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Set a Productive Mathematical Reliability Model for a System with Fuzzy Constant Failure Rates

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Abstract

Reliability Models based on Markov Chain (except Queuing models) have extensive application in reliability of electrical and electronically equipments. In this article, a system with three parallel and identical elements with fuzzy constant failure rate is analyzed and the results are in consideration of failure rates in of triangular fuzzy numbers.

Keywords: Fuzzy, Reliability, productivity, Confidence Interval, Constant Failure Rate.

1. Introduction

Nowadays, Reliability models are considered one of the most important applications of Markov Chains, and most electronic systems come across these models.
This is an extensive system and for every electrical system a specific model is designed and implemented and in these models Failure Rate is Constant.
Since these failure rates are driven from gathering data and usage of probability distribution functions, or the opinions of the experts on the matter, uncertainty is also, an obvious parameter.
Hence, in this article, one particular system with three parallel elements is reviewed and the results are in consideration of fixed and equal failure rate (λ).

To demonstrate the uncertainty in calculation of failure rate, these parameters are estimated through a triangular fuzzy number.

2. Notifications

The notification that used in this article is as followed:
λ : Failure rate of elements.
L : Lower limit of triangular fuzzy number related to failure rate of elements
M : Medium of triangular fuzzy number related to failure rate of elements.
U : Upper limit of triangular fuzzy number related to failure rate of elements.
Pu (t) : Probability of the system at the *t* moment to be in condition *u* .
RP (t): Probability of the functionality of the system.
MTTF : Mean time to failure.

3. Introduction to the discussed sample

In this article, a system that working with three parallel elements is considered. Assuming that the system will stop working when entire of elements have failed, we can consider the following eight conditions for the system:

State	Condition of first part	Condition of second part	Condition of third part
123	Working	Working	Working
12	Working	working	Failed
13	Working	Failed	Working
23	Failed	Working	Working
1	Working	Failed	Failed
2	Failed	Working	Failed
3	Failed	Failed	Working
0	Failed	Failed	Failed

Table 1: States of the system

And the flow diagram for this system will be as follow:

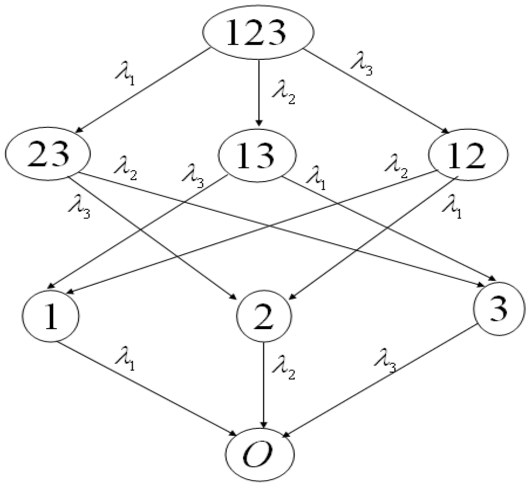

Fig 1: States of the system

In respect to the above descriptions, the probability that the system work at the moment is calculated as followed :

$$R_p(t) = 1 - P_0(t) \qquad (01)$$

We also know that:

$$P_{123}(t) + P_{12}(t) + \ldots + P_0 t = 1 \qquad (02)$$

In this system, the purpose is to find Pu（t ）. For the nods 123 through O in figure 1 we have:

$$P_{123}(t + \Delta t) = P_{123}(t) - (\lambda_1 + \lambda_2 + \lambda_3) \times \Delta t \times P_{123} \qquad (03)$$

$$P_{12}(t + \Delta t) = P_{12}(t) + \lambda_3 \times \Delta t \times P_{123}(t) - (\lambda_1 + \lambda_2) \times \Delta t \times P_{12}(t) \qquad (04)$$

$$P_{13}(t + \Delta t) = P_{13}(t) + \lambda_2 \times \Delta t \times P_{123}(t) - (\lambda_2 + \lambda_3) \times \Delta t \times P_{13}(t) \qquad (05)$$

$$P_{23}(t + \Delta t) = P_{23}(t) + \lambda_1 \times \Delta t \times P_{123}(t) - (\lambda_2 + \lambda_3) \times \Delta t \times P_{23}(t) \qquad (06)$$

$$P_1(t + \Delta t) = P_1(t) + \lambda_1 \times \Delta t \times P_{13}(t) + \lambda_2 \times \Delta t \times P_{12}(t) - \lambda_1 \Delta t \times P_1(t) \qquad (07)$$

$$P_2(t+\Delta t)=P_2(t)+\lambda_1\times\Delta t\times P_{12}(t)+\lambda_2\times\Delta t\times P_{23}(t)-\lambda_2\times\Delta t\times P_1(t) \quad (08)$$

$$P_3(t+\Delta t)=P_3(t)+\lambda_1\times\Delta t+P_{13}(t)+\lambda_2\times\Delta t\times P_{23}(t)-\lambda_3\times\Delta t\times P_1(t) \quad (09)$$

And by solving the equations (03)through (09)can calculate the values *Pu* (*t*), as followed:

$$P_{123}(t)=e^{-3\lambda t} \quad (10)$$

$$P_{12}(t)=P_{13}(t)=P_{23}(t)=e^{-2\lambda t}-e^{-3\lambda t} \quad (11)$$

$$p_1(t)=P_2(t)=P_3(t)=e^{-\lambda t}-2e^{-2\lambda t}+e^{-3\lambda t} \quad (12)$$

And the system MTTF is also calculated as followed:

$$MTTF = \int_0^{+\infty} Rp(t)dt = \frac{3}{\lambda}+\frac{3}{2\lambda}-\frac{1}{3\lambda} = \frac{11}{6\lambda} \quad (13)$$

4. Calculations with respect to Fuzzy failure Rates

Most important consideration is that the Values of λ, are not fixed. Since they are driven from collected data or the opinions of the experts, uncertainty of the value is an undeniable fact.

Most of the times, these failure rates considered as a known value or have a known distributions function.
Of course in this condition, failure rates are considered in the form of a Triangular Fuzzy Number as follows:

$$\bar{\lambda}=(L/M/U) \quad (14)$$

The α-cut of these failure rates can be calculated as follow:

$$\bar{\lambda}[\alpha]=[L+\alpha(M-L),U-\alpha(U-M)] = [A,B] \quad (15)$$

Now we can calculate the *Pu* (*t*), in Fuzzy condition by using extension principle [2].

Assume $\overline{P}_u[t,\alpha]=\left[P_{u(1)}(t,\alpha),P_{u(2)}(t,\alpha)\right]$, therefore we will have [3]:

$$P_{123\ (1)}(t,\alpha)=e^{-3[U-\alpha(U-M)]t} \tag{16}$$

$$P_{123\ (2)}(t,\alpha)=e^{-3[L+\alpha(M-L)]t} \tag{17}$$

$$P_{12(1)}(t,\alpha)=P_{13(1)}(t,\alpha)=P_{23(1)}= e^{-2[U-\alpha(U-M)]t}-e^{-3[L+\alpha(M-L)]t} \tag{18}$$

$$P_{12(2)}(t,\alpha)=P_{13(1)}(t,\alpha)=P_{23(2)}= e^{-2[L+\alpha(M-L)]t}-e^{-3[U-\alpha(U-M)]t} \tag{19}$$

$$P_{1(1)}(t)=P_{2(1)}(t)=P_{3(1)}(t)= e^{-[U-\alpha(U-M)]t}-2e^{-2[L+(M-L)]t}+ e^{-3[U-\alpha(U-M)]t} \tag{20}$$

$$P_{1(2)}(t)=P_{2(2)}(t)=P_{3(2)}(t)= e^{-[L+\alpha(M-L)]t}-2e^{-2[U-(U-M)]t}+ e^{-3[L+\alpha(U-M)]t} \tag{21}$$

$$\begin{aligned} Rp_{(1)}(t) &= e^{-3At}+3e^{-2At}-3e^{-3Bt} \\ &+3e^{-At}-6e^{-2Bt}+4e^{-3At} \\ &=4e^{-3At}\,3e^{-2At}+3e^{-At} \\ &-3e^{-3Bt}-6e^{-2Bt} \end{aligned} \tag{22}$$

$$
\begin{aligned}
&Rp_{(2)}(t) = e^{-3Bt} + 3e^{-2Bt} - 3e^{-3At} \\
&+ 3e^{-At} - 6e^{-2At} + 3e^{-3Bt} \\
&= 4e^{-3Bt} 3e^{-2Bt} + 3e^{-Bt} \\
&- 3e^{-3At} - 6e^{-2At}
\end{aligned}
\tag{23}
$$

Assume MTTF1[α]=[MTTF1(α),MTTF2 (α)], therefore we will have:

$$
\begin{aligned}
&MTTF_1(\alpha) = \int_0^{+\infty} R_{P(1)}(t, \alpha) dt = \\
&\frac{4}{3A} + \frac{3}{2A} + \frac{3}{A} - \frac{3}{3B} - \frac{6}{2B} \\
&= \frac{35}{6A} - \frac{4}{B}
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
&MTTF_2(\alpha) = \int_0^{+\infty} R_{P(2)}(t, \alpha) dt = \\
&\frac{4}{3B} + \frac{3}{2B} + \frac{3}{B} - \frac{3}{3B} - \frac{3}{3A} - \frac{6}{2A} \\
&= \frac{35}{6B} - \frac{4}{A}
\end{aligned}
\tag{25}
$$

$$
MTTF[\alpha] = \begin{bmatrix} \dfrac{35}{6[U + \alpha(U - M)]} - \\ \dfrac{4}{L + \alpha(M - L)}, \\ \dfrac{35}{6[L + \alpha(M - L)]} - \\ \dfrac{4}{u - \alpha(U - M)} \end{bmatrix}
\tag{26}
$$

And to calculate the Fuzzy Number related to the mean time to failure of system (MTTF) we have:

$$\overline{MTTF} = \left(L^{*} / M^{*} / U^{*}\right) \tag{27}$$

$$MTTF\ (0) = \left[\frac{35}{6U} - \frac{4}{L}, \frac{35}{6L} - \frac{4}{U}\right] \tag{28}$$

$$MTTF(1) = \left[\frac{11}{6M}, \frac{11}{6M}\right] \tag{29}$$

$$\overline{MTTF} = \left[\frac{35}{6U} - \frac{4}{L}, \frac{11}{6M}, \frac{35}{6L} - \frac{4}{U}\right] \tag{30}$$

5. Numerical Sample

Assume that based on an opinion of an expert the value of fuzzy triangular number of λ is as followed:

$$\overline{\lambda} = (0.05 / 0.07 / 0.11)$$

Then the MTTF in crisp condition is 26.19 days. And the fuzzy number of MTTF is:

$$\overline{MTTF} = (0,26.19,133.03)$$

The lower limit of fuzzy number was -26.96 and because of the bounds of the fuzzy number is not negative it changes too 0.

6. Conclusion and Further research

Because of using fuzzy failure rates, the system condition is more realistically than crisp condition, and these are the advantages of this model.
The system discussed in this article is one of hundreds of the actual existing systems that have already been produced based on definite parameters.

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