International Journal of Algebra, Vol. 20, 2026, no. 2, 81 - 87
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/ija.2026.92009

Consequence of Rank in Curve $y^2 = x^3 - pqx$

Shin-Wook Kim

Deokjin-gu, Songcheon 54823
I-Park Apt
Jeonju, Jeonbuk, Korea

Abstract

In curve $E_{-pq}: y^2 = x^3 - pqx$ with distinct primes p and q we can attain rank 2. The primes are gotten as various forms. The forms $p = Hu^4 + Iu^2v^2 + Kv^2$ and $q = H'u^4 + I'u^2v^2 + K'v^4$ are beginning of the forms. We pursue as many terms as possible in primes. In this article, we consider rank of curve with this point.

Mathematics Subject Classification: 11A41, 14G05

Keywords: Distinct odd primes, elliptic curve

1 Introduction

In [4] and [5] and [6] and [7] the author treated rank of elliptic curve $E_{-pq}: y^2 = x^3 - pqx$ where distinct odd primes p and q are consisted of 15 variables and 120 terms$\cdots\cdots (YY)$. The variables are $c, d, f, g, h, i, j, k, l, s, t, u, v, w, z$. The degree of c was 4 and other square parts were degree 2 and for multiplication part if there is variable c then, degrees were 2 and 1 respectively and if variable c is excluded then, degree was 1 in both variables in [4], [5], [6], [7]. Here, we access to primes differently. That is, we decide the degrees of squares are 2 and regard more terms as possible. Namely, we pursue more variables and more terms of primes p and $q \cdots\cdots (UU)$. As we gain primes (UU) an abstraction is strengthened in systematization of rank in E_{-pq}. In this article, we treat rank of $y^2 = x^3 - pqx$ where p and q are more terms than [4], [5], [6], [7]. For getting rank 2 in this curve primes are usually given as the forms $p \equiv 11(mod\ 16)$, $q \equiv 7(mod\ 16)$ or $p \equiv 3(mod\ 16)$, $q \equiv 15(mod\ 16)$. In any case, it is enough that we find the solutions of relating equations $3) N^2 = pM^4 - qe^4$ for Γ and $5) N^2 = 2pM^4 + 2qe^4$ for $\bar{\Gamma}$ due to [2] and [3]. Now $r4.4$ is in [3] and $w.i.A.B.C.D.E.F.G.H.I.J.K.L.O.P.Q.R.S.T.U.V.W.Z.1$ means that with integers A and B and C and D and E and F and G and H and I and J and K and L and O and P and Q and R and S and T and U and V and W and Z and $(A, B, C, D, E, F, G, H, I, J, K, L, O, P, Q, R, S, T, U, V, W, Z) = 1$.

2 New forms of primes

As we mentioned in the previous section, we treat more terms in primes than (YY). Here we use the variables from A to Z with excepting M and N and X and Y. Now we approach to the primes.

Theorem 2.1. In curve E_{-pq} take distinct odd primes p and q as $p = 51A^2 + B^2 + C^2 + D^2 + E^2 + F^2 + G^2 + H^2 + I^2 + J^2 + K^2 + L^2 + O^2 + P^2 + Q^2 + R^2 + S^2 + T^2 + U^2 + V^2 + W^2 + Z^2 + 14AB + 14AC + 14AD + 14AE + 14AF + 14AG + 14AH + 14AI + 14AJ + 14AK + 14AL + 14AO + 14AP + 14AQ + 14AR + 14AS + 14AT + 14AU + 14AV + 14AW + 14AZ + 2BC + 2BD + 2B \cdot E + 2BF + 2BG + 2BH + 2BI + 2BJ + 2BK + 2BL + 2BO + 2BP + 2BQ + 2BR + 2BS + 2BT + 2BU + 2BV + 2BW + 2BZ + 2CD + 2CE + 2CF + 2CG + 2CH + 2CI + 2CJ + 2CK + 2CL + 2CO + 2CP + 2CQ + 2CR + 2CS + 2CT + 2CU + 2CV + 2CW + 2CZ + 2DE + 2DF + 2DG + 2DH + 2DI + 2DJ + 2DK + 2DL + 2DO + 2DP + 2DQ + 2DR + 2DS + 2DT + 2DU + 2DV + 2DW + 2 \cdot DZ + 2EF + 2EG + 2EH + 2EI + 2EJ + 2EK + 2EL + 2EO + 2EP + 2EQ + 2ER + 2ES + 2ET + 2EU + 2EV + 2EW + 2EZ + 2FG + 2FH + 2FI + 2FJ + 2FK + 2FL + 2FO + 2FP + 2FQ + 2FR + 2FS + 2FT + 2FU + 2FV + 2FW + 2FZ + 2GH + 2GI + 2GJ + 2GK + 2GL + 2GO + 2GP + 2GQ + 2GR + 2GS + 2GT + 2GU + 2GV + 2GW + 2GZ + 2HI + 2HJ + 2HK + 2HL + 2HO + 2H \cdot P + 2HQ + 2HR + 2HS + 2HT + 2HU + 2HV + 2HW + 2HZ + 2IJ + 2IK + 2IL + 2IO + 2IP + 2IQ + 2IR + 2IS + 2IT + 2IU + 2IV + 2IW + 2IZ + 2JK + 2JL + 2JO + 2JP + 2JQ + 2JR + 2JS + 2JT + 2JU + 2JV + 2JW + 2JZ + 2K \cdot L + 2KO + 2KP + 2KQ + 2KR + 2KS + 2KT + 2KU + 2KV + 2KW + 2KZ + 2LO + 2LP + 2LQ + 2LR + 2LS + 2LT + 2LU + 2LV + 2LW + 2LZ + 2OP + 2OQ + 2OR + 2OS + 2OT + 2OU + 2OV + 2OW + 2OZ + 2PQ + 2PR + 2P \cdot S + 2PT + 2PU + 2PV + 2PW + 2PZ + 2QR + 2QS + 2QT + 2QU + 2QV + 2QW + 2QZ + 2RS + 2RT + 2RU + 2RV + 2RW + 2RZ + 2ST + 2SU + 2SV + 2SW + 2SZ + 2TU + 2TV + 2TW + 2TZ + 2UV + 2UW + 2UZ + 2VW + 2 \cdot VZ + 2WZ$ and $q = 47A^2 + B^2 + C^2 + D^2 + E^2 + F^2 + G^2 + H^2 + I^2 + J^2 + K^2 + L^2 + O^2 + P^2 + Q^2 + R^2 + S^2 + T^2 + U^2 + V^2 + W^2 + Z^2 + 14AB + 14 \cdot AC + 14AD + 14AE + 14AF + 14AG + 14AH + 14AI + 14AJ + 14AK + 14 \cdot AL + 14AO + 14AP + 14AQ + 14AR + 14AS + 14AT + 14AU + 14AV + 14 \cdot AW + 14AZ + 2BC + 2BD + 2BE + 2BF + 2BG + 2BH + 2BI + 2BJ + 2BK + 2BL + 2BO + 2BP + 2BQ + 2BR + 2BS + 2BT + 2BU + 2BV + 2BW + 2B$

$\cdot Z + 2CD + 2CE + 2CF + 2CG + 2CH + 2CI + 2CJ + 2CK + 2CL + 2CO + 2$
$\cdot CP + 2CQ + 2CR + 2CS + 2CT + 2CU + 2CV + 2CW + 2CZ + 2DE + 2DF$
$+2DG + 2DH + 2DI + 2DJ + 2DK + 2DL + 2DO + 2DP + 2DQ + 2DR + 2D$
$\cdot S + 2DT + 2DU + 2DV + 2DW + 2DZ + 2EF + 2EG + 2EH + 2EI + 2EJ +$
$2EK + 2EL + 2EO + 2EP + 2EQ + 2ER + 2ES + 2ET + 2EU + 2EV + 2EW$
$+2EZ + 2FG + 2FH + 2FI + 2FJ + 2FK + 2FL + 2FO + 2FP + 2FQ + 2FR$
$+2FS + 2FT + 2FU + 2FV + 2FW + 2FZ + 2GH + 2GI + 2GJ + 2GK + 2GL$
$+2GO + 2GP + 2GQ + 2GR + 2GS + 2GT + 2GU + 2GV + 2GW + 2GZ + 2HI$
$+2HJ + 2HK + 2HL + 2HO + 2HP + 2HQ + 2HR + 2HS + 2HT + 2HU + 2H$
$\cdot V + 2HW + 2HZ + 2IJ + 2IK + 2IL + 2IO + 2IP + 2IQ + 2IR + 2IS + 2IT$
$+2IU + 2IV + 2IW + 2IZ + 2JK + 2JL + 2JO + 2JP + 2JQ + 2JR + 2JS + 2J$
$\cdot T + 2JU + 2JV + 2JW + 2JZ + 2KL + 2KO + 2KP + 2KQ + 2KR + 2KS + 2$
$\cdot KT + 2KU + 2KV + 2KW + 2KZ + 2LO + 2LP + 2LQ + 2LR + 2LS + 2LT +$
$2LU + 2LV + 2LW + 2LZ + 2OP + 2OQ + 2OR + 2OS + 2OT + 2OU + 2OV$
$+2OW + 2OZ + 2PQ + 2PR + 2PS + 2PT + 2PU + 2PV + 2PW + 2PZ + 2Q$
$\cdot R + 2QS + 2QT + 2QU + 2QV + 2QW + 2QZ + 2RS + 2RT + 2RU + 2RV +$
$2RW + 2RZ + 2ST + 2SU + 2SV + 2SW + 2SZ + 2TU + 2TV + 2TW + 2TZ$
$+2UV + 2UW + 2UZ + 2VW + 2VZ + 2WZ$ $w.i.A.B.C.D.E.F.G.H.I.J.K.L.$
$O.P.Q.R.S.T.U.V.W.Z.1$, $p \equiv 11(mod\ 16)$ and $q \equiv 7(mod\ 16)$ then, we gain

$$rank(E_{-(51A^2+\cdots+14AB+\cdots+14AZ+\cdots+2WZ)(47A^2+\cdots+14AB+\cdots+14AZ+\cdots\cdots+2WZ)}(Q)) = 2.$$

Proo$f$. We write equation 3) as $N^2 = (51A^2 + B^2 + C^2 + D^2 + E^2 + F^2 + G^2$
$+H^2 + I^2 + J^2 + K^2 + L^2 + O^2 + P^2 + Q^2 + R^2 + S^2 + T^2 + U^2 + V^2 + W^2 +$
$Z^2 + 14AB + 14AC + 14AD + 14AE + 14AF + 14AG + 14AH + 14AI + 14A$
$\cdot J + 14AK + 14AL + 14AO + 14AP + 14AQ + 14AR + 14AS + 14AT + 14A$
$\cdot U + 14AV + 14AW + 14AZ + 2BC + 2BD + 2BE + 2BF + 2BG + 2BH + 2$
$\cdot BI + 2BJ + 2BK + 2BL + 2BO + 2BP + 2BQ + 2BR + 2BS + 2BT + 2BU +$
$2BV + 2BW + 2BZ + 2CD + 2CE + 2CF + 2CG + 2CH + 2CI + 2CJ + 2CK +$
$2CL + 2CO + 2CP + 2CQ + 2CR + 2CS + 2CT + 2CU + 2CV + 2CW + 2CZ +$
$2DE + 2DF + 2DG + 2DH + 2DI + 2DJ + 2DK + 2DL + 2DO + 2DP + 2DQ$
$+2DR + 2DS + 2DT + 2DU + 2DV + 2DW + 2DZ + 2EF + 2EG + 2EH + 2E$
$\cdot I + 2EJ + 2EK + 2EL + 2EO + 2EP + 2EQ + 2ER + 2ES + 2ET + 2EU + 2$
$\cdot EV + 2EW + 2EZ + 2FG + 2FH + 2FI + 2FJ + 2FK + 2FL + 2FO + 2FP +$
$2FQ + 2FR + 2FS + 2FT + 2FU + 2FV + 2FW + 2FZ + 2GH + 2GI + 2GJ +$
$2GK + 2GL + 2GO + 2GP + 2GQ + 2GR + 2GS + 2GT + 2GU + 2GV + 2GW$
$+2GZ + 2HI + 2HJ + 2HK + 2HL + 2HO + 2HP + 2HQ + 2HR + 2HS + 2H$
$\cdot T + 2HU + 2HV + 2HW + 2HZ + 2IJ + 2IK + 2IL + 2IO + 2IP + 2IQ + 2I$
$\cdot R + 2IS + 2IT + 2IU + 2IV + 2IW + 2IZ + 2JK + 2JL + 2JO + 2JP + 2JQ +$
$2JR + 2JS + 2JT + 2JU + 2JV + 2JW + 2JZ + 2KL + 2KO + 2KP + 2KQ + 2$
$\cdot KR + 2KS + 2KT + 2KU + 2KV + 2KW + 2KZ + 2LO + 2LP + 2LQ + 2LR$
$+2LS + 2LT + 2LU + 2LV + 2LW + 2LZ + 2OP + 2OQ + 2OR + 2OS + 2OT$
$+2OU + 2OV + 2OW + 2OZ + 2PQ + 2PR + 2PS + 2PT + 2PU + 2PV + 2P$

$\cdot W + 2PZ + 2QR + 2QS + 2QT + 2QU + 2QV + 2QW + 2QZ + 2RS + 2RT + 2RU + 2RV + 2RW + 2RZ + 2ST + 2SU + 2SV + 2SW + 2SZ + 2TU + 2TV + 2TW + 2TZ + 2UV + 2UW + 2UZ + 2VW + 2VZ + 2WZ)M^4 - (47A^2 + B^2 + C^2 + D^2 + E^2 + F^2 + G^2 + H^2 + I^2 + J^2 + K^2 + L^2 + O^2 + P^2 + Q^2 + R^2 + S^2 + T^2 + U^2 + V^2 + W^2 + Z^2 + 14AB + 14AC + 14AD + 14AE + 14AF + 14AG + 14AH + 14AI + 14AJ + 14AK + 14AL + 14AO + 14AP + 14AQ + 14AR + 14AS + 14AT + 14AU + 14AV + 14AW + 14AZ + 2BC + 2BD + 2B \cdot E + 2BF + 2BG + 2BH + 2BI + 2BJ + 2BK + 2BL + 2BO + 2BP + 2BQ + 2BR + 2BS + 2BT + 2BU + 2BV + 2BW + 2BZ + 2CD + 2CE + 2CF + 2CG + 2CH + 2CI + 2CJ + 2CK + 2CL + 2CO + 2CP + 2CQ + 2CR + 2CS + 2CT + 2CU + 2CV + 2CW + 2CZ + 2DE + 2DF + 2DG + 2DH + 2DI + 2DJ + 2DK + 2DL + 2DO + 2DP + 2DQ + 2DR + 2DS + 2DT + 2DU + 2DV + 2DW + 2 \cdot DZ + 2EF + 2EG + 2EH + 2EI + 2EJ + 2EK + 2EL + 2EO + 2EP + 2EQ + 2ER + 2ES + 2ET + 2EU + 2EV + 2EW + 2EZ + 2FG + 2FH + 2FI + 2FJ + 2FK + 2FL + 2FO + 2FP + 2FQ + 2FR + 2FS + 2FT + 2FU + 2FV + 2FW + 2FZ + 2GH + 2GI + 2GJ + 2GK + 2GL + 2GO + 2GP + 2GQ + 2GR + 2GS + 2GT + 2GU + 2GV + 2GW + 2GZ + 2HI + 2HJ + 2HK + 2HL + 2HO + 2H \cdot P + 2HQ + 2HR + 2HS + 2HT + 2HU + 2HV + 2HW + 2HZ + 2IJ + 2IK + 2IL + 2IO + 2IP + 2IQ + 2IR + 2IS + 2IT + 2IU + 2IV + 2IW + 2IZ + 2JK + 2JL + 2JO + 2JP + 2JQ + 2JR + 2JS + 2JT + 2JU + 2JV + 2JW + 2JZ + 2K \cdot L + 2KO + 2KP + 2KQ + 2KR + 2KS + 2KT + 2KU + 2KV + 2KW + 2KZ + 2LO + 2LP + 2LQ + 2LR + 2LS + 2LT + 2LU + 2LV + 2LW + 2LZ + 2OP + 2OQ + 2OR + 2OS + 2OT + 2OU + 2OV + 2OW + 2OZ + 2PQ + 2PR + 2P \cdot S + 2PT + 2PU + 2PV + 2PW + 2PZ + 2QR + 2QS + 2QT + 2QU + 2QV + 2QW + 2QZ + 2RS + 2RT + 2RU + 2RV + 2RW + 2RZ + 2ST + 2SU + 2SV + 2SW + 2SZ + 2TU + 2TV + 2TW + 2TZ + 2UV + 2UW + 2UZ + 2VW + 2 \cdot VZ + 2WZ)e^4$. Taking $M = e = 1$ then, we confront to $(51A^2 + B^2 + C^2 + \cdots \cdots + Z^2 + 14AB + \cdots\cdots\cdots + 14AZ + 2BC + 2BD + \cdots\cdots + 2WZ) - (47A^2 + B^2 + C^2 + \cdots\cdots + Z^2 + 14AB + \cdots\cdots\cdots + 14AZ + 2BC + 2BD + \cdots\cdots + 2WZ) = 51A^2 - 47A^2 = 4A^2$. Whence, the triple $(1, 1, 2A)$ satisfies the solution of this equation, thus there comes that $\#\alpha(\Gamma) = 4$. Next, equation 5) for $\bar{\Gamma}$ is $N^2 = 2 \cdot (51A^2 + B^2 + C^2 + D^2 + E^2 + F^2 + G^2 + H^2 + I^2 + J^2 + K^2 + L^2 + O^2 + P^2 + Q^2 + R^2 + S^2 + T^2 + U^2 + V^2 + W^2 + Z^2 + 14AB + 14AC + 14AD + 14AE + 14AF + 14AG + 14AH + 14AI + 14AJ + 14AK + 14AL + 14AO + 14AP + 14AQ + 14AR + 14AS + 14AT + 14AU + 14AV + 14AW + 14AZ + 2BC + 2 \cdot BD + 2BE + 2BF + 2BG + 2BH + 2BI + 2BJ + 2BK + 2BL + 2BO + 2BP + 2BQ + 2BR + 2BS + 2BT + 2BU + 2BV + 2BW + 2BZ + 2CD + 2CE + 2CF + 2CG + 2CH + 2CI + 2CJ + 2CK + 2CL + 2CO + 2CP + 2CQ + 2CR + 2CS + 2CT + 2CU + 2CV + 2CW + 2CZ + 2DE + 2DF + 2DG + 2DH + 2DI + 2DJ + 2DK + 2DL + 2DO + 2DP + 2DQ + 2DR + 2DS + 2DT + 2DU + 2DV + 2D \cdot W + 2DZ + 2EF + 2EG + 2EH + 2EI + 2EJ + 2EK + 2EL + 2EO + 2EP + 2EQ + 2ER + 2ES + 2ET + 2EU + 2EV + 2EW + 2EZ + 2FG + 2FH + 2FI + 2FJ + 2FK + 2FL + 2FO + 2FP + 2FQ + 2FR + 2FS + 2FT + 2FU + 2FV + 2FW + 2FZ + 2GH + 2GI + 2GJ + 2GK + 2GL + 2GO + 2GP + 2GQ + 2GR$

$$
\begin{aligned}
&+2GS + 2GT + 2GU + 2GV + 2GW + 2GZ + 2HI + 2HJ + 2HK + 2HL + 2H \\
&\cdot O + 2HP + 2HQ + 2HR + 2HS + 2HT + 2HU + 2HV + 2HW + 2HZ + 2IJ \\
&+2IK + 2IL + 2IO + 2IP + 2IQ + 2IR + 2IS + 2IT + 2IU + 2IV + 2IW + 2I \\
&\cdot Z + 2JK + 2JL + 2JO + 2JP + 2JQ + 2JR + 2JS + 2JT + 2JU + 2JV + 2JW + \\
&2JZ + 2KL + 2KO + 2KP + 2KQ + 2KR + 2KS + 2KT + 2KU + 2KV + 2KW \\
&+2KZ + 2LO + 2LP + 2LQ + 2LR + 2LS + 2LT + 2LU + 2LV + 2LW + 2LZ \\
&+2OP + 2OQ + 2OR + 2OS + 2OT + 2OU + 2OV + 2OW + 2OZ + 2PQ + 2P \\
&\cdot R + 2PS + 2PT + 2PU + 2PV + 2PW + 2PZ + 2QR + 2QS + 2QT + 2QU + \\
&2QV + 2QW + 2QZ + 2RS + 2RT + 2RU + 2RV + 2RW + 2RZ + 2ST + 2SU \\
&+2SV + 2SW + 2SZ + 2TU + 2TV + 2TW + 2TZ + 2UV + 2UW + 2UZ + 2V \\
&\cdot W + 2VZ + 2WZ)M^4 + 2(47A^2 + B^2 + C^2 + D^2 + E^2 + F^2 + G^2 + H^2 + I^2 \\
&+J^2 + K^2 + L^2 + O^2 + P^2 + Q^2 + R^2 + S^2 + T^2 + U^2 + V^2 + W^2 + Z^2 + 14A \\
&\cdot B + 14AC + 14AD + 14AE + 14AF + 14AG + 14AH + 14AI + 14AJ + 14A \\
&\cdot K + 14AL + 14AO + 14AP + 14AQ + 14AR + 14AS + 14AT + 14AU + 14A \\
&\cdot V + 14AW + 14AZ + 2BC + 2BD + 2BE + 2BF + 2BG + 2BH + 2BI + 2BJ \\
&+2BK + 2BL + 2BO + 2BP + 2BQ + 2BR + 2BS + 2BT + 2BU + 2BV + 2B \\
&\cdot W + 2BZ + 2CD + 2CE + 2CF + 2CG + 2CH + 2CI + 2CJ + 2CK + 2CL + 2 \\
&\cdot CO + 2CP + 2CQ + 2CR + 2CS + 2CT + 2CU + 2CV + 2CW + 2CZ + 2DE \\
&+2DF + 2DG + 2DH + 2DI + 2DJ + 2DK + 2DL + 2DO + 2DP + 2DQ + 2D \\
&\cdot R + 2DS + 2DT + 2DU + 2DV + 2DW + 2DZ + 2EF + 2EG + 2EH + 2EI + \\
&2EJ + 2EK + 2EL + 2EO + 2EP + 2EQ + 2ER + 2ES + 2ET + 2EU + 2EV + \\
&2EW + 2EZ + 2FG + 2FH + 2FI + 2FJ + 2FK + 2FL + 2FO + 2FP + 2FQ + \\
&2FR + 2FS + 2FT + 2FU + 2FV + 2FW + 2FZ + 2GH + 2GI + 2GJ + 2GK + \\
&2GL + 2GO + 2GP + 2GQ + 2GR + 2GS + 2GT + 2GU + 2GV + 2GW + 2GZ \\
&+2HI + 2HJ + 2HK + 2HL + 2HO + 2HP + 2HQ + 2HR + 2HS + 2HT + 2H \\
&\cdot U + 2HV + 2HW + 2HZ + 2IJ + 2IK + 2IL + 2IO + 2IP + 2IQ + 2IR + 2IS \\
&+2IT + 2IU + 2IV + 2IW + 2IZ + 2JK + 2JL + 2JO + 2JP + 2JQ + 2JR + 2J \\
&\cdot S + 2JT + 2JU + 2JV + 2JW + 2JZ + 2KL + 2KO + 2KP + 2KQ + 2KR + 2 \\
&\cdot KS + 2KT + 2KU + 2KV + 2KW + 2KZ + 2LO + 2LP + 2LQ + 2LR + 2LS \\
&+2LT + 2LU + 2LV + 2LW + 2LZ + 2OP + 2OQ + 2OR + 2OS + 2OT + 2OU \\
&+2OV + 2OW + 2OZ + 2PQ + 2PR + 2PS + 2PT + 2PU + 2PV + 2PW + 2P \\
&\cdot Z + 2QR + 2QS + 2QT + 2QU + 2QV + 2QW + 2QZ + 2RS + 2RT + 2RU + \\
&2RV + 2RW + 2RZ + 2ST + 2SU + 2SV + 2SW + 2SZ + 2TU + 2TV + 2TW \\
&+2TZ + 2UV + 2UW + 2UZ + 2VW + 2VZ + 2WZ)e^4
\end{aligned}
$$

. There are terms $102A^2$ and $94A^2$ in coefficients of M^4 and e^4. Thus, we can consider square $196A^2 \cdots\cdots (RR)$. Next, there are terms $28AB$ and $28AC$ and $28AD$ and $28AE$ and $28AF$ and $28AG$ and $28AH$ and $28AI$ and $28AJ$ and $28AK$ and $28AL$ and $28AO$ and $28AP$ and $28AQ$ and $28AR$ and $28AS$ and $28AT$ and $28AU$ and $28AV$ and $28AW$ and $28AZ$ in both coefficients. And remaining terms from $4BC$ to $4WZ$ has common coefficient $4 \cdots\cdots (SS)$. Resultanly, in determining the value N we consider its component as $14A$ from (RR) and choosing $2B$ and $2C$ and $2D$ and $2E$ and $2F$ and $2G$ and $2H$ and $2I$ and $2J$ and $2K$ and $2L$ and $2O$ and $2P$ and $2Q$ and $2R$ and $2S$ and $2T$ and $2U$ and $2V$ and $2W$ and $2Z$ in consideration of (SS). Then, the value N is decided as $14A + 2B + 2C + 2D + 2E + 2F + 2G +$

$2H + 2I + 2J + 2K + 2L + 2O + 2P + 2Q + 2R + 2S + 2T + 2U + 2V + 2W$ $+2Z$. And all above treatment is done under the hypothesis of $M = e = 1$. Therefore, we have that $\#\bar{\alpha}(\bar{\Gamma}) = 4$ and $r4.4$. Consequently, we accomplished the proof. □

In above, we obtained primes comprised of 22 variables and 253 terms. Compared with consequences in [4], [5], [6], [7] the numbers of variables have increased by seven and that of terms is changed from 120? into 253. The numbers 22 and 253 are not absolute values. There can be derived primes of more numbers of variables and terms than it. Resultantly, we have more complex forms than [4], [5], [6], [7]. And the degrees satisfied the conditions as we mentioned in the previous. The coefficients of A^2 are both odds as 51 and 47 and other coefficients of squares are all 1. These are similar results in [4], [5], [6], [7]. But the difference is numbers of variables. In above, the number is even and in [4], [5], [6], [7] it is odd. In theoretically, it doesn't have meaning. But in treating the examples it affects. As we will see in next section, here the value of Z is given as even and so in 22 variables one variable is even and other 21 varaibles are odd and resultantly the primes p and q became odd since other terms of primes were even due to coefficients 14 and 2. Meanwhile in [4], [5], [6], [7] the numbers of variables are 15 and so the value z is taken as odd, hence there are 15 variables and addition of odds and other terms became an odd. Outwardly, the coefficients 51 and 47 are crucial in determining the primes but if we take into account more then, it isn't. Another coefficient 14 also plays an important role. In the range of 51, 47 and 14 coefficient 14 depends 51 and 47 due to the calculation of $2 \cdot 51 + 2 \cdot 47 = 196$ but in from the perspective of odd primes the number 14 affects to primes itself. Furthermore, we should notice squares B^2 and C^2 and D^2 and E^2 and F^2 and G^2 and H^2 and I^2 and J^2 and K^2 and L^2 and O^2 and P^2 and Q^2 and R^2 and S^2 and T^2 and U^2 and V^2 and W^2 and Z^2. The coefficients of all these are 1 but if there is changed into some odds then, it also changes other terms. If F^2 became $9F^2$ then, it also changes the terms which has F as a factor and as a result the forms of primes should be considered again.

Remark 2.2. In two primes p and q there are terms from $2BC$ to $2WZ$ whose coefficients are 2. These terms functions even parts in primes from an overall perspective. It is fixed and thus it is enough that we only take into account square parts which are consisted of odds for getting odd primes. In [5], [6], [7] the terms from $\pm 2df$ to $2wz$ play a similar role.

3 Examples

In this section, we regard examples of theorem.
Primality was taken by [1].
Now we gain examples of theorem 2.1 as follows:

$(p, q, A, B, C, D, E, F, G, H, I, J, K, L, O, P, Q, R, S, T, U, V, W, Z)$:

(13691, 13687, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 90) and

(56171, 56167, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 210).

In above, if $(B, C, D, E, F, G, H, I, J, K, L, O, P, Q, R, S, T, U, V, W, Z)$ is taken as (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 6) then, (p, q) is given as (1091, 1087) but these are the forms $p \equiv 3(mod\ 16)$, $q \equiv 15(mod\ 16)$.

References

[1] C. Caldwell, http://primes.utm.edu/curios/includes/primetest.php.

[2] S. W. Kim, Different odd primes in curve $y^2 = x^3 - pqx$, *Far East J. Math. Sci.(FJMS),* **107** (2018), 155-165. https://doi.org/10.17654/ms107010155

[3] S. W. Kim, Enumeration in ranks of various elliptic curves $y^2 = x^3 \pm Ax$, *Int. J. of Algebra,* **14** (2020), 139-162. https://doi.org/10.12988/ija.2020.91250

[4] S. W. Kim, Compositions of distinct primes in elliptic curve $y^2 = x^3 - pqx$, *Int. J. of Algebra,* **18** (2024), 105-116. https://doi.org/10.12988/ija.2024.91862

[5] S. W. Kim, Elliptic curve $y^2 = x^3 - pqx$ with rank, *Int. J. of Algebra,* **18** (2024), 135-143. https://doi.org/10.12988/ija.2024.91915

[6] S. W. Kim, Numeration of rank in elliptic curve $y^2 = x^3 - pqx$, *Int. J. of Algebra,* **18** (2024), 145-154. https://doi.org/10.12988/ija.2024.91916

[7] S. W. Kim, Forms of primes in elliptic curve $y^2 = x^3 - pqx$, *Int. J. of Algebra,* **18** (2024), 155-166. https://doi.org/10.12988/ija.2024.91917

Received: January 1, 2026; Published: February 3, 2026