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Mappings of the Direct Product of B-algebras

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Abstract

In this paper, we introduce two canonical mappings of the direct product of B-algebras and we obtain some of their properties.

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1 Introduction

In [7], J. Neggers and H.S. Kim introduced the notion of B-algebras in 2002. A *B-algebra* $\mathbf{A} = (A; *, 0)$ is an algebra of type (2, 0), that is, a nonempty set A together with a binary operation $*$ and a constant 0 satisfying the following axioms for all $x, y, z \in A$:
(I) $x * x = 0$,
(II) $x * 0 = x$,
(III) $(x * y) * z = x * (z * (0 * y))$.
A B-algebra $\mathbf{A}$ is *commutative* if $x * (0 * y) = y * (0 * x)$ for all $x, y \in A$. In [5], H.S. Kim and H.G. Park characterized commutativity of B-algebras. In [8], J. Neggers and H.S. Kim introduced the notions of subalgebras and normality in B-algebras, and established their properties. A nonempty subset N of A is called a *subalgebra* of $\mathbf{A}$ if $x * y \in N$ for any $x, y \in N$. By (I), 0 is always

an element of a subalgebra. A nonempty subset N of A is called a *normal subalgebra* of $\mathbf{A}$ if $(x * a) * (y * b) \in N$ for any $x * y, a * b \in N$. A. Walendziak [10] characterized normality in B-algebras. J. Neggers and H.S. Kim used the concept of normality in B-algebras to construct quotient B-algebras. That is, given a normal subalgebra N of a B-algebra $\mathbf{A}$, the relation $\sim_N$ is defined by $x \sim_N y$ if and only if $x * y \in N$ for any $x, y \in A$. Then $\sim_N$ is a congruence relation of $\mathbf{A}$. For $x \in A$, we write xN for the congruence class containing x, that is, $xN = \{y \in A : x \sim_N y\}$. Denote $A/N = \{xN : x \in A\}$ and define $*'$ on A/N by $xN *' yN = (x * y)N$. Note that $xN = yN$ if and only if $x \sim_N y$. The algebra $\mathbf{A}/N = (A/N; *', N)$ is a B-algebra, and is called the *quotient B-algebra* of $\mathbf{A}$ modulo N. The concept of B-homomorphism was also introduced by J. Neggers and H.S. Kim. A map $\varphi : A \to B$ is called a *B-homomorphism* if $\varphi(x * y) = \varphi(x) * \varphi(y)$ for any $x, y \in A$. The *kernel of* φ, denoted by ker φ, is defined to be the set $\{x \in A : \varphi(x) = 0_B\}$. The ker φ is a normal subalgebra of $\mathbf{A}$, and ker $\varphi = \{0_A\}$ if and only if φ is one-one. A B-homomorphism φ is called a *B-monomorphism*, *B-epimorphism*, or *B-isomorphism* if φ is one-one, onto, or a bijection, respectively. In [4], J.A.V. Lingcong and J.C. Endam introduced and established the direct product of B-algebras. In this paper, we introduced and established two canonical mappings of the direct product of B-algebras.

2 Direct Product of B-algebras

The results in this section are found in [4].

Example 2.1 The algebra $(\mathbb{Z}; *, 0)$ is a B-algebra, where $*$ is defined by $x * y = x - y$ for all $x, y \in \mathbb{Z}$.

Example 2.2 [7] Let $A = \{0, 1, 2, 3, 4, 5\}$ be a set with the following table:

$*$	0	1	2	3	4	5
0	0	2	1	3	4	5
1	1	0	2	4	5	3
2	2	1	0	5	3	4
3	3	4	5	0	2	1
4	4	5	3	1	0	2
5	5	3	4	2	1	0

Then $(A; *, 0)$ is a B-algebra.

Let $\mathbf{A} = (A; *, 0_A)$ and $\mathbf{B} = (B; *, 0_B)$ be B-algebras. Define the direct product of $\mathbf{A}$ and $\mathbf{B}$ to be the structure $\mathbf{A} \times \mathbf{B} = (A \times B; \circledast, (0_A, 0_B))$, where

$A \times B$ is the set $\{(a,b) : a \in A \text{ and } b \in B\}$ and whose binary operation $\circledast$ is given by $(a_1,b_1) \circledast (a_2,b_2) = (a_1 * a_2, b_1 * b_2)$. Note that the binary operation $\circledast$ is componentwise. Thus, the properties (I), (II), and (III) of $\mathbf{A} \times \mathbf{B}$ follow from those of $\mathbf{A}$ and $\mathbf{B}$. Hence, the following theorem easily follows.

Theorem 2.3 *[4] The direct product of two B-algebras is also a B-algebra.*

Now, we extend this direct product to any finite family of B-algebras. Let $\mathrm{I}_n = \{1, 2, \ldots, n\}$ and let $\{\mathbf{A}_i = (A_i; *, 0_i) \colon i \in \mathrm{I}_n\}$ be a finite family of B-algebras. Define the direct product of B-algebras $\mathbf{A}_1, \ldots, \mathbf{A}_n$ to be the structure $\prod_{i=1}^{n} \mathbf{A}_i = \left(\prod_{i=1}^{n} A_i; \circledast, (0_1, \ldots, 0_n) \right)$, where

$$\prod_{i=1}^{n} A_i = A_1 \times \cdots \times A_n = \{(a_1, \ldots, a_n) : a_i \in A_i, i \in \mathrm{I}_n\}$$

and whose operation $\circledast$ is given by

$$(a_1, \ldots, a_n) \circledast (b_1, \ldots, b_n) = (a_1 * b_1, \ldots, a_n * b_n).$$

Obviously, $\circledast$ is a binary operation on $\prod_{i=1}^{n} \mathbf{A}_i$.

Corollary 2.4 *[4] If $\{\boldsymbol{A}_i = (A_i;\, *,\, 0_i) \colon i \in I_n\}$ is a family of B-algebras, then $\prod_{i=1}^{n} \boldsymbol{A}_i$ is a B-algebra.*

Theorem 2.5 *[4] Let $\{\varphi_i \colon A_i \to B_i : i \in I_n\}$ be a family of B-homomorphisms. If φ is the map $\prod_{i=1}^{n} A_i \to \prod_{i=1}^{n} B_i$ given by $(a_1, \ldots, a_n) \mapsto (\varphi_1(a_1), \ldots, \varphi_n(a_n))$, then φ is a B-homomorphism with $\ker \varphi = \prod_{i=1}^{n} \ker \varphi_i$, $\varphi(\prod_{i=1}^{n} A_i) = \prod_{i=1}^{n} \varphi_i(A_i)$. Furthermore, φ is a B-monomorphism (respectively, B-epimorphism) if and only if each φ_i is a B-monomorphism (respectively, B-epimorphism).*

Theorem 2.6 *[4] Let $\{\boldsymbol{A}_i = (A_i;\, *,\, 0_i) \colon i \in I_n\}$ be a family of B-algebras and let J_i be a normal subalgebra of $\boldsymbol{A}_i$ for each $i \in I_n$. Then $\prod_{i=1}^{n} J_i$ is a normal subalgebra of $\prod_{i=1}^{n} \boldsymbol{A}_i$ and $\prod_{i=1}^{n} A_i \,/ \prod_{i=1}^{n} J_i \cong \prod_{i=1}^{n} (A_i / J_i)$.*

3 Mappings of the Direct Product

This section presents two canonical mappings of the direct product of any finite family of B-algebras and provides some of their properties.

Theorem 3.1 *Let* $\{\boldsymbol{A}_i = (A_i; *, 0_i) \colon i \in I_n\}$ *be a family of B-algebras. Then* $f_k \colon \prod_{i=1}^{n} A_i \to A_k$ *given by* $(a_1, \ldots, a_k, \ldots, a_n) \mapsto a_k$ *is a B-epimorphism of B-algebras for each* $k \in I_n$.

Proof: For each $k \in \mathrm{I}_n$, define $f_k : \prod_{i=1}^{n} A_i \to A_k$ by $f_k((a_1, \ldots, a_k, \ldots, a_n)) = a_k$ for all $(a_1, \ldots, a_k, \ldots, a_n) \in \prod_{i=1}^{n} A_i$. Let $(a_1, \ldots, a_k, \ldots, a_n)$, $(b_1, \ldots, b_k, \ldots, b_n)$ be elements of $\prod_{i=1}^{n} A_i$. If $(a_1, \ldots, a_k, \ldots, a_n) = (b_1, \ldots, b_k, \ldots, b_n)$, then $a_i = b_i$ for each $i \in \mathrm{I}_n$. It follows that

$$\begin{aligned} f_k((a_1, \ldots, a_k, \ldots, a_n)) &= a_k \\ &= b_k \\ &= f_k((b_1, \ldots, b_k, \ldots, b_n)). \end{aligned}$$

Hence, f_k is well-defined. If $(a_1, \ldots, a_k, \ldots, a_n), (b_1, \ldots, b_k, \ldots, b_n) \in \prod_{i=1}^{n} A_i$, then

$$\begin{aligned} & f_k((a_1, \ldots, a_k, \ldots, a_n) \circledast (b_1, \ldots, b_k, \ldots, b_n)) \\ &\quad = f_k((a_1 * b_1, \ldots, a_k * b_k, \ldots, a_n * b_n)) \\ &\quad = a_k * b_k \\ &\quad = f_k((a_1, \ldots, a_k, \ldots, a_n)) * f_k((b_1, \ldots, b_k, \ldots, b_n)). \end{aligned}$$

Thus, f_k is a B-homomorphism. If $c_k \in A_k$, then $(0_1, \ldots, c_k, \ldots, 0_n) \in \prod_{i=1}^{n} A_i$ and $f_k((0_1, \ldots, c_k, \ldots, 0_n)) = c_k$. Therefore, f_k is onto and so f_k is a B-epimorphism. □

The maps f_k in Theorem 3.1 are called the *canonical projections* of the direct product. The following theorem relates the direct product $\prod_{i=1}^{n} \mathbf{A}_i$ and its canonical projections.

Theorem 3.2 *Let* $\{\boldsymbol{A}_i = (A_i; *, 0_i) \colon i \in I_n\}$ *be a family of B-algebras. Then there exists a B-algebra* $\boldsymbol{D}$*, together with a family of B-homomorphisms*

$\{f_i\colon D \to A_i : i \in I_n\}$ with the following property: for any B-algebra $\boldsymbol{C}$ and a family of B-homomorphisms $\{\varphi_i\colon C \to A_i : i \in I_n\}$, there exists a unique B-homomorphism $\varphi\colon C \to D$ such that $f_i \circ \varphi = \varphi_i$ for all $i \in I_n$. Furthermore, D is uniquely determined up to B-isomorphism.

Proof: Let $\{\mathbf{A}_i = (A_i; *, 0_i)\colon i \in \mathrm{I}_n\}$ be a family of B-algebras. Then $\prod_{i=1}^{n}\mathbf{A}_i$ is a B-algebra by Corollary 2.4. Let $\mathbf{D} = \prod_{i=1}^{n}\mathbf{A}_i$ and let $\{f_i : D \to A_i : i \in \mathrm{I}_n\}$ be the family of canonical projections. Suppose that $\mathbf{C}$ is any B-algebra and $\{\varphi_i : C \to A_i : i \in \mathrm{I}_n\}$ a family of B-homomorphisms. Define $\varphi : C \to D$ by $\varphi(c) = (\varphi_1(c), \ldots, \varphi_i(c), \ldots, \varphi_n(c))$ for all $c \in C$. If $c, d \in C$, then

$$\begin{aligned}
\varphi(c * d) &= (\varphi_1(c * d), \ldots, \varphi_i(c * d), \ldots, \varphi_n(c * d)) \\
&= (\varphi_1(c) * \varphi_1(d), \ldots, \varphi_i(c) * \varphi_i(d), \ldots, \varphi_n(c) * \varphi_n(d)) \\
&= (\varphi_1(c), \ldots, \varphi_i(c), \ldots, \varphi_n(c)) \circledast (\varphi_1(d), \ldots, \varphi_i(d), \ldots, \varphi_n(d)) \\
&= \varphi(c) \circledast \varphi(d).
\end{aligned}$$

Hence, φ is a B-homomorphism. Moreover, $f_i \circ \varphi = \varphi_i$ for all $i \in \mathrm{I}_n$ since $(f_i \circ \varphi)(c) = f_i(\varphi(c)) = f_i((\varphi_1(c), \ldots, \varphi_i(c), \ldots, \varphi_n(c))) = \varphi_i(c)$. To show that φ is unique, let $\varphi'\colon C \to D$ be another B-homomorphism such that $f_i \circ \varphi' = \varphi_i$ for all $i \in \mathrm{I}_n$. If $c \in C$, then $(f_i \circ \varphi)(c) = \varphi_i(c) = (f_i \circ \varphi')(c)$. By the definition of φ, $\varphi(c) = (\varphi_1(c), \ldots, \varphi_i(c), \ldots, \varphi_n(c))$ and assume that $\varphi'(c) = (a_1, \ldots, a_i, \ldots, a_n)$. Thus, for each $i \in \mathrm{I}_n$,

$$\begin{aligned}
a_i &= f_i((a_1, \ldots, a_i, \ldots, a_n)) \\
&= f_i(\varphi'(c)) \\
&= (f_i \circ \varphi')(c) \\
&= (f_i \circ \varphi)(c) \\
&= f_i(\varphi(c)) \\
&= f_i((\varphi_1(c), \ldots, \varphi_i(c), \ldots, \varphi_n(c))) \\
&= \varphi_i(c).
\end{aligned}$$

Hence, $\varphi(c) = (\varphi_1(c), \ldots, \varphi_i(c), \ldots, \varphi_n(c)) = (a_1, \ldots, a_i, \ldots, a_n) = \varphi'(c)$. Therefore, φ is unique.

Suppose that a B-algebra $\mathbf{D}'$ has the same property as $\mathbf{D}$ with the family of B-homomorphisms $\{f_i' : D' \to A_i : i \in \mathrm{I}_n\}$. If we apply this property for $\mathbf{D}$ to the family of B-homomorphisms $\{f_i' : D' \to A_i : i \in \mathrm{I}_n\}$ and also apply it for $\mathbf{D}'$ to the family of B-homomorphisms $\{f_i : D \to A_i : i \in \mathrm{I}_n\}$, then we obtain unique B-homomorphisms $\alpha : D' \to D$ and $\beta : D \to D'$ such that $f_i \circ \alpha = f_i'$ and $f_i' \circ \beta = f_i$ for all $i \in \mathrm{I}_n$. Thus, $\alpha \circ \beta : D \to D$ is a unique

B-homomorphism such that $f_i \circ (\alpha \circ \beta) = f_i$ for all $i \in \mathrm{I}_n$. Since $\mathrm{id}_D : D \to D$ is a B-homomorphism such that $f_i \circ \mathrm{id}_D = f_i$ for all $i \in \mathrm{I}_n$, $\alpha \circ \beta = \mathrm{id}_D$ by uniqueness. A similar argument shows that $\beta \circ \alpha = \mathrm{id}_{D'}$. Therefore, β is an B-isomorphism, that is, D is uniquely determined up to B-isomorphism. □

Theorem 3.3 *Let $\{\boldsymbol{A}_i = (A_i;\, *,\, 0_i)\colon\ i \in I_n\}$ be a family of B-algebras. Then $g_k\colon A_k \to \prod_{i=1}^{n} A_i$ given by $a_k \mapsto (0_1, \ldots, 0_{k-1}, a_k, 0_{k+1}, \ldots, 0_n)$ is a B-monomorphism of B-algebras for each $k \in I_n$.*

Proof: Let $\{\mathbf{A}_i = (A_i;\, *,\, 0_i) : i \in \mathrm{I}_n\}$ be a family of B-algebras. For each $k \in \mathrm{I}_n$, define $g_k : A_k \to \prod_{i=1}^{n} A_i$ by $g_k(a_k) = (0_1, \ldots, 0_{k-1}, a_k, 0_{k+1}, \ldots, 0_n)$ for all $a_k \in A_k$. Let $a_k, b_k \in A_k$. If $a_k = b_k$, then

$$\begin{aligned} g_k(a_k) &= (0_1, \ldots, 0_{k-1}, a_k, 0_{k+1}, \ldots, 0_n) \\ &= (0_1, \ldots, 0_{k-1}, b_k, 0_{k+1}, \ldots, 0_n) \\ &= g_k(b_k). \end{aligned}$$

Hence, g_k is well-defined. If $a_k, b_k \in A_k$, then

$$\begin{aligned} g_k(a_k * b_k) &= (0_1, \ldots, 0_{k-1}, a_k * b_k, 0_{k+1}, \ldots, 0_n) \\ &= (0_1 * 0_1, \ldots, 0_{k-1} * 0_{k-1}, a_k * b_k, 0_{k+1} * 0_{k+1}, \ldots, 0_n * 0_n) \\ &= (0_1, \ldots, 0_{k-1}, a_k, 0_{k+1}, \ldots, 0_n) \circledast (0_1, \ldots, 0_{k-1}, b_k, 0_{k+1}, \ldots, 0_n) \\ &= g_k(a_k) \circledast g_k(b_k). \end{aligned}$$

Therefore, g_k is a B-homomorphism. If $g_k(a_k) = g_k(b_k)$, then

$$\begin{aligned} (0_1, \ldots, 0_{k-1}, a_k, 0_{k+1}, \ldots, 0_n) &= g_k(a_k) \\ &= g_k(b_k) \\ &= (0_1, \ldots, 0_{k-1}, b_k, 0_{k+1}, \ldots, 0_n). \end{aligned}$$

Hence, $a_k = b_k$. Thus, g_k is one-to-one and so g_k is a B-monomorphism. □

The maps g_k in Theorem 3.3 are called the *canonical injections*.

Theorem 3.4 *Let $\{\boldsymbol{A}_i = (A_i;\, *,\, 0_i)\colon\ i \in I_n\}$ be a family of B-algebras. For each $k \in I_n$, if g_k is the canonical injection, then $g_k(A_k)$ is a normal subalgebra of $\prod_{i=1}^{n} \boldsymbol{A}_i$ and $\prod_{i=1}^{n} A_i \,/\, g_k(A_k) \cong \prod_{i \neq k} A_i$.*

Proof: Note that $g_k(A_k) = \{(0_1, \ldots, 0_{k-1}, a_k, 0_{k+1}, \ldots, 0_n) : a_k \in A_k\}$. It is easy to see that $g_k(A_k)$ is a normal subalgebra of $\prod_{i=1}^{n} \mathbf{A}_i$.

Define φ_k: $\prod_{i=1}^{n} A_i \ / \ g_k(A_k) \to \prod_{i \neq k} A_i$ given by

$$\varphi_k((a_1, \ldots, a_n)g_k(A_k)) = (a_1, \ldots, a_{k-1}, a_{k+1}, \ldots, a_n)$$

for all $(a_1, \ldots, a_n)g_k(A_k) \in \prod_{i=1}^{n} A_i \ / \ g_k(A_k)$.

Let $(a_1, \ldots, a_k, \ldots, a_n)g_k(A_k)$, $(b_1, \ldots, b_k, \ldots, b_n)g_k(A_k) \in \prod_{i=1}^{n} A_i \ / \ g_k(A_k)$. Suppose that $(a_1, \ldots, a_k, \ldots, a_n)g_k(A_k) = (b_1, \ldots, b_k, \ldots, b_n)g_k(A_k)$. Then $(a_1, \ldots, a_k, \ldots, a_n) \sim_{g_k(A_k)} (b_1, \ldots, b_k, \ldots, b_n)$, that is,

$$(a_1{*}b_1, \ldots, a_k{*}b_k, \ldots, a_n{*}b_n) = (a_1, \ldots, a_k, \ldots, a_n) \circledast (b_1, \ldots, b_k, \ldots, b_n) \in g_k(A_k)$$

so that $a_i * b_i = 0_i$ for all $i \neq k$. Hence, $a_i = b_i$ for all $i \neq k$ and so

$$\begin{aligned} \varphi_k((a_1, \ldots, a_k, \ldots, a_n)g_k(A_k)) &= (a_1, \ldots, a_{k-1}, a_{k+1}, \ldots, a_n) \\ &= (b_1, \ldots, b_{k-1}, b_{k+1}, \ldots, b_n) \\ &= \varphi_k((b_1, \ldots, b_k, \ldots, b_n)g_k(A_k)). \end{aligned}$$

This shows that φ_k is well-defined. Moreover,

$$\begin{aligned} &\varphi_k((a_1, \ldots, a_k, \ldots, a_n)g_k(A_k) *' (b_1, \ldots, b_k, \ldots, b_n)g_k(A_k)) \\ &\quad = \varphi_k(((a_1, \ldots, a_k, \ldots, a_n) \circledast (b_1, \ldots, b_k, \ldots, b_n))g_k(A_k)) \\ &\quad = \varphi_k((a_1 * b_1, \ldots, a_k * b_k, \ldots, a_n * b_n)g_k(A_k)) \\ &\quad = (a_1 * b_1, \ldots, a_{k-1} * b_{k-1}, a_{k+1} * b_{k+1}, \ldots, a_n * b_n) \\ &\quad = (a_1, \ldots, a_{k-1}, a_{k+1}, \ldots, a_n) \circledast (b_1, \ldots, b_{k-1}, b_{k+1}, \ldots, b_n) \\ &\quad = \varphi_k((a_1, \ldots, a_k, \ldots, a_n)g_k(A_k)) \circledast \varphi_k((b_1, \ldots, b_k, \ldots, b_n)g_k(A_k)). \end{aligned}$$

This shows that φ_k is a B-homomorphism.
If $\varphi_k((a_1, \ldots, a_k, \ldots, a_n)g_k(A_k)) = \varphi_k((b_1, \ldots, b_k, \ldots, b_n)g_k(A_k))$, then

$$\begin{aligned} (a_1, \ldots, a_{k-1}, a_{k+1}, \ldots, a_n) &= \varphi_k((a_1, \ldots, a_k, \ldots, a_n)g_k(A_k)) \\ &= \varphi_k((b_1, \ldots, b_k, \ldots, b_n)g_k(A_k)) \\ &= (b_1, \ldots, b_{k-1}, b_{k+1}, \ldots, b_n). \end{aligned}$$

Thus, $a_i = b_i$ for all $i \neq k$ so that $a_i * b_i = 0_i$ for all $i \neq k$. Hence, $(a_1, \ldots, a_k, \ldots, a_n) \circledast (b_1, \ldots, b_k, \ldots, b_n) = (a_1 * b_1, \ldots, a_k * b_k, \ldots, a_n * b_n)$ is an element of $g_k(A_k)$, that is, $(a_1, \ldots, a_k, \ldots, a_n) \sim_{g_k(A_k)} (b_1, \ldots, b_k, \ldots, b_n)$ so that $(a_1, \ldots, a_k, \ldots, a_n)g_k(A_k) = (b_1, \ldots, b_k, \ldots, b_n)g_k(A_k)$. This shows that φ_k is one-to-one.

If $(a_1, \ldots, a_{k-1}, a_{k+1}, \ldots, a_n) \in \prod_{i \neq k} A_i$, then $a_i \in A_i$ for all $i \neq k$ so that $(a_1, \ldots, a_{k-1}, 0_k, a_{k+1}, \ldots, a_n) \in \prod_{i=1}^{n} A_i$ since $0_k \in A_k$. It follows that $(a_1, \ldots, a_{k-1}, a_{k+1}, \ldots, a_n) = \varphi_k((a_1, \ldots, a_{k-1}, 0_k, a_{k+1}, \ldots, a_n) g_k(A_k))$, where $(a_1, \ldots, a_{k-1}, 0_k, a_{k+1}, \ldots, a_n) g_k(A_k) \in \prod_{i=1}^{n} A_i \ / \ g_k(A_k)$. Hence, φ_k is onto. Therefore, φ_k is an B-isomorphism, that is, $\prod_{i=1}^{n} A_i \ / \ g_k(A_k) \cong \prod_{i \neq k} A_i$. □

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