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On Pseudo δ-Open Fuzzy Sets and Pseudo Fuzzy δ-Continuous Functions

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Abstract

In this paper, introducing new fuzzy topologies on a fts (X, τ) their interrelations with τ is discussed. The notion of pseudo fuzzy δ-continuous function is also introduced and studied in terms of these new topologies.

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1 Introduction and Preliminaries

In this article, a couple of new fuzzy topologies are introduced with the help of strong α-level topology on a fts (X, τ). In the first section, we discuss their interrelations with the given fuzzy topology τ. Finally, introducing the notion of pseudo fuzzy δ-continuous function we characterize them in terms of these newly developed fuzzy topologies.

Let X be a non empty set and I be the closed interval $[0, 1]$. A fuzzy set μ

[7] on X is a function on X into I and the collection of all fuzzy sets on X is denoted by I^X. The support of a fuzzy set μ, denoted by $supp\mu$, is the crisp set $\{x \in X : \mu(x) > 0\}$. A fuzzy set with a singleton as its support is called a fuzzy point, denoted by x_α, and defined as,

$$x_\alpha(z) = \begin{cases} \alpha, & for \text{ z = x} \\ 0, & otherwise \end{cases}$$

A collection $\tau \subseteq I^X$ is called a *fuzzy topology* [2] on X if
(i) $0, 1 \in \tau$
(ii) $\forall \mu_1, \mu_2, ..., \mu_n \in \tau \Rightarrow \wedge\mu_i \in \tau$
(iii) $\mu_\alpha \in \tau$, $\forall \alpha \in \Lambda$ (where Λ is an index set) $\Rightarrow \vee\mu_\alpha \in \tau$
Then (X, τ) is called a fuzzy topological space (fts, for short).
Let X and Y be two nonempty sets and $f : X \to Y$ be any function. If A and B are fuzzy sets on X and Y respectively then $f(A)$ and $f^{-1}(B)$ are respective fuzzy sets on Y and X, given by [7],

$$f(A)(y) = \begin{cases} sup\{A(x) : x \in f^{-1}(y)\}, & iff^{-1}(y) \neq \phi \\ 0, & otherwise \end{cases}$$

and

$$f^{-1}(B)(x) = B(f(x))$$

Let f be a function from a fts X to a fts Y. Then f is fuzzy continuous (fuzzy δ-continuous) iff $f^{-1}(U)$ is fuzzy open (respectively, fuzzy δ-open) set in X for each fuzzy open (respectively, fuzzy δ-open) set U in Y. [5]
For each $i \in \Lambda$, if $f_i : X \to (Y_i, \tau_i)$ are the functions from a set X into fts (Y_i, τ_i), then the smallest fuzzy topology on X for which the functions $f_i, i \in \Lambda$ are fuzzy continuous is called initial fuzzy topology on X generated by the collection of functions $\{f_i, i \in \Lambda\}$. [4]
For a fuzzy set μ in X, the set $\mu^\alpha = \{x \in X : \mu(x) > \alpha\}$ is called the strong α-level set of X. In a fts $(X, \tau), w(\tau)$ denotes the collection of all lower semi continuous functions from X into I, i.e $w(\tau) = \{\mu : \mu^{-1}(a, 1] \in \tau, \forall a \in [0, 1]\}$ is a laminated fuzzy topology on X. In a fuzzy topological space (X, τ), for each $\alpha \in I_1 = [0, 1)$, the collection $i_\alpha(\tau) = \{\mu^{-1}(\alpha, 1] : \mu \in \tau\}$ is a topology on X and is called strong α-level topology. [4]
A fuzzy set μ on a fts (X, τ) is called fuzzy regular open if $\mu = int(cl\mu)$ and its complement is fuzzy regular closed. A fuzzy δ-open set is the union of some fuzzy regular open sets.

2 New fuzzy topologies from old

We begin this section with an example showing that in a fts (X, τ) if μ is fuzzy regular open then μ^α need not be regular open in the corresponding topological space $(X, i_\alpha(\tau)), \alpha \in I_1$ and also μ^α may be regular open in $(X, i_\alpha(\tau)), \alpha \in I_1$ inspite of μ not being fuzzy regular open in the fts (X, τ).

Example 2.1 Let X be a set with at least two elements. Fix an element $y \in X$. Clearly, $\tau = \{0, 1, A\}$ is a fuzzy topology on X, where A is defined as
$$A(x) = \begin{cases} 0.5, & \text{for } x = y \\ 0.3, & otherwise. \end{cases}$$
The fuzzy closed sets in (X, τ) are $0, 1$ and $1 - A$ where
$$(1 - A)(x) = \begin{cases} 0.5, & \text{for } x = y \\ 0.7, & otherwise. \end{cases}$$
Clearly $A \leq 1 - A$ and hence $int(clA) = A$. i.e A is fuzzy regular open in (X, τ). Now, in the corresponding topological space $(X, i_\alpha(\tau))$, $\alpha \in I_1$, the open sets are Φ, X and A^α where $A^\alpha = \begin{cases} X, & \text{for } \alpha < 0.3 \\ \{y\}, & \text{for } 0.3 \leq \alpha < 0.5 \\ \Phi, & \text{for } \alpha \geq 0.5. \end{cases}$
For, $0.3 \leq \alpha < 0.5$ the closed sets in $(X, i_\alpha(\tau))$ are Φ, X and $X - \{y\}$. It is clear that $int(clA^\alpha) = X$. Hence, A^α is not regular open in $(X, i_\alpha(\tau))$ for $0.3 \leq \alpha < 0.5$.

Example 2.2 Let $X = \{x, y, z\}$. Define fuzzy sets μ, γ and η as follows: $\mu(a) = 0.4$, $\gamma(a) = 0.55$ and $\eta(a) = 0.6$, $\forall a \in X$. If $\tau = \{0, 1, \mu, \gamma, \eta\}$ then (X, τ) is a fts. The closed fuzzy sets are $(1-\mu) = \eta$, $(1-\gamma)(a) = 0.45$, $\forall a \in X$ and $(1 - \eta) = \mu$. Here, $cl(\gamma) = \eta$ and $int(cl\gamma) = \eta$. Hence γ is not fuzzy regular open fuzzy set. But, $\gamma^\alpha = \{x : \gamma(x) > \alpha\}$. For $\alpha \geq 0.55$, $\gamma^\alpha = \Phi$, which is regular open. For $\alpha < 0.55$, $\gamma^\alpha = \{x, y, z\} = X$, which is also regular open. Hence for all $\alpha \in I_1, \gamma^\alpha$ is regular open in$(X, i_\alpha(\tau))$.

In view of these examples we define the following:

Definition 2.1 *A fuzzy open set (fuzzy closed set) μ on a fts (X, τ) is said to be pseudo regular open (respectively, pseudo regular closed) fuzzy set if the strong α-level set μ^α is regular open (respectively, regular closed) in $(X, i_\alpha(\tau))$, $\forall \alpha \in I_1$.*

The following example establishes that pseudo regular closed and pseudo regular open fuzzy sets are not complements of each other.

Example 2.3 Let $X = \{x, y, z, w\}, \tau = \{0, 1, \mu\}$ where μ is defined as $\mu(x) = 0.1, \mu(y) = 0.2, \mu(z) = 0.3, \mu(w) = 0.4$. Clearly, (X, τ) is a fts.
If $\alpha = 0.3$, $i_\alpha(\tau) = \{\Phi, X, \mu^\alpha\}$ and $\mu^\alpha = \{x \in X : \alpha < \mu(x) \leq 1\} = \{w\}$.
Closed sets in $(X, i_\alpha(\tau))$ are Φ, X and $\{x, y, z\}$. Here we shall show that μ is not pseudo regular open but its complement $(1 - \mu)$ is pseudo regular closed fuzzy set in (X, τ). As the smallest closed set containg μ^α is X, $cl(\mu^\alpha) = X$.

So $int(cl\mu^\alpha) = X \neq \mu^\alpha$. Hence μ^α is not regular open in $(X, i_\alpha(\tau))$.
This shows that μ is not pseudo regular open in (X, τ).
Here, $(1-\mu)(x) = 0.9, (1-\mu)(y) = 0.8, (1-\mu)(z) = 0.7, (1-\mu)(w) = 0.6$.
$(1-\mu)^\alpha = \{x \in X : \alpha < (1-\mu)(x) \leq 1\}$.
For, $\alpha \geq 0.6,\ i_\alpha(\tau) = \{\Phi, X\}$ and so $cl[int(1-\mu)^\alpha] = X$.
Also for $\alpha < 0.6, (1-\mu)^\alpha = X$ and so $cl[int(1-\mu)^\alpha] = X$, whatever be $i_\alpha(\tau)$.
Hence, in any case $(1-\mu)^\alpha$ is regular closed in $(X, i_\alpha(\tau))$. This shows $(1-\mu)$ is pseudo regular closed fuzzy set in (X, τ).

Definition 2.2 *A fuzzy set μ on a fts (X, τ) is said to be pseudo δ-open (respectively, pseudo δ-closed) fuzzy set if the strong α-level set μ^α is δ-open (respectively, δ-closed) in $(X, i_\alpha(\tau))$, $\forall \alpha \in I_1$.*

Theorem 2.1 *The collection of all pseudo δ-open fuzzy sets on a fts (X, τ) forms a fuzzy topology on X.*
proof. Straightforward.

Definition 2.3 *The fuzzy topology as obtained in the above theorem is called pseudo δ-fuzzy topology (in short ps-δ fuzzy topology) on X. The complements of the members of ps-δ fuzzy topology are known as $ps-\delta$-closed fuzzy sets.*

Theorem 2.2 *In a fts (X, τ) union of pseudo regular open fuzzy sets is pseudo δ-open.*
Proof. Let $\mu = \vee\{\mu_i : i \in \Lambda\}$, where μ_i is pseudo regular open fuzzy sets in a fts (X, τ), for each $i \in \Lambda$. Here, μ_i^α is regular open in $(X, i_\alpha(\tau)), \forall i \in \Lambda$. As, $(\vee\mu_i)^\alpha = \cup\mu_i^\alpha$ is δ-open in $(X, i_\alpha(\tau)), \forall \alpha \in I_1$, μ is pseudo δ-open fuzzy set in (X, τ).

The following example shows that the converse of the Theorem(2.2) is not true in general. i.e Any pseudo δ-open fuzzy set on a fts (X, τ) need not be expressible as union of pseudo regular open fuzzy sets.

Example 2.4 Let $X = \{x, y, z\}$ and the topology τ generated by μ , γ , η where $\mu(x) = 0.4, \mu(y) = 0.4, \mu(z) = 0.5$, $\gamma(x) = 0.4$,$\gamma(y) = 0.6$, $\gamma(z) = 0.4$ and $\eta(x) = 0.5$, $\eta(y) = 0.5$, $\eta(z) = 0.6$.
Consider $i_\alpha(\tau)$, for each α as follows:
Case 1: For $\alpha < 0.4$, $\mu^\alpha = \gamma^\alpha = \eta^\alpha = X$ and hence $i_\alpha(\tau) = \{X, \Phi\}$. Consequently, $\mu^\alpha, \gamma^\alpha$ and η^α are all regular open.
Case 2: For $\alpha \geq 0.6$, $\mu^\alpha = \gamma^\alpha = \eta^\alpha = \Phi$ and hence $i_\alpha(\tau) = \{X, \Phi\}$. Consequently, $\mu^\alpha, \gamma^\alpha$ and η^α are all regular open.
Case 3: For $0.4 \leq \alpha < 0.5$, $\mu^\alpha = \{z\}$, $\gamma^\alpha = \{y\}$, $\eta^\alpha = X$ and hence $i_\alpha(\tau) = \{X, \Phi, \{y\}, \{z\}, \{y, z\}\}$. We observe that $int(cl\mu^\alpha) = \mu^\alpha$, $int(cl\gamma^\alpha) = \gamma^\alpha$ and $int(cl\eta^\alpha) = \eta^\alpha$, proving all of them to be regular open.

But $(\mu \vee \gamma)^\alpha = \{y, z\}$ is not regular open as $int(cl\{y, z\}) = X \neq \{y, z\}$.
Case 4: For $0.5 \leq \alpha < 0.6$, $\mu^\alpha = \Phi$, $\gamma^\alpha = \{y\}$, $\eta^\alpha = \{z\}$ and hence $i_\alpha(\tau) = \{X, \Phi, \{y\}, \{z\}, \{y, z\}\}$. In this case too all μ^α,γ^α and η^α are regular open but $(\gamma \vee \eta)^\alpha = \{y, z\}$ is not so.
Now we consider a fuzzy set K on X as follows: $K(x) = 0.4$, $K(y) = 0.6$ and $K(z) = 0.6$. Clearly $K^\alpha = \begin{cases} X, & \text{for } \alpha < 0.4 \\ \{y, z\}, & \text{for } 0.4 \leq \alpha < 0.6 \\ \Phi, & \text{for } \alpha \geq 0.6. \end{cases}$

Hence, $K^\alpha = \begin{cases} X, & \text{for } \alpha < 0.4 \\ \mu^\alpha \cup \gamma^\alpha, & \text{for } 0.4 \leq \alpha < 0.5 \\ \gamma^\alpha \cup \eta^\alpha, & \text{for } 0.5 \leq \alpha < 0.6 \\ \Phi, & \text{for } \alpha \geq 0.6. \end{cases}$

and so, K^α is δ-open in $(X, i_\alpha(\tau))$, $\forall \alpha \in I_1$. Therefore, K is pseudo δ-open fuzzy set in (X, τ). It can be shown easily that K is neither a pseudo regular open fuzzy set in (X, τ) nor is expressible as union of pseudo regular open fuzzy sets in (X, τ).

It is clear from the above example that $A^\alpha = B^\alpha$, for some $\alpha \in I_1$ does not imply $A = B$. However we have the following Theorem:

Theorem 2.3 *If A and B are two fuzzy sets in a fts (X, τ) such that $A^\alpha = B^\alpha$, $\forall \alpha \in I_1$ then $A = B$.*
Proof. For $\alpha = 0, A^0 = B^0 \Rightarrow A(x) > 0$ iff $B(x) > 0$. Hence $A(x) = 0$ iff $B(x) = 0$. Suppose, $y \in Y$ is such that $A(y) > 0, B(y) > 0$ and $A(y) \neq B(y)$. Let $A(y) = \alpha_1$ and $B(y) = \alpha_2$. Without any loss of generality let us take $\alpha_1 > \alpha_2$. Since $A(y) = \alpha_1 > \alpha_2$, $y \in A^{\alpha_2}$, but $y \notin B^{\alpha_2}$ i.e $A^{\alpha_2} \neq B^{\alpha_2}$, which is a contradiction. Hence, forall $y \in Y$, $A(y) = B(y)$ i.e. $A = B$.

Theorem 2.4 *If $\{\mu_i\}$be a collection of all pseudo regular open fuzzy sets on a fts (X, τ), then*

1. $0, 1 \in \{\mu_i\}$.
2. $\forall \mu_1, \mu_2 \in \{\mu_i\} \Rightarrow \mu_1 \wedge \mu_2 \in \{\mu_i\}$.

Proof. 0 and 1 are pseudo regular open fuzzy sets in a fts (X, τ). Hence $0, 1 \in \{\mu_i\}$. Let $\mu_1, \mu_2 \in \{\mu_i\}$. $\Rightarrow$ $\mu_1^\alpha, \mu_2^\alpha$ are regular open in $(X, i_\alpha(\tau))$.
$\Rightarrow \mu_1^\alpha \cap \mu_2^\alpha$ is regular open in $(X, i_\alpha(\tau))$.
$\Rightarrow (\mu_1 \wedge \mu_2)^\alpha = \mu_1^\alpha \cap \mu_2^\alpha$ is regular open in $(X, i_\alpha(\tau))$.
$\Rightarrow \mu_1 \wedge \mu_2$ is pseudo regular open fuzzy set in (X, τ).

Remark 2.1 In view of Theorem (2.4) the collection of all pseudo regular open fuzzy sets on (X, τ) generates a fuzzy topology called pseudo regular open fuzzy topology (in short $ps - ro$ fuzzy topology) on X. The members of this topology are termed as $ps - ro$ open fuzzy sets and their complements as $ps - ro$ closed fuzzy sets.

Theorem 2.5 *In a fts (X, τ), $ps - ro$ fuzzy topology is coarser than τ.*
Proof. Straightforward.

Theorem 2.6 *In a fts (X, τ), $ps - ro$ fuzzy topology is coarser than $ps - \delta$ fuzzy topology.*
Proof. Let $\mu \in ps - ro$ fuzzy topology. So, $\mu = \bigvee_i \gamma_i$ where γ_i's are pseudo regular open fuzzy sets on (X, τ). Hence $\mu^\alpha = (\bigvee_i \gamma_i)^\alpha = \bigcup_i \gamma_i^\alpha$ is δ-open in $(X, i_\alpha(\tau))$, $\forall \alpha \in I_1$. This shows that $\mu \in ps - \delta$ fuzzy topology on X.

Remark 2.2 In view of Example (2.4), in general $ps - ro$ fuzzy topology is strictly coarser than $ps - \delta$ fuzzy topology in a fts (X, τ).

3 Pseudo fuzzy δ-continuous functions

Definition 3.1 *A function f from a fts X to a fts Y is pseudo fuzzy δ-continuous iff $f^{-1}(U)$ is pseudo δ-open fuzzy set in X for each pseudo δ-open fuzzy set U in Y.*

Theorem 3.1 *A function $f : (X, \tau) \to (Y, \sigma)$ is pseudo fuzzy δ-continuous then $f : (X, i_\alpha(\tau)) \to (Y, i_\alpha(\sigma))$, is δ-continuous for each $\alpha \in I_1$, where $(X, \tau), (Y, \sigma)$ are fts.*
Proof. Let v be a δ-open set in $i_\alpha(\sigma)$. As every δ-open set is open, $v \in i_\alpha(\sigma)$ and so there exist $\mu \in \sigma$ such that $v = \mu^\alpha$. Now,

$$\begin{aligned} f^{-1}(\mu^\alpha) &= \{x \in X : f(x) \in \mu^\alpha\} \\ &= \{x \in X : \mu(f(x)) > \alpha\} \\ &= \{x \in X : (\mu f)(x) > \alpha\} \\ &= \{x \in X : (f^{-1}(\mu))(x) > \alpha\} \\ &= \{x \in X : x \in (f^{-1}(\mu))^\alpha\} \\ &= (f^{-1}(\mu))^\alpha \end{aligned}$$

Consider a fuzzy set ζ on X given by

$$\zeta(x) = \begin{cases} 1 & if\ \mu(x) > \alpha \\ \alpha & otherwise \end{cases}$$

Then

$$\zeta^\beta = \begin{cases} \mu^\alpha & if\ \beta \geq \alpha \\ Y & \beta < \alpha \end{cases}$$

Consequently, ζ^β is δ-open for all $\beta \in I_1$. Hence, ζ is a pseudo δ-open on Y. Since, f is pseudo fuzzy δ-continuous, $f^{-1}(\zeta)$ is pseudo δ-open fuzzy set in X. Now, $(f^{-1}(\zeta))^\alpha = f^{-1}(\zeta^\alpha) = f^{-1}(\mu^\alpha) = f^{-1}(v)$. Hence $f^{-1}(v)$ is δ-open set whenever v is so. This proves that f is δ-continuous for each $\alpha \in I_1$.

Theorem 3.2 *A function $f : (X, i_\alpha(\tau)) \to (Y, i_\alpha(\sigma))$, is δ-continuous for each $\alpha \in I_1$, where $(X, \tau), (Y, \sigma)$ are fts then $f : (X, \tau) \to (Y, \sigma)$ is pseudo fuzzy δ-continuous.*

Proof. Let μ be any fuzzy pseudo δ-open set in (Y, σ). μ^α is δ-open in $(Y, i_\alpha(\sigma))$. By the δ-continuity of $f : (X, i_\alpha(\tau)) \to (Y, i_\alpha(\sigma)), f^{-1}(\mu^\alpha) = (f^{-1}(\mu))^\alpha$ is δ-open in $(X, i_\alpha(\tau))$. Hence $f^{-1}(\mu)$ is pseudo δ-open fuzzy set in (X, τ), proving f to be pseudo fuzzy δ-continuous.

Combining Theorems (3.1) and (3.2) we get:

Theorem 3.3 *A function $f : (X, \tau) \to (Y, \sigma)$ is pseudo fuzzy δ-continuous iff $f : (X, i_\alpha(\tau)) \to (Y, i_\alpha(\sigma))$, is δ-continuous for each $\alpha \in I_1$, where $(X, \tau), (Y, \sigma)$ are fts.*

Theorem 3.4 *If a function $f : (X, \tau) \to (Y, \sigma)$ is pseudo fuzzy δ-continuous then $f^{-1}(\mu)$ is pseudo δ-closed fuzzy set in (X, τ), for all pseudo δ-closed fuzzy set μ in (Y, σ).*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be pseudo fuzzy δ-continuous.
$\Rightarrow f : (X, i_\alpha(\tau)) \to (Y, i_\alpha(\sigma))$ is δ-continuous for each $\alpha \in I_1$. Now μ is pseudo δ-closed fuzzy set in (Y, σ). Hence, $\forall \alpha \in I_1$, μ^α is δ-closed in $(Y, i_\alpha(\sigma))$, that is $(Y - \mu^\alpha)$ is δ-open in $(Y, i_\alpha(\sigma))$. Now,

$$\begin{aligned} f^{-1}(Y - \mu^\alpha) &= \{x \in X : f(x) \notin \mu^\alpha\} \\ &= \{x \in X : \mu(f(x)) \leq \alpha\} \\ &= \{x \in X : (\mu f)(x)) \leq \alpha\} \\ &= X - \{x \in X : (\mu f)(x)) > \alpha\} \\ &= X - \{x \in X : (f^{-1}(\mu))(x) > \alpha\} \\ &= X - (f^{-1}(\mu))^\alpha \end{aligned}$$

As, $f^{-1}(Y - \mu^\alpha)$ is δ-open, $(f^{-1}(\mu))^\alpha$ is δ-closed in $(X, i_\alpha(\tau))$, $\forall \alpha \in I_1$. Hence $f^{-1}(\mu)$ is pseudo δ-closed fuzzy set in (Y, σ).

As a pseudo δ-closed set need not be the complement of a pseudo δ-open set, the converse of the Theorem (3.4) may not hold true. However, the following theorem characterizes pseudo fuzzy δ-continuous functions in terms of $ps - \delta$-closed fuzzy sets.

Theorem 3.5 *A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is pseudo fuzzy δ-continuous iff $f^{-1}(\mu)$ is $ps - \delta$-closed fuzzy set in a fts (X, τ), where μ is ps-δ-closed fuzzy set in a fts (Y, σ).*

Proof. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be pseudo fuzzy δ-continuous.
Then $f : (X, i_\alpha(\tau)) \rightarrow (Y, i_\alpha(\sigma))$ is δ-continuous for each $\alpha \in I_1$.
Let μ be $ps - \delta$-closed fuzzy set in a fts (Y, σ).
$\Rightarrow 1 - \mu$ is pseudo δ-open fuzzy set in (Y, σ).
$\Rightarrow (1 - \mu)^\alpha$ is δ-open and so $f^{-1}((1 - \mu)^\alpha) = (f^{-1}(1 - \mu))^\alpha$ is δ-open fuzzy set in $(X, i_\alpha(\tau)), \forall \alpha \in I_1$. This shows that $f^{-1}(1 - \mu)$ is pseudo δ-open fuzzy set in (X, τ). Now,

$$\begin{aligned}
(1 - f^{-1}(1 - \mu))(x) &= 1 - f^{-1}(1 - \mu)(x) \\
&= 1 - (1 - \mu)(f(x)) \\
&= \mu(f(x)) \\
&= f^{-1}(\mu)(x).
\end{aligned}$$

Hence, $f^{-1}(\mu)$ is $ps - \delta$-closed fuzzy set in (X, τ).
Conversely, Let μ be any pseudo δ-open and so $(1 - \mu)$ is $ps - \delta$-closed fuzzy set in (Y, σ). As, $f^{-1}(1 - \mu)$ is $ps - \delta$-closed, $1 - f^{-1}(1 - \mu)$ is pseudo δ-open fuzzy set in $(X, i_\alpha(\tau)), \forall \alpha \in I_1$. Again, $f^{-1}(\mu) = 1 - f^{-1}(1 - \mu), f^{-1}(\mu)$ is pseudo δ-open fuzzy set in (X, τ). Hence f is pseudo fuzzy δ-continuous.

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