

# Resolving a Misconception in Combining Inequalities

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## Abstract

This article addresses a common misconception about combining inequalities. Inequality is used in algebra to show magnitude comparison, as well as in set theory and mathematical programming to represent a set of values satisfying inequality. It is a widespread misconception that two inequalities can be combined under any circumstances. In this article, we prove that when inequality represents a set of values, combining inequalities would fail to maintain the set they represent and would result in a larger set. Therefore, even though it is valid when inequality is used to represent relation of magnitude comparison, combining inequalities is not valid when inequality is used to represent a set of values. Six examples are diagnosed in applications of solving simultaneous equations and constructing constraints in linear and integer programming, and pitfalls due to invalidly combining inequalities are indicated in those examples.

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## 1. Introduction

We in this article investigate a common misconception about combining inequalities.

Inequality  $A \geq B$  indicates amount  $A$  is greater than or equal to amount  $B$ . This is what inequality means in *algebra* [4]. For example,  $5 > 3$  and  $4 = 4$ . Another example is  $x > 7$  which tells value  $x$  is larger than 7.

Inequality is also used to represent a set of values [5]. Let  $f(x)$  be a function of variable  $x$ . Inequality  $f(x) \geq 0$  represents a set of numbers that for each member  $x$  in that set, inequality  $f(x) \geq 0$  holds. For example, if  $f(x) = x + 2$ , then inequality  $x + 2 \geq 0$  represents a set of values of  $x$ , which is  $\{x + 2 \geq 0\} = \{x \geq -2\}$ . If  $f(x) = 3x - 15$ , then inequality  $3x - 15 \geq 0$  refers to a set of values of  $x$  which is  $\{3x - 15 \geq 0\} = \{x \geq 5\}$ .

From inequalities  $A \geq B$  and  $C \geq D$ , we have  $A + C \geq B + D$  by adding the two larger parts,  $A$  and  $C$ , and adding two smaller parts,  $B$  and  $D$ . This is so-called *combining inequalities*.  $A + C \geq B + D$  is called the *combination of inequalities*  $A \geq B$  and  $C \geq D$ .

A widespread misconception is that two inequalities can always be combined under any circumstances with a plausible reason that sum of two larges is greater than sum of two smalls. There are many examples seemingly supporting the misconception. Given  $4 > 2$  and  $4 > 3$  which both hold, for instance, we can combine them to have a new inequality  $8 > 5$  which also holds. And combining  $x > 2$  and  $y > 4$ , we have  $x + y > 6$ . If  $x > 2$  and  $y > 4$  hold, then  $x + y > 6$  also holds, because if a number  $x$  is greater than 2 and another number  $y$  is greater than 4, then sum of  $x$  and  $y$  must be greater than sum of 2 and 4.

However, despite it looks valid in many examples, combining inequalities may lead to weird outcomes in some cases. Suppose that both inequalities  $x > 2$  and  $x > 3$  hold true. After combining them we have  $x > 2.5$ . If the combination were valid, then a value satisfying  $x > 2.5$  would satisfy  $x > 2$  and  $x > 3$ . But it is not true. For example, value  $x = 2.8$  satisfies  $x > 2.5$ , but it does not satisfy  $x > 3$ ! It seems that the operation of combining inequalities has “surreptitiously” altered something that inequalities represent.

This article scrutinizes the misconception that underlies the weird outcomes of combining inequalities and clarifies the covert misunderstandings. We prove in Section 2 that it is not valid to combine inequalities in the circumstance that inequality represents a set of values, despite that it is valid when inequality is used for comparison of the sizes of numbers. Particularly, combination of inequalities may “expand” the set represented by the inequalities. In Section 3, we diagnose six examples and pathologically point out the misconceptions that give rise to the

fallacious outcomes. Section 4 summarizes the applications of inequalities where combination of inequalities is prohibitive.

## 2. Consequences of Combining Inequalities that Represent Sets

If inequality is used for comparison of numbers, as in algebra, combining inequalities is valid. For example, if  $A > B$  and  $C > D$  hold, with  $A, B, C,$  and  $D$  being numbers, then  $A + C > B + D$  holds. It is no brainer to see it is true since sum of two larges is greater than sum of two smalls.

However, in the circumstance that inequality is used for representing a set of numbers, combining inequalities is no longer a valid operation. In this section, we explore the consequences of combining two inequalities representing sets.

Let inequality  $f(x) \geq 0$  represent set  $\{f(x) \geq 0\}$  which contains all the values of  $x$  satisfying  $f(x) \geq 0$ . Let  $g(x) \geq 0$  represent set  $\{g(x) \geq 0\}$  which contains all the values of  $x$  satisfying  $g(x) \geq 0$ . Simultaneous inequalities,  $f(x) \geq 0$  and  $g(x) \geq 0$ , represent the set of  $x$  values,  $\{f(x) \geq 0\} \cap \{g(x) \geq 0\}$ , in which every  $x$  satisfies both  $f(x) \geq 0$  and  $g(x) \geq 0$ . We call the set  $\{f(x) \geq 0\} \cap \{g(x) \geq 0\}$  intersection set of  $\{f(x) \geq 0\}$  and  $\{g(x) \geq 0\}$ .

By combining inequalities  $f(x) \geq 0$  and  $g(x) \geq 0$ , we have inequality  $f(x) + g(x) \geq 0$ . Set  $\{f(x) + g(x) \geq 0\}$  contains all  $x$ 's that satisfy the combination of  $f(x)$  and  $g(x)$ , i.e.  $\{f(x) + g(x) \geq 0\}$  arises from combining inequalities  $f(x) \geq 0$  and  $g(x) \geq 0$ . We call the set  $\{f(x) + g(x) \geq 0\}$  combination set.

We show in this section that the set represented by two simultaneous inequalities does not remain the same after combining the inequalities. That is, intersection set  $\{f(x) \geq 0\} \cap \{g(x) \geq 0\}$  is different from combination set  $\{f(x) + g(x) \geq 0\}$  in general. Therefore, combining inequality is not a valid operation in the circumstance of sets.

Lemma 1 shows that intersection set is a subset of combination set. That is, the set associated with two simultaneous inequalities is enclosed in the set associated with the combined inequality.

### Lemma 1.

$$\{f(x) \geq 0\} \cap \{g(x) \geq 0\} \subseteq \{f(x) + g(x) \geq 0\}.$$

*Proof:*

To prove  $A \subseteq B$ , we need to show that if an element in set  $A$  then it is in set  $B$ .

Suppose an element  $x=c$  is in set  $\{f(x) \geq 0\} \cap \{g(x) \geq 0\}$ . So, it must hold that  $f(c) \geq 0$  and  $g(c) \geq 0$ . Note that  $f(c) \geq 0$  and  $g(c) \geq 0$  here just tell the fact that

amounts  $f(c)$  and  $g(c)$  are both greater than or equal to 0. Hence it must hold that  $f(c)+g(c)\geq 0$  because the sum of two larges is greater than the sum of two smalls. Since inequality  $f(c)+g(c)\geq 0$  holds,  $c$  is in set  $\{f(x)+g(x)\geq 0\}$ . That is, for any  $x=c$ , if it is in set  $\{\{f(x)\geq 0\}\cap\{g(x)\geq 0\}\}$ , then it must be in set  $\{f(x)+g(x)\geq 0\}$ .

Therefore,  $\{\{f(x)\geq 0\}\cap\{g(x)\geq 0\}\} \subseteq \{f(x)+g(x)\geq 0\}$ .

*End of proof.*

Lemma 2 below indicates that combination set  $\{f(x)+g(x)\geq 0\}$  is not a subset of intersection set  $\{\{f(x)\geq 0\}\cap\{g(x)\geq 0\}\}$ . That is, the set associated with the combined inequality is not a subset of that associated with the two inequalities before combination.

**Lemma 2.**

It is false that  $\{\{f(x)\geq 0\}\cap\{g(x)\geq 0\}\} \supseteq \{f(x)+g(x)\geq 0\}$ .

*Proof:*

To show a general statement false, we need one and only one counterexample for which the statement is false.

Let us have an example in which  $f(x)=x-2$  and  $g(x)=x-3$ .

With this example, the intersection set is:  $\{\{f(x)\geq 0\}\cap\{g(x)\geq 0\}\} = \{x-2\geq 0\}\cap\{x-3\geq 0\} = \{x-3\geq 0\}$ ; while the combination set is:  $\{f(x)+g(x)\geq 0\} = \{(x-2)+(x-3)\geq 0\} = \{2x-5\geq 0\} = \{x-2.5\geq 0\}$ .

Element  $x=2.7$  is in the combination set  $\{x-2.5\geq 0\}$ , but it is not in the intersection set  $\{x-3\geq 0\}$ . So, it is false that  $\{x-2.5\geq 0\} \subseteq \{x-3\geq 0\}$ . That is, for the example  $[f(x)=x-2$  and  $g(x)=x-3]$ , it is false that  $\{f(x)+g(x)\geq 0\} \subseteq \{\{f(x)\geq 0\}\cap\{g(x)\geq 0\}\}$ . Hence,  $[f(x)=x-2, g(x)=x-3]$  is a counterexample to disprove  $\{\{f(x)\geq 0\}\cap\{g(x)\geq 0\}\} \supseteq \{f(x)+g(x)\geq 0\}$ .

Therefore,  $\{\{f(x)\geq 0\}\cap\{g(x)\geq 0\}\} \supseteq \{f(x)+g(x)\geq 0\}$  is a false proposition.

*End of proof.*

Two sets are *identical*, if the elements contained in one set are contained in another set, and vice versa. Given two sets  $\{S\}$  and  $\{T\}$ , if  $\{S\} \subseteq \{T\}$  and  $\{T\} \subseteq \{S\}$ , then  $\{S\}$  is identical to  $\{T\}$ ; but if  $\{S\} \subseteq \{T\}$  but  $\{T\}$  not  $\subseteq \{S\}$ , then  $\{S\}$  and  $\{T\}$  are not identical. Now, we have Theorem 1 by putting Lemma 1 and Lemma 2 together.

**Theorem 1.**

Set  $\{\{f(x)\geq 0\}\cap\{g(x)\geq 0\}\}$  is not identical to set  $\{f(x)+g(x)\geq 0\}$ .

*Proof:*

Set  $\{A\}$  is identical to set  $\{B\}$  if and only if  $\{A\} \subseteq \{B\}$  and  $\{A\} \supseteq \{B\}$ . Although  $\{\{f(x)\geq 0\}\cap\{g(x)\geq 0\}\} \subseteq \{f(x)+g(x)\geq 0\}$  by Lemma 1, it does not hold that  $\{\{f(x)\geq 0\}\cap\{g(x)\geq 0\}\} \supseteq \{f(x)+g(x)\geq 0\}$  by Lemma 2. Therefore, we conclude that  $\{\{f(x)\geq 0\}\cap\{g(x)\geq 0\}\}$  is *not identical* to  $\{f(x)+g(x)\geq 0\}$ .

*End of proof.*

Theorem 1 indicates that intersection set and combination set are not identical. That is, the set associated with two simultaneous inequalities does not remain the same after combining the two inequalities. Combining two inequalities therefore is not valid in the circumstance where the inequalities represent sets, as shown in Corollary 1 below.

**Corollary 1.**

Combining inequalities does not maintain the set represented by the inequalities.

*Proof:*

This conclusion is directly from Theorem 1.

*End of proof.*

Let symbol  $|\{S\}|$  denote the *cardinality* of set  $\{S\}$ , which is the size of set  $\{S\}$  in terms of number of elements in  $\{S\}$ . Corollary 2 below is about the cardinalities of intersection set and combination set.

**Corollary 2.**

$|\{f(x)+g(x)\geq 0\}| \geq |\{\{f(x)\geq 0\}\cap\{g(x)\geq 0\}\}|$ , where  $|\{S\}|$  denotes the cardinality of set S.

*Proof:*

Since  $\{\{f(x)\geq 0\}\cap\{g(x)\geq 0\}\} \subseteq \{f(x)+g(x)\geq 0\}$  by Lemma 1, hence  $|\{f(x)+g(x)\geq 0\}| \geq |\{\{f(x)\geq 0\}\cap\{g(x)\geq 0\}\}|$ .

*End of proof.*

This corollary says that, after two inequalities are combined, the combination set could be larger, but never smaller, than the intersection set. The set associated with the combined inequality is larger than or equal to the set associated with the two simultaneous inequalities before the combination.

Next corollary explicitly indicates that all the above lemmas, theorems, and corollaries about inequalities apply to equations.

**Corollary 3.**

Lemma 1, Lemma 2, Theorem 1, Corollary 1, and Corollary 2 apply to not only inequalities but also equations.

*Proof:*

Equations  $f(x)=0$  and  $g(x)=0$  are special cases of  $f(x)\geq 0$  and  $g(x)\geq 0$ , respectively. Therefore, all the lemmas and theorems about  $f(x)\geq 0$  and  $g(x)\geq 0$  are applicable to  $f(x)=0$  and  $g(x)=0$ .

*End of proof.*

### 3. Illustrative Examples - Diagnostic Analysis

In the last section, we have showed that combining inequalities is not valid in the circumstance where inequalities are used to represent sets of values. Between sets, *union* and *intersection* are valid operations, while “combining inequalities” is not at all.

The examples below come from solving simultaneous equation systems, linear programming, and integer programming. In these examples, combination of inequalities gives rise to fallacious outcomes. We use the theory developed in the last section to pathologically diagnose what has caused errors and confusions.

#### Example 1.

Consider simultaneous equations:

$$X^2-4 = 0, \quad (1.1)$$

$$X+2 = 0. \quad (1.2)$$

Combining them gives:

$$(X^2-4)+(X+2) = 0, \text{ which results in} \\ X^2+X-2 = 0. \quad (1.3)$$

Solutions to equation (1.3) are:

$$X=-2 \text{ and } X=1. \quad (1.4)$$

The system of simultaneous equations (1.1) and (1.2) has only one solution which is  $X=-2$ . After combining the equations, the new equation (1.3) has two solutions,  $X=-2$  and  $X=1$  as in (1.4), in which  $X=1$  is not a solution to the original system of (1.1) and (1.2).

Solving a system of simultaneous equations is to find *a set* of variable values satisfying all equations. Combining equations in this circumstance of dealing with “set” does not guarantee maintaining the original set of solutions defined by equations (1.1) and (1.2). By Lemma 1, the original solution set defined by (1.1) and (1.2), which is  $\{-2\}$ , is a subset of the solution set (1.4) which is derived from the combined equation (1.3), which is  $\{-2, 1\}$ . Combining two equations in this example expands the solution set from  $\{-2\}$  to  $\{-2, 1\}$ .  $X=1$  is a solution of (1.3), but not a solution to simultaneous (1.1) and (1.2). The extraneous solution comes from combining the two equations.

#### Example 2.

Given a system of simultaneous inequalities:

$$X>2, \quad (2.1)$$

$$X>3. \quad (2.2)$$

Combine (2.1) and (2.2) by adding them up:

$$2X>5 \quad (2.3)$$

After simplifying:

$$X > 2.5. \quad (2.4)$$

The solution set defined by simultaneous inequalities (2.1) and (2.2) is  $X > 3$ . After combining the two inequalities, we have  $X > 2.5$  as in (2.4), which is not the solution set of the original system (2.1) and (2.2). For example,  $X = 2.8$  satisfies (2.4), but it does not satisfy both (2.1) and (2.2).

If we took two inequalities  $[X > 2, X > 3]$  as a system of algebraic relations, we could legally add the two up and have  $X > 2.5$ , which would make sense that if a number  $X$  which made both inequalities  $X > 2$  and  $X > 3$  hold then it would make inequality  $X > 2.5$  hold. There is nothing wrong here in this sense of algebra. Inequality relation  $X > 2.5$  is an implication of relations  $X > 2$  and  $X > 3$ . For instance,  $X = 3.2$  makes both  $X > 2$  and  $X > 3$  hold, so it makes  $X > 2.5$  hold.

However, when we consider “*solutions*” of inequalities (2.1) and (2.2), those inequalities represent “sets” instead of just algebraic relations. In this circumstance, combining two inequalities is no longer a valid operation as indicated in Corollary 1. Combining two inequalities does not maintain equivalence of sets according to Theorem 1. In this example, the original set is  $\{X > 2 \text{ and } X > 3\} = \{X > 2\} \cap \{X > 3\} = \{X > 3\}$ . After adding the two inequalities up, the set becomes  $\{X + X > 2 + 3\} = \{X > 2.5\}$ .  $\{X > 3\} \neq \{X > 2.5\}$  because we carried out an illegal operation between sets. And  $\{X > 3\}$  is a subset of  $\{X > 2.5\}$  as Lemma 1 indicates.

Confusion in this example comes from our false assumption that combining inequalities is by default valid in any circumstances.

### Example 3.

Given:

$$5 \geq X \geq 2. \quad (3.1)$$

Split (3.1) into two inequalities:

$$5 \geq X, \quad (3.2)$$

$$X \geq 2. \quad (3.3)$$

Multiply (3.3) by  $(-1)$ , and flip the resulted inequality up:

$$-2 \geq -X. \quad (3.4)$$

Add (3.2) and (3.4) up:

$$5 - 2 \geq X - X \quad (3.5)$$

After simplifying:

$$3 \geq 0. \quad (3.6)$$

(3.6) is a tautology which holds for any  $X$ . If all the operations in (3.2) through (3.5) were valid, then tautology (3.6) would be equivalent to (3.1). But it is obviously not, since tautology (3.6) does not have the restrictive information that  $X$  is between 2 and 5 as given in (3.1).

It is the operation “adding (3.2) and (3.4) up” that has caused the fallacy and made the restrictive information lost. Each inequality in this example represents a set rather than simply a comparison between two numbers. Inequality (3.1) defines the set of  $X$ 's values which is  $[2, 5]$ . While tautology (3.6) is associated with the set of  $X$ 's values  $(-\infty, +\infty)$ . Combining inequalities in this example makes the set of  $X$ 's values expand from original  $[2, 5]$  to  $(-\infty, +\infty)$ . Obviously, set  $[2, 5]$  is a subset of set  $(-\infty, +\infty)$ , which is consistent with what Lemma 1 indicates.

#### Example 4.

This example is from integer programming.

Use inequalities with binary variables to represent this proposition:

If A is “on” then either B or C must be “on”, but not both; if A is “off” then B and C can be either “on” or “off”. (4.1)

Three binary variables are defined:

$X_a=1$  if A is on,  $X_a=0$  if A is off.

$X_b=1$  if B is on,  $X_b=0$  if B is off.

$X_c=1$  if C is on,  $X_c=0$  if C is off.

Proposition (4.1) is represented by two inequalities with binary variables  $X_a$ ,  $X_b$ , and  $X_c$ :

$$X_a \leq X_b + X_c, \quad (4.2)$$

$$X_b + X_c \leq 1. \quad (4.3)$$

Combine (4.2) and (4.3):

$$X_a + X_b + X_c \leq X_b + X_c + 1. \quad (4.4)$$

After simplifying:

$$X_a \leq 1. \quad (4.5)$$

Simultaneous inequalities (4.2) and (4.3) are correct representation of proposition (4.1). But after combining (4.2) and (4.3), we have  $X_a \leq 1$  as in (4.5), which obviously does not represent the same set defined by (4.2) and (4.3). The original restrictive information about B and C in (4.1) is lost in (4.5). Inequality (4.5) does not represent restriction (4.1).

Mistake here is caused by combining inequalities (4.2) and (4.3). The feasible solution region defined by (4.2) and (4.3) expands to (4.5) in the process of combining the two inequalities, as indicated in Lemma 1 and Corollary 2.

#### Example 5.

In this example, an infeasible linear program becomes feasible after combining constraints.

Given constraints of a linear program:

$$X + 0.5Y \leq 1, \quad (5.1)$$

$$2X+Y \geq 3, \quad (5.2)$$

Multiply (5.1) by (-1):

$$-X-0.5Y \geq -1. \quad (5.3)$$

Add (5.2) and (5.3) up:

$$X+0.5Y \geq 2 \quad (5.4)$$

It can be easily verified that the linear program with constraints (5.1) and (5.2) is infeasible since (5.1) and (5.2) are inconsistent. However, after combining (5.1) and (5.2), the feasible solution region represented by (5.4) is obviously non-empty, and therefore the linear program becomes feasible.

Inequalities (5.1) and (5.2) define an empty set  $\emptyset$  of  $X$ 's feasible values. Combining the two inequalities is an invalid operation since the inequalities represent "sets". After combining the inequalities, the original empty feasible region becomes non-empty. An empty set  $\emptyset$  is a subset of any non-empty set, hence what happens in this example is in accordance with what is indicated in Lemma 1.

#### Example 6.

Given constraints of a linear program:

$$X+2Y \leq 6, \quad (6.1)$$

$$2X+Y \leq 6, \quad (6.2)$$

Adding (6.1) and (6.2):

$$3X+3Y \leq 12. \quad (6.3)$$

After simplification:

$$X+Y \leq 4. \quad (6.4)$$

After combining constraints (6.1) and (6.2), the feasible region of the linear program changes from (6.1) and (6.2) to (6.4). For example, solution ( $X=0, Y=4$ ) is not feasible according to (6.1) and (6.2), but it is feasible according to (6.4).

What happens here can be explained by Lemma 1: The set defined by simultaneous inequalities is a subset of the set defined by the combination of those inequalities. Hence, solution ( $X=0, Y=4$ ) is feasible for the combined constraint (6.4) but not feasible for the original constraints (6.1) and (6.2). In this example, combining inequalities makes the feasible region larger.

## 4. Summary and Remarks

An equation or inequality may bear two meanings. One is the relationship between two amounts. Another is representation of a set of values. For example,  $x=y$  tells  $x$ 's value is same as  $y$ 's value, or it tells the *set* of points in an  $x$ - $y$  coordinate system such that  $x$ 's value is same as  $y$ 's value.

Given that  $x \geq a$  and  $y \geq b$  hold, it seems a no-brainer that  $x+y \geq a+b$  holds since sum of two larges is greater than sum of two smalls. However, as we have proved in Section 2 that if  $x \geq a$  and  $y \geq b$  represent two sets  $\{x > a\}$  and  $\{y > b\}$ , then  $x+y \geq a+b$  does not make sense. In set theory, between two sets  $\{x \geq a\}$  and  $\{y \geq b\}$ , “*union*”  $\{x \geq a\} \cup \{y \geq b\}$  and “*intersection*”  $\{x \geq a\} \cap \{y \geq b\}$  are valid / legal operations [1], while “*combination*”  $\{x \geq a\} + \{y \geq b\}$  is not defined therefore not legal at all. Particularly, “*combination set*”  $\{x+y \geq a+b\}$  is not same as “*intersection set*”  $\{\{x \geq a\} \cap \{y \geq b\}\}$ , and  $\{\{x \geq a\} \cap \{y \geq b\}\}$  is a subset of  $\{x+y \geq a+b\}$ , as proved in Section 2.

We started learning equation and inequality as early as in elementary school in which comparison of two amounts was by default focused. Such a focus was maintained until analytical geometry and linear programming in which an equation or inequality is used to represent a set of values. Unfortunately, mathematical textbooks fail to warn the readers that it is “illegal” to combine inequalities when they represent sets. That gives rise to the widespread misconception that combining equation/inequality is always a valid operation. The six examples in Section 3 have shown the pitfalls of combining inequalities in solving simultaneous equations and formulating linear programs and integer programs.

Solving simultaneous equations is to figure out the set of solutions satisfying all equations. In those circumstances, equations are viewed as “sets”, therefore combining equations is not valid theoretically. Practically, we combine equations to eliminate variables in the solution process, which is dubbed as “*elimination method*”. One must bear in mind that combining equations may expand the solution set. Therefore, after solving simultaneous equations by using elimination method, it is necessary to check whether extraneous solutions have emerged in the solution process.

In linear/integer programming, feasible region is defined by inequalities. Every inequality represents a set [3]. Therefore, never combine constraints to simplify the problem, since combining inequalities will possibly alter the feasible solution region.

The problem of reasoning in a knowledge base or a database in artificial intelligence can be converted to an issue of verifying consistency of a system of simultaneous inequalities [2], which is a computationally hard NP-complete problem. Never combine inequalities for the purpose of simplification, since it is prohibitive in this circumstance where inequalities represent sets.

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