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# Orthogonality of Left Reverse Derivation and Symmetric Left Reverse Biderivation in Semiprime Rings

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**Abstract**

This paper gives the notion of orthogonality between the left reverse derivation and symmetric left reverse biderivation of a semiprime ring. We prove that if R is a semiprime ring, $B$ is a reverse biderivation and $d$ is a derivation of R are orthogonal if and only if any one of the following equivalent conditions hold for every $x, y, z \in R$.
(i)$B(x,y)d(z) + d(x)B(z,y) = 0$ (ii) $dB = 0$
(iii) $d(x)B(x,y) = 0$ (iv) $dB$ is a biderivation.

**Keywords**: Semiprimering, Derivation, Reverse derivation, Left reverse derivation, Left reverse biderivation

## Introduction

In [2] Bresar.M and Vukman.J introduce the idea of orthogonality for a pair of derivations (d, g) in semiprime rings. Abdul Rhman and H. Majeed worked on orthogonal reverse derivations in semiprime ring in [1]. C. Jaya Subba Reddy and Ramoorthy Reddy. B studied orthogonal symmetric bi derivations and orthogonal symmetric Bi-$(\sigma,\tau)$on semiprime rings in [4,7]. In [5] C. Jaya Subba Reddy and Ramoorthy Reddy. B have proved results on orthogonal symmetric reverse biderivations in semiprime rings.In [3] Daif.M.N, Tammam, M.S., El-Sayiad, Haetinger are focused on orthogonal derivations and biderivations. Kleinfeld. E Proved results on standard and Accessible ring in [9].In [10] P.SVijaya Lakshmi, K.Suvarna illustrated on orthogonality of derivations and biderivations in semiprime accessible rings in [8].C.Jaya Subba Reddy, Krishna Moorthy analysed results on orthogonal symmetric reverse Bi-$(\sigma,\tau)$ in semiprime rings.C.Jaya Subba Reddy, G. Venkata Bhaskara Rao and B. Ramoorthy Reddy evaluated results on symmetric reverse biderivations in prime and semiprime rings in [6]. In this paper my aim is to prove new results on orthogonality of reverse derivation and reverse bi derivation in semiprime rings.

## Preliminaries

Throughout this paper R will be an associative ring. A ring R is said to be 2-torsion free if $2x = 0, \forall x \in R$ implies $x = 0$.A ring R is said to be prime if $xRy = 0$ implies $x = 0$ or $y = 0$, and R is said to be semiprime if $xRx = 0$ implies $x = 0$. An additive map $d: R \to R$ is called as reverse derivation if $d(xy) = d(y)x + yd(x)$. An additive map $d: R \to R$ is called as left reverse biderivation then $d(xy) = xd(y) + yd(x), \forall\, x, y, z \in R$. A mapping $B(.,.): R \times R \to R$ is symmetric mapping if $B(x, y) = B(y, x), \forall x, y \in R$. A symmetric bi additive mapping $B(.,.): R \times R \to R$ is called as bi derivation if $B(xy, z) = B(x, z)y + xB(y, z)$,$B(x, yz) = B(x, y)z + yB(x, z)$.
An additive map $B(.,.): R \times R \to R$ is called as reverse biderivation then $(x, yz) = B(x, z)y + zB(x, y), B(xy, z) = B(y, z)x + yB(x, z)$ .
An additive map $B: R \times R \to R$ is termed as left reverse biderivations then $B(x, yz) = yB(x, z) + zB(x, y), B(xy, z) = xB(y, z) + yB(x, z),\ \ \forall x, y \in R$. Let two derivations d and g are orthogonal then $d(x)Rg(y) = 0$ or $g(y)Rd(x) = 0$. Let R be semiprime ring, then derivation $d$ and biderivation $B$ are called as orthogonal, if $d(x)RB(y, z) = 0 = B(y, z)Rd(x), \forall x, y \in R$.

**Lemma-1**: **[[2], Lemma 1]]** Let R be a 2-torsion free semiprime ring and a, b $\in R$. Then the following are equivalent**:**
$i)\ arb = 0\ , \forall r \in R\ \ ii)\ bra = 0, \forall r \in R$ iii) $arb + bra = 0, \forall r \in R$
If any one of the following equivalent conditions hold $ab = ba = 0$

**Lemma 2** [**[10, Lemma 3**]: Let R is a 2-torsion free semi prime ring. If an additive map $d$ on R and a biadditive mapping $B: R \times R \to R$ satisfy $B(x, y)Rd(x) = 0, \forall x, y, z \in R$, then $B(x, y)Rd(z) = 0$

**Lemma 3:** Let $d$ be a left reverse derivation and $B$ be a symmetric left reverse bi derivation of a 2-torsion semiprime ring R. Then the following identities hold for every $x, y, z \in R$.
$dB(xy\,, z) = d\,(xB(y,z) + yB(x,z))$
$dB(xy,z) = d(xB(y,z)) + d(yB(x,z))$
$dB(xy,z) = B(y,z)d(x) + xdB(y,z) + ydB(x,z) + B(x,z)d(y)$

## Main Results

**Theorem1**: Let R be a 2 torsion free semiprime ring. A symmetric left reverse bi derivation $B$ and symmetric left reverse derivation $d$ are orthogonal if and only if $B(x,y)d(z) + d(x)B(z,y) = 0$

**Proof:** Let R be a 2-torsion free semiprime ring, then
$B(x,y)d(z) + d(x)B(z,y) = 0$ (1)
Replace $z$ by $xz$ in (1)
$B(x,y)d(xz) + d(x)B(xz,y) = 0$
$B(x,y)\big(xd(z) + zd(x)\big) + d(x)\big(xB(z,y) + zB(x,y)\big) = 0$
$B(x,y)xd(z) + B(x,y)zd(x) + d(x)xB(z,y) + d(x)zB(x,y) = 0, \forall x,y,z \in R$
$B(x,y)Rd(z) + B(x,y)Rd(x) + d(x)RB(z,y) + d(x)RB(x,y) = 0$
$B(x,y)Rd(x) + d(x)RB(x,y) = 0$
By lemma (1)
$B(x,y)Rd(x) = 0$ (2)
By lemma (2)
$B(x,y)Rd(z) = 0$
$\therefore$ $d$ and $B$ are orthogonal.
Conversely, if $d$ and $B$ are orthogonal then $d(x)RB(y,z) = 0$ also
$B(x,y)Rd(z) = 0, \forall x,y,z \in R$.
Since $1 \in R$, $d(x)B(y,z) = 0$ also $B(x,y)d(z) = 0$ (3)
$\therefore$ $d(x)B(y,z) + B(x,y)d(z) = 0$ ,$\forall\, x,y,z \in R$.

**Theorem 2**: Let R be a 2-torsion free semiprime ring. A left reverse bi derivation $B$ and a left reverse derivation $d$ are orthogonal if and only if $B(x,y)d(x) = 0, \forall\, x,y \in R$.

**Proof:** Let $B(x,y)d(x) = 0$ (4)
Then to prove that $d$ and $B$ are orthogonal
Replace $y$ by $zy$ in (4)
$B(x,zy)d(x) = 0$
$(zB(x,y) + yB(x,z))d(x) = 0$
$zB(x,y)d(x) + yB(x,z)d(x) = 0$ (5)
Substituting (4) in (5)
$yB(x,z)d(x) = 0$
Since $y \neq 0 \in R$, then $B(x,z)d(x) = 0$ (6)

Substituting $x$ by $x + y$ in $B(x, z)$ of equation (6), we get
$B(x + y, z)d(x) = 0$
$(B(x, z) + B(y, z))d(x) = 0$
$B(x, z)d(x) + B(y, z)d(x) = 0$ (7)
From (6)
$B(y, z)d(x) = 0$ (8)
Replace $x$ by $xz$ in (8)
$B(y, z)d(xz) = 0$
$B(y, z)(xd(z) + zd(x)) = 0$ ,$\forall\, x, y, z \in R$
$B(y, z)Rd(z) + B(y, z)Rd(x) = 0$
By orthogonality condition
$B(y, z)Rd(z) = 0$ (by symmetry)
$B(z, y)Rd(z) = 0$ (9)
Replace $z$ by $z + x$ in (9)
$B(z + x, y)Rd(z + x) = 0$
$(B(z, y) + B(x, y))R(d(z) + d(x)) = 0$
$B(z, y)Rd(z) + B(z, y)Rd(x) + B(x, y)Rd(z) + B(x, y)Rd(x) = 0$ (10)
Substituting (2) and (9) in (10), we get
$B(z, y)Rd(x) + B(x, y)Rd(z) = 0$
$B(z, y)Rd(x) = 0$ (By orthogonality condition$B(x, y)Rd(z) = 0$)
$B(y, z)Rd(x) = 0$
$\therefore$ $d$ and $B$ are orthogonal.
Conversely, if $d$ and $B$ are orthogonal then $B(x, y)Rd(z) = 0$,$\forall\, x, y, z \in R$ (11)
Replace $z$ by $x$ in (11) we get
$B(x, y)Rd(x) = 0$ (By symmetry)
Since 1$\in R$, $B(x, y)d(x) = 0$.

**Theorem 3**: Let R be a 2-torsion free semiprime ring. A left reverse biderivation $B$ and a left reverse derivation $d$ are orthogonal if and only if $dB = 0$.

**Proof:** we assume that $d$ and $B$ are reverse derivation and reverse biderivation then $dB = 0$
By Lemma (3)
$B(y, z)d(x) + B(x, z)d(y) = 0$ (12)
Substitute (8) in (12), we get
$B(x, z)d(y) = 0$
Substitute $z = y$ and $y = x$ we get
$B(x, y)d(x) = 0$
By theorem (2)
$\therefore$ $d$ and $B$ are orthogonal.
Conversely, if $d$ and $B$ are orthogonal then $(x)rB(y, z) = 0$ ,$\forall\, x, y, z, r \in R$
$d(d(x)rB(y, z)) = 0$
$d(x)d(rB(y, z)) + rB(y, z)d(d(x)) = 0$
$d(x)(rdB(y, z) + B(y, z)d(r)) + rB(y, z)d(x)d(x) = 0$

The sum of second and third term becomes zero as $d$ and $B$ are orthogonal.
$d(x)rdB(y,z) = 0, \forall\ x,y,z \in R$.
Put $x = B(y,z)$ in above equation, we get
$dB(y,z)rdB(y,z) = 0$
Since R is a semi prime $(y,z) = 0\ ,\forall\ x,y,z \in R$
Hence $dB = 0$.

**Theorem 4**: Let R be a 2-torsion free semiprime ring. A left reverse derivation $d$ and a left reverse biderivation $B$ are orthogonal if and only if $dB$ is a left biderivation.

**Proof**: Let $d$ and $B$ are left reverse derivation and left reverse biderivation then $dB$ is a left biderivation $i.e\ dB(xy,z) = ydB(x,z) + xdB(y,z)$
By lemma (3)
$$B(y,z)d(x) + B(x,z)d(y) = 0 \qquad (13)$$
From (8)
$B(x,z)d(y) = 0$
Substituting $z$ by $y$ and $y$ by $x$ in above equation we get
$B(x,y)d(x) = 0$
From theorem (2)
$\therefore\ d$ and $B$ are orthogonal.
Conversely, if $d$ and $B$ are orthogonal to prove that $dB$ is a left biderivation
By lemma (3), we get
$dB(xy,z) = B(y,z)d(x) + xdB(y,z) + ydB(x,z) + B(x,z)d(y)$
From (13), we get
$dB(xy,z) = x\ dB(y,z) + ydB(x,z)$
$\therefore\ dB$ is a left biderivation

**Theorem 5**: Let R be a 2-torsion free semiprime ring. Then a left reverse derivation $d$ and a left reverse biderivation $B$ are orthogonal if and only if the following conditions are equivalent:
(i)$B(x,y)d(z) + d(x)B(z,y) = 0$ (ii)$B(x,y)d(x) = 0$ (iii)$dB = 0$ (iv) $dB$ is a biderivation.

**Proof:** It follows easily from theorems 1,2,3,4.

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