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Notes on Anti L-Fuzzy Subfield of a Field

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Abstract

In this paper, we made an attempt to study the algebraic nature of anti L-fuzzy subfield of a field.

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INTRODUCTION

After the introduction of fuzzy sets by L.A.Zadeh[16], several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subgroups, anti-fuzzy subgroups, fuzzy fields and fuzzy linear spaces was introduced by Biswas.R[4, 5]. In this paper, we introduce the some theorems in anti L-fuzzy subfield of a field.

1. PRELIMINARIES

1.1 Definition: Let X be a non-empty set and L be a complete lattice. A L-**fuzzy subset A** of X is a function A: $X \rightarrow L$.

1.2 Definition: Let $(F, +, \cdot)$ be a field. A L-fuzzy subset A of F is said to be an anti L-fuzzy subfield(ALFSF) of F if the following conditions are satisfied:

(i) $A(x+y) \leq A(x) \vee A(y)$, for all x and y in F,

(ii) $A(-x) \leq A(x)$, for all x in F,

(iii) $A(xy) \leq A(x) \vee A(y)$, for all x and y in F,

(iv) $A(x^{-1}) \leq A(x)$, for all x in $F - \{0\}$, where 0 is the additive identity element of F.

1.3 Definition: Let A and B be any two fuzzy subsets of sets G and H, respectively. The anti-product of A and B, denoted by A x B, is defined as

$A \times B = \{\langle (x, y), A \times B(x, y) \rangle$ / for all x in G and y in H$\}$, where

$A \times B(x, y) = A(x) \vee B(y)$, for all x in G and y in H.

1.4 Definition: Let A be a fuzzy subset in a set S, the **anti-strongest fuzzy relation** on S, that is a fuzzy relation on A is $V = \{\langle (x, y), V(x, y) \rangle$ / x and y in S$\}$ given by $V(x, y) = A(x) \vee A(y)$, for all x and y in S.

2 PROPERTIES OF ANTI L-FUZZY SUBFIELDS

2.1 Theorem: Let A be an anti-fuzzy subfield of a field $(F, +, \cdot)$. If $A(x) > A(y)$, for some x and y in F, then $A(x+y) = A(x) = A(y+x)$, for all x and y in F and $A(xy) = A(x) = A(yx)$, for all x and y in $F-\{0\}$.

Proof : We have $A(x) > A(y)$, for some x and y in F, $A(x+y) \leq A(x) \vee A(y)$ $A(x)$; $A(x) \leq A(x+y) \vee A(-y) = A(x+y)$. Therefore, $A(x+y) = A(x)$, for all x and y in F. Now, $A(xy) \leq A(x) \vee A(y) = A(x)$; $A(x) \leq A(xy) \vee A(y^{-1}) = A(xy)$. Therefore, $A(xy) = A(x)$, for all x and y in $F - \{0\}$.

2.2 Theorem: Let A be an anti-fuzzy subfield of a field $(F, +, \cdot)$. If $A(x) < A(y)$, for some x and y in F, then $A(x+y)=A(y)=A(y+x)$, for all x and y in F and $A(xy) = A(y) = A(yx)$, for all x and y in $F-\{0\}$.

Proof: It is trivial.

2.3 Theorem: Let A be an anti-fuzzy subfield of a field (F, +, ·) such that In A= {α }, where α in L. If A = B∩C, where B and C are anti-fuzzy subfields of F, then either B ⊆ C or C ⊆ B.

Proof: Case (i) : Assume that B(x) > C(x) and B(y) < C(y), for some x and y in F. Then, α = A(x) = B(x) ∧ C(x)= C(x) < B(x). Therefore, α < B(x). And, α = A(y) = B(y) ∧ C(y) = B(y) < C(y). Therefore, α < C(y). So that, C(x) < C(y) and B(y) < B(x). Hence B(x+ y) = B(x), for all x and y in F and B(x y) = B(x), for all x and y in F−{0} and C(x+ y) = C(y), for all x and y in F and C(x y) = C(y), for all x and y in F−{0}, by Theorem 2.1 and 2.2. But then, α = A(x+ y) = B(x+ y) ∧ C(x+ y) = B(x) ∧ C(y) > α -------(1). And α = A(x y)= B(x y) ∧ C(x y) = B(x) ∧ C(y) > α --------(2). It is a contradiction by (1), (2). Therefore, either B ⊆ C or C ⊆ B is true.

2.4 Theorem: If A and B are anti-fuzzy subfields of the fields G and H, respectively, then the anti-product A×B is an anti-fuzzy subfield of G×H.

Proof: Let x_1 and x_2 be in G, y_1 and y_2 be in H. Then $(x1, y_1)$ and (x_2, y_2) are in G x H. Now, A×B $[(x_1, y_1) - (x_2, y_2)] = A(x_1- x_2) \vee B(y_1- y_2) \leq \{A(x_1) \vee A(x_2)\} \vee \{ B(y_1) \vee B(y_2)\}=$ A×B$(x_1, y_1) \vee$ A×B (x_2, y_2). Therefore, A×B$[(x1, y1) - (x2, y_2)] \leq$ A×B$(x_1, y_1) \vee$ A×B(x_2, y_2), for all x1 and x2 in G and y1 and y2 in H. And, A×B$[(x_1, y_1)(x_2, y_2)^{-1}] = A(x_1x_2^{-1}) \vee B(y_1y_2^{-1}) \leq\{A(x_1)\vee A(x_2)\}\vee\{ B(y_1) \vee B(y_2)\}=$ A×B$(x_1, y_1) \vee$ A×B(x_2, y_2). Therefore, A×B$[(x_1, y_1)(x_2, y_2)^{-1}] \leq$ A×B$(x_1, y_l) \vee$ A×B(x_2, y_2), for all x1 and x2 in G −{0} and y1 and y2 in H− $\{0^1\}$. Hence anti-product A × B is an anti-fuzzy subfield of G×H.

2.5 Theorem: Let A and B be fuzzy subsets of the fields G and H, respectively. Suppose that 0, 1and 0^l, 1^l are the identity elements of G and H, respectively. If the anti-product A × B is an anti-fuzzy subfield of G×H, then at least one of the following two statements must hold.

(i) B $(0^l) \leq A(x)$, for all x in G and B $(1^l) \leq A(x)$, for all x in G−{0},

(ii) $A(0) \leq B(y)$, for all y in H and $A(1) \leq B(y)$, for all y in H−$\{0^l\}$.

Proof: Let the anti-product A×B be an anti-fuzzy subfield of G×H. By contraposition, suppose that none of the statements (i) and (ii) holds. Then we can find a in G and b in H such that $A(a) < B(0^{I})$, $A(a) < B(1^{I})$ and $B(b) < A(0)$, $B(b) < A(1)$. We have, $A \times B(a, b) = A(a) \vee B(b) < A(0) \vee B(0^{I}) = A \times B(0, 0^{I})$. And, $A \times B(a, b) < A(1) \vee B(1^{I}) = A \times B(1, 1^{I})$.Thus anti-product AxB is not an anti-fuzzy subfield of GxH. Hence either $B(0^{I}) \leq A(x)$, for all x in G and $B(1^{I}) \leq A(x)$, for all x in G–{0} or $A(0) \leq B(y)$, for all y in H and $A(1) \leq B(y)$, for all y in $H-\{0^{I}\}$.

2.6 Theorem: Let A and B be fuzzy subsets of the fields G and H, respectively and the anti-product AxB is an anti-fuzzy subfield of GxH. Then the following are true:

(i) if $A(x) \geq B(0^{I})$, $A(x) \geq B(1^{I})$, then A is an anti-fuzzy subfield of G.

(ii) if $B(x) \geq A(0)$, $B(x) \geq A(1)$, then B is an anti-fuzzy subfield of H.

(iii) either A is an anti-fuzzy subfield of G or B is an anti-fuzzy subfield of H, where 0, 1and 0^{I}, 1^{I} are the identity elements of G and H, respectively.

Proof: Then $(x, 0^{I})$, $(x, 1^{I})$ and $(y, 0^{I})$, $(y, 1^{I})$ are in GxH. Now, using the property $A(x) \geq B(0^{I})$, $A(x) \geq B(1^{I})$, for all x in G, we get, $A(x-y) = A(x-y) \vee B(0^{I}+0^{I}) = A \times B[(x, 0^{I})+(-y, 0^{I})] \leq A \times B(x, 0^{I}) \vee A \times B(-y, 0^{I}) = \{A(x) \vee B(0^{I})\} \vee \{A(-y) \vee B(0^{I})\} = A(x) \vee A(-y) \leq A(x) \vee A(y)$. Therefore, $A(x-y) \leq A(x) \vee A(y)$, for all x and y in G. And, $A(x y^{-1}) = A(x y^{-1}) \vee B(1^{I} 1^{I}) = A \times B[(x, 1^{I})(y^{-1}, 1^{I})] \leq A \times B(x, 1^{I}) \vee A \times B(y^{-1}, 1^{I}) = A(x) \vee A(y^{-1}) \leq A(x) \vee A(y)$. Therefore, $A(x y^{-1}) \leq A(x) \vee A(y)$, for all x and y in G–{0}. Hence A is an anti-fuzzy subfield of G. Now, $B(x) \geq A(0)$, for all x in H and $B(x) \geq A(1)$, for all x in $H-\{0^{I}\}$, we get, $B(x-y) = B(x-y) \vee A(0+0) = A \times B[(0, x)+(0, -y)] \leq A \times B(0, x) \vee A \times B(0, -y) = B(x) \vee B(-y) \leq B(x) \vee B(y)$. Therefore, $B(x-y) \leq B(x) \vee B(y)$, for all x and y in H. And, $B(x y^{-1}) = B(x y^{-1}) \vee A(1.1) = A \times B[(1, x)(1, y^{-1})] \leq A \times B(1, x) \vee A \times B(1, y^{-1}) = \{A(1) \vee B(x)\} \vee \{A(1) \vee B(y^{-1})\} = B(x) \vee B(y^{-1}) \leq B(x) \vee B(y)$. Therefore, $B(x y^{-1}) \leq B(x) \vee B(y)$, for all x and y in $H-\{0^{I}\}$. Hence B is an anti-fuzzy subfield of H. And (iii) is clear.

2.7 Theorem: Let A be a fuzzy subset of a field (F, +,.) and V be the anti-strongest fuzzy relation of F. Then A is an anti-fuzzy subfield of F if and only if V is an anti-fuzzy subfield of F×F.

Proof: Suppose that A is an anti-fuzzy subfield of F. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in F×F. We have, $V(x–y) = V(x_1–y_1, x_2–y_2) \leq \{A(x_1) \vee A(y_1)\} \vee \{A(x_2) \vee A(y_2)\} = V(x_1, x_2) \vee V(y_1, y_2) = V(x) \vee V(y)$. Therefore, $V(x–y) \leq V(x) \vee V(y)$, for all x and y in F×F. And $V(x y^{-1}) = A(x_1 y_1^{-1}) \vee A(x_2 y_2^{-1}) \leq \{A(x_1) \vee A(y_1)\} \vee \{A(x_2) \vee A(y_2)\} = V(x_1, x_2) \vee V(y_1, y_2) = V(x) \vee V(y)$. Therefore, $V(x y^{-1}) \leq V(x) \vee V(y)$, for all x and y in $F×F– \{(0, 0)\}$. Conversely, assume that V is an anti-fuzzy subfield of F×F, then for any $x= (x_1, x_2)$ and $y = (y_1, y_2)$ are in F×F, we have $A(x_1– y_1) \vee A(x_2 – y_2)= V[(x_1, x_2) – (y_1, y_2)] = V(x– y) \leq V(x) \vee V(y) = V(x_1, x_2) \vee V(y_1, y_2)=\{A(x_1) \vee A(x_2)\} \vee \{A(y_1) \vee A(y_2)\}$. If we put $x_2 = y_2 = 0$, we get, $A(x_1 – y_1) \leq A(x_1) \vee A(y_1)$, for all x_1 and y_1 in F. And $A(x_1 y_1^{-1}) \vee A(x_2 y_2^{-1}) = V[(x_1, x_2)(y_1, y_2)^{-1}] = V(xy^{-1}) \leq V(x) \vee V(y)= \{A(x_1) \vee A(x_2)\} \vee \{A(y_1) \vee A(y_2)\}$. If we put $x_2 = y_2 = 1$, we get, $A(x_1 y_1^{-1}) \leq A(x_1) \vee A(y_1)$, for all x_1 and y_1 in $F−\{0\}$.

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