in$ IFαO(X) for every $B \in \sigma$

(iii) *intuitionistic fuzzy pre continuous* (IFP continuous) mapping if $f^{-1}(B) \in$ IFPO(X) for every $B \in \sigma$

Every IF continuous mapping is an IFα-continuous mapping and every IFα-continuous mapping is an IFS continuous mapping as well as an IFP continuous mapping, but the separate converses may not be true in general [7]

Definition 2.15:[7] Let $c(\alpha, \beta)$ be an IFP of an IFTS (X, τ). An IFS A of X is called an intuitionistic fuzzy neighborhood (IFN for short) of $c(\alpha, \beta)$ if there exists an IFOS B in X such that $c(\alpha, \beta) \in B \subseteq A$.

Definition 2.16:[11] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an *intuitionistic fuzzy generalized continuous* (IFG continuous for short) mapping if $f^{-1}(B) \in$ IFGC(X) for every IFCS B in Y.

Every IF continuous mapping is an IFG continuous mapping but the converse may not be true in general[11]

Definition 2.17: [13] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an *intuitionistic fuzzy semi-pre continuous* (IFSP continuous for short) mapping if $f^{-1}(B) \in$ IFSPO(X) for every $B \in \sigma$

Every IFS continuous mapping and IFP continuous mappings are IFSP continuous mapping but the converses may not be true in general[13].

Definition 2.18: Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an *intuitionistic fuzzy semi-pre irresolute* (IFSP irresolute for short) mapping if $f^{-1}(B)$ is an IFSPCS of X for every IFSPCS B in Y.

Definition 2.19:[9] If every IFGSPCS in (X, τ) is an IFSPCS in (X, τ), then the space can be called as an intuitionistic fuzzy semi- pre $T_{1/2}$ space (IFSP$T_{1/2}$ space for short).

Definition 2.20:[11] An IFTS (X, τ) is said to be IF$T_{1/2}$ space if every IFGCS in (X, τ) is an IFCS in (X, τ).

Theorem 2.21: For any IFS A in an IFTS (X, τ) where X is an IFSP$T_{1/2}$ space, $A \in$ IFGSPO(X) if and only if for every IFP $c(\alpha , \beta) \in A$, there exists an IFGSPOS B in X such that $c(\alpha , \beta) \in B \subseteq A$.
Proof: Necessity: If $A \in$ IFGSPO(X), then we can take B = A so that $c(\alpha , \beta) \in B \subseteq A$ for every IFP $c(\alpha , \beta) \in A$.
Sufficiency: Let A be an IFS in X and assume that there exists $B \in$ IFGSPO(X) such that $c(\alpha, \beta) \in B \subseteq A$. Since X is an IFSP$T_{1/2}$ space, B is an IFSPOS of X. Then $A = \cup_{c(\alpha, \beta) \in A}\{ c(\alpha , \beta) \} \subseteq \cup_{c(\alpha, \beta)\in A} B \subseteq A$. Therefore $A = \cup_{c(\alpha, \beta)\in A} B$ is an IFSPOS [13] and hence A is an IFGSPOS in X. Thus $A \in$ IFGSPO(X).

3. Intuitionistic fuzzy generalized semi-pre continuous mappings

In this section we introduce intuitionistic fuzzy generalized semi-pre continuous mapping and investigate some of its properties.

Definition 3.1: A mapping f: (X, τ) → (Y,σ) is called an *intuitionistic fuzzy generalized semi-pre continuous* (IFGSP continuous for short) mapping if $f^{-1}(V)$ is an IFGSPCS in (X, τ) for every IFCS V of (Y, σ).

For the sake of simplicity, we shall use the notation A= $\langle$ x, (μ_a, μ_b), (ν_a, ν_b) $\rangle$ instead of A = $\langle$ x, (a/μ_a, b/μ_b), (a/ν_a, b/ν_b) $\rangle$ in the following examples.
Similarly we shall use the notation B = $\langle$ y, (μ_u, μ_v), (ν_u, ν_v) $\rangle$ instead of B = $\langle$ y, (u/μ_u, v/μ_v), (u/ν_u, v/ν_v) $\rangle$ in the following examples.

Example 3.2 : Let X = { a, b }, Y = { u, v } and G_1 = $\langle$ x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) $\rangle$, G_2 = $\langle$ y, (0.2_u, 0.1_v), (0.8_u, 0.9_v) $\rangle$. Then τ = {$0_\sim$, G_1, $1_\sim$}and σ = { $0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFGSP continuous mapping.

Theorem 3.3: Every IF continuous mapping is an IFGSP continuous mapping but not conversely.
Proof : Let f: (X, τ) → (Y,σ) be an IF continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFCS in X. Since every IFCS is an IFGSPCS, $f^{-1}(V)$ is an IFGSPCS in X. Hence f is an IFGSP continuous mapping.

Example 3.4: In Example 3.2, f is an IFGSP continuous mapping but not an IF continuous mapping, since G_2 = $\langle$ y, (0.2_u, 0.1_v), (0.8_u, 0.9_v) $\rangle$ is an IFOS in Y but $f^{-1}(G_2)$ = $\langle$ x, (0.2_a, 0.1_b), (0.8_a, 0.9_b) $\rangle$ is not an IFOS in X.

Theorem 3.5: Every IFG continuous mapping is an IFGSP continuous mapping but not conversely.
Proof: Let f: (X, τ) → (Y,σ) be an IFG continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFGCS in X. Since every IFGCS is an IFGSPCS, $f^{-1}(V)$ is an IFGSPCS in X. Hence f is an IFGSP continuous mapping.

Example 3.6: Let X = {a, b}, Y = {u, v} and G_1 = $\langle$ x, (0.6_a, 0.7_b), (0.4_a, 0.3_b) $\rangle$, G_2 = $\langle$ y, (0.5_u, 0.7_v), (0.5_u, 0.3_v) $\rangle$. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFGSP continuous mapping but not an IFG continuous mapping, since G_2^c = $\langle$ y, (0.5_u, 0.3_v), (0.5_u, 0.7_v) $\rangle$ is an IFCS in Y but $f^{-1}(G_2^c)$ = $\langle$ x, (0.5_a, 0.3_b),(0.5_a, 0.7_b) $\rangle \subseteq G_1$ is not an IFGCS in X, since $cl(f^{-1}(G_2^c)) = 1_\sim \not\subseteq G_1$.

Theorem 3.7: Every IFS continuous mapping is an IFGSP continuous mapping but not conversely.
Proof: Let f: $(X, \tau) \to (Y,\sigma)$ be an IFS continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSCS in X. Since every IFSCS is an IFGSPCS, $f^{-1}(V)$ is an IFGSPCS in X. Hence f is an IFGSP continuous mapping.

Example 3.8: In Example 3.6, f is an IFGSP continuous mapping but not an IFS continuous mapping, since $G_2 = \langle y, (0.5_u, 0.7_v), (0.5_u, 0.3_v) \rangle$ is an IFOS in Y but $f^{-1}(G_2) = \langle x, (0.5_a, 0.7_b), (0.5_a, 0.3_b) \rangle \nsubseteq cl(int(f^{-1}(G_2))) = 0_\sim$

Theorem 3.9: Every IFP continuous mapping is an IFGSP continuous mapping but not conversely.
Proof: Let f: $(X, \tau) \to (Y,\sigma)$ be an IFP continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFPCS in X. Since every IFPCS is an IFGSPCS, $f^{-1}(V)$ is an IFGSPCS in X. Hence f is an IFGSP continuous mapping.

Example 3.10: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$, $G_2 = \langle x, (0.4_a, 0.1_b), (0.6_a, 0.7_b)\rangle$, $G_3 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.2_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFGSP continuous mapping but not an IFP continuous mapping. Since G_3 is an IFOS in Y but $f^{-1}(G_3) = \langle x, (0.6_a, 0.7_b), (0.4_a, 0.2_b)\rangle \nsubseteq int(cl(f^{-1}(G_3))) = G_1$.

Theorem 3.11: Every IFSP continuous mapping is an IFGSP continuous mapping but not conversely.
Proof: Let f: $(X, \tau) \to (Y,\sigma)$ be an IFSP continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSPCS in X. Since every IFSPCS is an IFGSPCS, $f^{-1}(V)$ is an IFGSPCS in X. Hence f is an IFGSP continuous mapping.

Example 3.12: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.6_a, 0.7_b), (0.2_a, 0.3_b) \rangle$, $G_2 = \langle x, (0.2_a, 0.1_b), (0.7_a, 0.8_b)\rangle$, $G_3 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, and let $G_4 = \langle y, (0.5_u, 0.3_v), (0.5_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, G_3, 1_\sim\}$ and $\sigma = \{0_\sim, G_4, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFGSP continuous mapping but not an IFSP continuous mapping, since G_4 is an IFOS in Y but $f^{-1}(G_4) = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle \nsubseteq cl(int(cl(f^{-1}(G_4)))) = G_1^c$.

Theorem 3.13: Every IFα- continuous mapping is an IFGSP continuous mapping but not conversely.

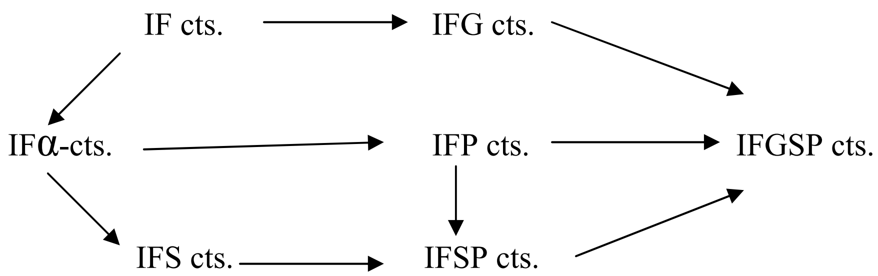
Proof: Let f: $(X, \tau) \to (Y,\sigma)$ be an IFα- continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFαCS in X. Since every IFαCS is an IFGSPCS, $f^{-1}(V)$ is an IFGSPCS in X. Hence f is an IFGSP continuous mapping.

Example 3.14: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle$, $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.2_v) \rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFGSP continuous mapping but not an IFα- continuous mapping, since G_2 is an IFOS in Y but $f^{-1}(G_2) = \langle x, (0.6_a, 0.7_b), (0.4_a, 0.2_b) \rangle \not\subseteq int(cl(int(f^{-1}(G_2)))) = G_1$.

Theorem 3.15: Let $f: (X, \tau) \to (Y,\sigma)$ be a mapping where $f^{-1}(V)$ is an IFRCS in X for every IFCS in Y. Then f is an IFGSP continuous mapping but not conversely.
Proof: Let A be an IFCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFGSPCS, $f^{-1}(V)$ is an IFGSPCS in X. Hence f is an IFGSP continuous mapping.

Example 3.16: In Example 3.14, f is an IFGSP continuous mapping but not a mapping as defined in Theorem 3.15, since G_2^c is an IFCS in Y but $f^{-1}(G_2^c) = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.7_b) \rangle \neq cl(int(f^{-1}(G_2^c))) = 0_\sim$

The relation between various types of intuitionistic fuzzy continuity is given in the following diagram. In this diagram 'cts.' means continuous.



The reverse implications are not true in general in the above diagram.

Theorem 3.17: A mapping $f: (X, \tau) \to (Y, \sigma)$ is an IFGSP continuous mapping if and only if the inverse image of each IFOS in Y is an IFGSPOS in X.
Proof: The proof is obvious since $f^{-1}(A^c) = (f^{-1}(A))^c$.

Theorem 3.18: If $f: (X, \tau) \to (Y,\sigma)$ is an IFGSP continuous mapping then for each IFP $c(\alpha,\beta)$ of X and each $A \in \sigma$ such that $f(c(\alpha, \beta)) \in A$, there exists an IFGSPOS B of X such that $c(\alpha, \beta) \in B$ and $f(B) \subseteq A$.
Proof : Let $c(\alpha,\beta)$ be an IFP of X and $A \in \sigma$ such that $f(c(\alpha,\beta)) \in A$. Put $B = f^{-1}(A)$. Then by hypothesis B is an IFGSPOS in X such that $c(\alpha,\beta) \in B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

Theorem 3.19: If f : (X, τ) → (Y, σ) is an IFGSP continuous mapping then for each IFP c(α,β) of X and each A ∈ σ such that f(c(α,β)) $_q$ A, there exists an IFGSPOS B of X such that c(α,β) $_q$ B and f(B) ⊆ A.

Proof : Let c(α,β) be an IFP of X and A∈σ such that f(c(α,β)) $_q$ A. Put B = f^{-1}(A). Then by hypothesis B is an IFGSPOS in X such that c(α,β) $_q$ B and f (B) = f(f^{-1}(A)) ⊆ A.

Theorem 3.20: Let f : (X, τ) → (Y, σ) be an IFGSP continuous mapping, then f is an IFSP continuous mapping if X is an IFSP$T_{1/2}$ space.

Proof: Let V be an IFCS in Y. Then f^{-1}(V) is an IFGSPCS in X, by hypothesis. Since X is an IFSP$T_{1/2}$ space, f^{-1}(V) is an IFSPCS in X. Hence f is an IFSP continuous mapping.

Theorem 3.21: Let f : (X, τ) → (Y, σ) be an IFGSP continuous mapping and g: (Y, σ) → (Z, γ) is an IFG continuous mapping and Y is an IF$T_{1/2}$ space, then g ∘ f : (X, τ) → (Z, γ) is an IFGSP continuous mapping.

Proof: Let V be an IFCS in Z. Then g^{-1}(V) is an IFGCS in Y, by hypothesis. Since Y is an IF$T_{1/2}$ space, g^{-1}(V) is an IFCS in Y. Therefore f^{-1}(g^{-1}(V)) is an IFGSPCS in X, by hypothesis. Hence g ∘ f is an IFGSP continuous mapping.

Theorem 3.22: Let f : (X, τ) → (Y, σ) be an IFGSP continuous mapping and g: (Y, σ) → (Z, γ) is an IF continuous mapping, then g ∘ f : (X, τ) → (Z, γ) is an IFGSP continuous mapping.

Proof: Let V be an IFCS in Z. Then g^{-1}(V) is an IFCS in Y, by hypothesis. Since f is an IFGSP continuous mapping, f^{-1}(g^{-1}(V)) is an IFGSPCS in X. Hence g ∘ f is an IFGSP continuous mapping.

Theorem 3.23: Let f : (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are IFSP$T_{1/2}$ spaces.

(i) f is an IFGSP continuous mapping
(ii) f^{-1}(B) is an IFGSPOS in X for each IFOS B in Y
(iii) for every IFP c(α, β) in X and for every IFOS B in Y such that f(c(α, β)) ∈ B, there exists an IFGSPOS A in X such that c(α, β) ∈ A and f(A) ⊆ B.

Proof: (i) ⇒ (ii) is obvious from the Theorem 3.17.

(ii) ⇒ (iii) Let B be any IFOS in Y and let c(α, β) ∈ X. Given f(c(α, β)) ∈ B. By hypothesis f^{-1}(B) is an IFGSPOS in X. Take A = f^{-1}(B). Now c(α, β) ∈ f^{-1}(f(c(α, β))). Therefore f^{-1}(f(c(α, β))) ∈ f^{-1}(B) = A. This implies c(α, β) ∈ A and f(A) = f(f^{-1}(B)) ⊆ B.

(iii) ⇒ (i) Let A be an IFCS in Y. Then its complement, say B is an IFOS in Y. Let c(α, β) ∈ X and f(c(α, β)) ∈ B. Then there exists an IFGSPOS, say C = f^{-1}(B) in X such that c(α, β) ∈ C and f(C) ⊆ B. Therefore f^{-1}(B) is an IFGSPOS in

X, by Theorem 2.21. That is $f^{-1}(A^c)$ is an IFGSPOS in X and hence $f^{-1}(A)$ is an IFGSPCS in X. Thus f is an IFGSP continuous mapping.

Theorem 3.24: Let f : (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are $IFSPT_{1/2}$ spaces.

(i) f is an IFGSP continuous mapping

(ii) for each IFP c(α, β) in X and every IFN A of f(c(α, β)), there exists an IFGSPOS B in X such that $c(\alpha, \beta) \in B \subseteq f^{-1}(A)$.

(iii) for each IFP c(α, β) in X and for every IFN A of f(c(α, β)), there exists an IFGSPOS B in X such that $c(\alpha, \beta) \in B$ and $f(B) \subseteq A$.

Proof: (i) ⇒ (ii) Let $c(\alpha, \beta) \in X$ and let A be an IFN of f(c(α, β)). Then there exists an IFOS C in Y such that $f(c(\alpha, \beta)) \in C \subseteq A$. Since f is an IFGSP continuous mapping, $f^{-1}(C) = B$ (say), is an IFGSPOS in X and $c(\alpha, \beta) \in B \subseteq f^{-1}(A)$.

(ii) ⇒ (iii) Let $c(\alpha, \beta) \in X$ and let A be an IFN of f(c(α, β)). Then there exists an IFGSPOS B in X such that $c(\alpha, \beta)) \in B \subseteq f^{-1}(A)$, by hypothesis. Therefore $c(\alpha, \beta)) \in B$ and $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.

(iii) ⇒ (i) Let B be an IFOS in Y and let $c(\alpha, \beta) \in f^{-1}(B)$. Then $f(c(\alpha, \beta)) \in B$. Therefore B is an IFN of f(c(α, β)). Since B is IFOS, by hypothesis there exists an IFGSPOS A in X such that $c(\alpha, \beta) \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$.Therefore $f^{-1}(B)$ is an IFGSPOS in X, by Theorem 2.20. Hence f is an IFGSP continuous mapping.

Theorem 3.25: Let f : (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y that satisfies $f^{-1}(int(B)) \subseteq cl(int(cl(f^{-1}(B))))$ for every IFS B in Y. Then f is an IFGSP continuous mapping.

Proof: Let B be an IFOS in Y. Then int(B) = B. By hypothesis $f^{-1}(B) \subseteq cl(int(cl(f^{-1}(B))))$. This implies $f^{-1}(B)$ is an IFSPOS in X. Therefore it is an IFGSPOS in X and hence f is an IFGSP continuous mapping, by Theorem 3.17.

Theorem 3.26: Let f : (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X is an $IFSPT_{1/2}$ space.

(i) f is an IFGSP continuous mapping

(ii) If B is an IFOS in Y then $f^{1}(B)$ is an IFGSPOS in X

(iii) $f^{-1}(int(B)) \subseteq cl(int(cl(f^{-1}(B))))$ for every IFS B in Y.

Proof: (i) ⇒ (ii) is obviously true by Theorem 3.17.

(ii) ⇒ (iii) Let B be any IFS in Y. Then int(B) is an IFOS in Y. Then $f^{-1}(int(B))$ is an IFGSPOS in X. Since X is an $IFSPT_{1/2}$ space, $f^{-1}(int(B))$ is an IFSPOS in X. Therefore $f^{-1}(int(B)) \subseteq cl(int(cl(f^{-1}(int(B))))) \subseteq cl(int(cl(f^{-1}(B))))$.

(iii) ⇒ (i) Let B be an IFCS in Y. Then its complement, say A is an IFOS in Y, then int(A) = A. Now by hypothesis $f^{-1}(int(A)) \subseteq cl(int(cl(f^{-1}(A))))$. This implies $f^{-1}(A) \subseteq cl(int(cl(f^{-1}(A))))$. Hence $f^{-1}(A)$ is an IFSPOS in X. Since every IFSPOS is an IFGSPOS, $f^{-1}(A)$ is an IFGSPOS in X. Thus $f^{-1}(B)$ is an IFGSPCS in X, since $f^{-1}(A) = f^{-1}(B^c)$. Hence f is an IFGSP continuous mapping.

Theorem 3.27: Let f : (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are $IFSPT_{1/2}$ spaces.
(i) f is an IFGSP continuous mapping
(ii) $int(cl(int(f^{-1}(B)))) \subseteq f^{-1}(spcl(B))$ for each IFCS B in Y.
(iii)$f^{-1}(spint (B)) \subseteq cl(int(cl(f^{-1}(B))))$ for each IFOS B of Y
(iv)$f(int(cl(int(A)))) \subseteq cl(f(A))$ for each IFS A of X.
Proof: (i) ⇒ (ii) Let B be an IFCS in Y. Then $f^{-1}(B)$ is an IFGSPCS in X. Since X is $IFSPT_{1/2}$ space, $f^{-1}(B)$ is an IFSPCS. Therefore $int(cl(int(f^{-1}(B)))) \subseteq f^{-1}(B) = f^{-1}(spcl(B))$
(ii) ⇒ (iii) can be easily proved by taking complement in (ii).
(iii)⇒(iv) Let A ∈ X. Then B = f(A) and therefore $A \subseteq f^{-1}(B)$. Here int(f(A)) = int(B) is an IFOS in Y. Then (iii) implies that $f^{-1}(spint(int(B))) \subseteq cl(int(cl(f^{-1}(int(B))))) \subseteq cl(int(cl(f^{-1}(B))))$. Now $(cl(int(cl(A^c))))^c \subseteq (cl(int(cl(f^{-1}(B)^c))))^c \subseteq (f^{-1}(spint(int(B^c))))^c$. Therefore $int(cl(int(A))) \subseteq f^{-1}(spcl(cl(B)))$. Now $f(int(cl(int(A)))) \subseteq f(f^{-1}(spcl(cl(B)))) = cl(B) = cl(f(A))$.
(iv)⇒ (i) Let B be any IFCS in Y, then $f^{-1}(B)$ is an IFS in X. By hypothesis $f(int(cl(int(f^{-1}(B))))) \subseteq cl(f(f^{-1}(B))) = cl(B) = B$. Now $int(cl(int(f^{-1}(B)))) \subseteq f^{1}(f (int(cl(int(f^{-1}(B)))))) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFSPCS and hence it is an IFGSPCS in X. Thus f is an IFGSP continuous mapping.

Theorem 3.28: A mapping f: (X, τ) → (Y, σ) is an IFGSP continuous mapping if $cl(int(cl(f^{-1}(A)))) \subseteq f^{-1}(cl(A))$ for every IFS A in Y.
Proof: Let A be an IFOS in Y then A^c is an IFCS in Y. By hypothesis, $cl(int(cl(f^{-1}(A^c)))) \subseteq f^{-1}(cl(A^c)) = f^{-1}(A^c)$, since A^c is an IFCS. Now $(int(cl(int(f^{-1}(A)))))^c = cl(int(cl(f^{-1}(A^c)))) \subseteq f^{-1}(A^c) = (f^{-1}(A))^c$. This implies $f^{-1}(A) \subseteq int(cl(int(f^{-1}(A))))$. Hence $f^{-1}(A)$ is an IFαOS and hence it is an IFGSPOS. Therefore f is an IFGSP continuous mapping, by Theorem 3.17.

Theorem 3.29: Let f : (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X is an $IFSPT_{1/2}$ space.
(i) f is an IFGSP continuous mapping
(ii $f^{-1}(B)$ is an IFGSPCS in X for every IFCS B in Y
(iii) $int(cl(int(f^{-1}(A)))) \subseteq f^{1}(cl(A))$ for every IFS A in Y.
Proof : (i) ⇒ (ii) is obvious from the Definition 3.1.
(ii) ⇒ (iii) Let A be an IFS in Y. Then cl(A) is an IFCS in Y. By hypothesis, $f^{-1}(cl(A))$ is an IFGSPCS in X. Since X is an $IFSPT_{1/2}$ space, $f^{-1}(cl(A))$ is an IFSPCS. Therefore $int(cl(int(f^{-1}(cl(A))))) \subseteq f^{-1}(cl(A))$. Now $int(cl(int(f^{-1}(A)))) \subseteq int(cl(int(f^{-1}(cl(A))))) \subseteq f^{-1}(cl(A))$.
(iii) ⇒ (i) Let A be an IFCS in Y. By hypothesis $int(cl(int(f^{-1}(A)))) \subseteq f^{-1}(cl(A)) = f^{-1}(A)$. This implies $f^{-1}(A)$ is an IFSPCS in X and hence it is an IFGSPCS. Thus f is an IFGSP continuous mapping.

4. Intuitionistic fuzzy generalized semi-pre irresolute mappings

In this section we introduce intuitionistic fuzzy generalized semi-pre irresolute mappings and study some of their properties.

Definition 4.1: A mapping f: (X, τ) → (Y,σ) is called *intuitionistic fuzzy generalized semi-pre irresolute* (IFGSP irresolute) mapping if $f^{-1}(V)$ is an IFGSPCS in (X, τ) for every IFGSPCS V of (Y, σ).

Theorem 4.2: Let f : (X, τ) → (Y, σ) be an IFGSP irresolute mapping, then f is an IFGSP continuous mapping but not conversely.

Proof: Let f be an IFGSP irresolute mapping. Let V be any IFCS in Y. Then V is an IFGSPCS and by hypothesis $f^{-1}(V)$ is an IFGSPCS in X. Hence f is an IFGSP continuous mapping.

Example 4.3 : Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.6_a, 0.7_b), (0.2_a, 0.1_b) \rangle$, $G_2 = \langle x, (0.3_a, 0.2_b), (0.2_a, 0.2_b) \rangle$, $G_3 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v) \rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y,σ) by f (a) = u and f (b) = v. Then f is an IFGSP continuous mapping but not an IFGSP irresolute mapping, since the IFS $A = \langle y, (0.5_u, 0.3_v), (0.2_u, 0.1_v) \rangle$ is an IFGSPCS in Y but $f^{-1}(A) = \langle x, (0.5_a, 0.3_b), (0.2_a, 0.1_b) \rangle \subseteq G_1$ is not an IFGSPCS in X, since $spcl(f^{-1}(A)) = 1_\sim \not\subseteq G_1$.

Theorem 4.4 : A mapping f: (X, τ) → (Y, σ) is an IFGSP irresolute mapping if and only if the inverse image of each IFGSPOS in Y is an IFGSPOS in X.

Proof: The proof is obvious from the Definition 4.1, since $f^{-1}(A^c) = (f^{-1}(A))^c$.

Theorem 4.5: Let f : (X, τ) → (Y, σ) be an IFGSP irresolute mapping, then f is an IFSP irresolute mapping if X is an $IFSPT_{1/2}$ space.

Proof: Let V be an IFSPCS in Y. Then V is an IFGSPCS in Y. Therefore $f^{-1}(V)$ is an IFGSPCS in X, by hypothesis. Since X is an $IFSPT_{1/2}$ space, $f^{-1}(V)$ is an IFSPCS in X. Hence f is an IFSP irresolute mapping.

Theorem 4.6: Let f : (X, τ) → (Y, σ) and g: (Y, σ) → (Z, γ) be IFGSP irresolute mappings, then $g \circ f$: (X, τ) → (Z, γ) is an IFGSP irresolute mapping.

Proof: Let V be an IFGSPCS in Z. Then $g^{-1}(V)$ is an IFGSPCS in Y. Since f is an IFGSP irresolute, $f^{-1}(g^{-1}(V))$ is an IFGSPCS in X, by hypothesis. Hence $g \circ f$ is an IFGSP irresolute mapping.

Theorem 4.7: Let f : (X, τ) → (Y, σ) be an IFGSP irresolute mapping and g: (Y, σ) → (Z, γ) is an IFGSP continuous mapping, then $g \circ f$: (X, τ) → (Z, γ) is an IFGSP continuous mapping.

Proof: Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFGSPCS in Y. Since f is an IFGSP irresolute mapping, $f^{-1}(g^{-1}(V))$ is an IFGSPCS in X. Hence $g \circ f$ is an IFGSP continuous mapping.

Theorem 4.8: Let f : (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are $IFSPT_{1/2}$ spaces.

(i) f is an IFGSP irresolute mapping
(ii) $f^{-1}(B)$ is an IFGSPOS in X for each IFGSPOS in Y
(iii) $f^{-1}(spint(B)) \subseteq spint(f^{-1}(B))$ for each IFS B of Y
(iv) $spcl(f^{-1}(B)) \subseteq f^{-1}(spcl(B))$ for each IFS B of Y

Proof: (i) ⇒ (ii) is obvious from the Theorem 4.4.
(ii) ⇒ (iii) Let B be any IFS in Y and $spint(B) \subseteq B$. Also $f^{-1}(spint(B)) \subseteq f^{-1}(B)$. Since spint(B) is an IFSPOS in Y, it is an IFGSPOS in Y. Therefore $f^{-1}(spint(B))$ is an IFGSPOS in X, by hypothesis. Since X is an $IFSPT_{1/2}$ space, $f^{-1}(spint(B))$ is an IFSPOS in X. Hence $f^{-1}(spint(B)) = spint(f^{-1}(spint(B))) \subseteq spint(f^{-1}(B))$.
(iii) ⇒ (iv) is obvious by taking complement in (iii).
(iv)⇒ (i) Let B be an IFGSPCS in Y. Since Y is an $IFSPT_{1/2}$ space, B is an IFSPCS in Y and spcl(B) = B. Hence $f^{-1}(B) = f^{-1}(spcl(B)) \supseteq spcl(f^{-1}(B))$. Therefore $spcl(f^{-1}(B)) = f^{-1}(B)$. This implies $f^{1}(B)$ is an IFSPCS and hence it is an IFGSPCS in X. Thus f is an IFGSP irresolute mapping.

Theorem 4.9: Let f : (X, τ) → (Y, σ) be an IFGSP irresolute mapping from an IFTS X into an IFTS Y. Then $f^{-1}(B) \subseteq spint(f^{-1}(cl(int(cl(B)))))$ for every IFGSPOS B in Y, if X and Y are $IFSPT_{1/2}$ spaces.
Proof: Let B be an IFGSPOS in Y. Then by hypothesis $f^{-1}(B)$ is an IFGSPOS in X. Since X is an $IFSPT_{1/2}$ space, $f^{-1}(B)$ is an IFSPOS in X. Therefore $spint(f^{-1}(B)) = f^{-1}(B)$. Since Y is an $IFSPT_{1/2}$ space, B is an IFSPOS in Y and $B \subseteq cl(int(cl(B)))$. Now, $f^{-1}(B) = spint(f^{-1}(B))$ implies, $f^{-1}(B) \subseteq spint(f^{-1}(cl(int(cl(B)))))$.

Theorem 4.10: Let f : (X, τ) → (Y, σ) be an IFGSP irresolute mapping from an IFTS X into an IFTS Y, then $f^{-1}(B) \subseteq spint(cl(int(cl(f^{-1}(B)))))$ if X is an $IFSPT_{1/2}$ space.

Proof: Let B be an IFGSPOS in Y. Then by hypothesis $f^{-1}(B)$ is an IFGSPOS in X. Since X is an $IFSPT_{1/2}$ space, $f^{-1}(B)$ is an IFSPOS in X. Therefore $spint(f^{-1}(B)) = f^{-1}(B)$ and $f^{-1}(B) \subseteq cl(int(cl(f^{-1}(B))))$. Hence $f^{-1}(B) \subseteq spint(cl(int(cl(f^{-1}(B)))))$.

REFERENCES

[1] Andrijevic, D., **Semi pre open sets**, Mat. Vesnik, 1986, 24-32.

[2] Atanassov, K., **Intuitionistic fuzzy sets**, Fuzzy Sets and Systems, 1986, 87- 96.

[3] Chang, C., **Fuzzy topological spaces,** J. Math. Anal. Appl., 1968, 182- 190.

[4] Coker, D., **An introduction to intuitionistic fuzzy topological space**, Fuzzy sets and systems, 1997, 81-89.

[5] Dontchev, J., **On generalizing semi-pre open sets,** Mem.Fac.Sci.Kochi Univ. Ser.A. Math., 1995, 35-48.

[6] Gurcay, H., Coker, D., and Haydar, Es. A., **On fuzzy continuity in intuitionistic fuzzy topological spaces**, The J. fuzzy mathematics, 1997, 365-378.

[7] Joung Kon Jeon, Young Bae Jun, and Jin Han Park, **Intuitionistic fuzzy alpha-continuity and intuitionistic fuzzy pre continuity**, International Journal of Mathematics and Mathematical Sciences, 2005, 3091-3101.

[8] Saraf, R. K., Govindappa Navalagi and Meena Khanna, **On fuzzy semi-pre generalized closed sets,** Bull. Malays. Sci. Soc., 2005, 19-30.

[9] Santhi, R. and Jayanthi, D., **Intuitionistic fuzzy generalized semi-pre closed sets** (accepted)

[10] Thakur, S. S. and Singh, S., **On fuzzy semi-pre open sets and fuzzy semi- pre continuity,** Fuzzy sets and systems, 1998, 383-391.

[11] Thakur, S. S and Rekha Chaturvedi, **Regular** g**eneralized closed sets in intuitionistic fuzzy topological spaces,** Universitatea Din Bacau,Studii Si Cercetari Stiintifice, Seria: Matematica, 2006, 257-272.

[12] Turnali, N and Coker, D., **Fuzzy connectedness in intuitionistic fuzzy topological spaces**, Fuzzy sets and systems, 2000, 369-375.

[13] Young Bae Jun and Seok- Zun Song, **Intuitionistic fuzzy semi-pre open sets and Intuitionistic semi-pre continuous mappings**, jour. of Appl. Math & computing, 2005, 467-474.

[14] Zadeh, L. A., **Fuzzy sets**, Information and control, 1965, 338-353.

Received: January, 2010